NUSYM15

# Covariance Analysis of Transport Model Parameters relevant for symmetry energy

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# Outline

1, symmetry energy constraints

2, developments of transport model for covariance analysis

3, Covariance analysis on the HIC observables

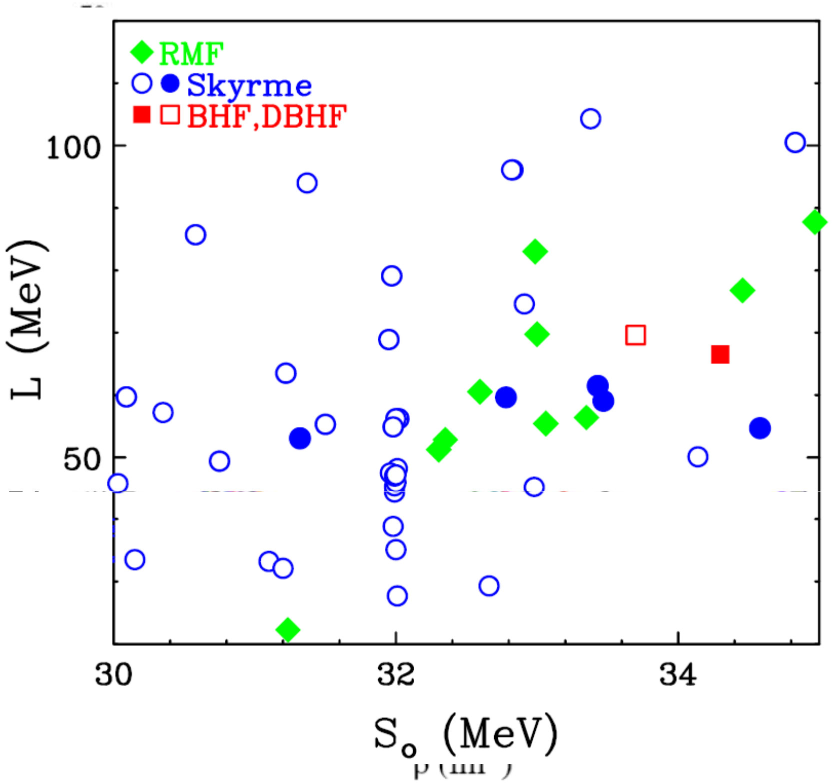
- CI-$R_2$(n/p), CI-DR(n/p), CI-$R_{21}$(n/n),CI-$R_{21}$(p/p), $R_{diff}$

4, Summary and outlook

# Isospin asymmetric Equation of State

$$E(\rho,\delta)=E(\rho,\delta=0)+S(\rho)\delta^2+O(\delta^4)$$

*It is a fundamental properties of nuclear matter, and is very important for understanding*
- *properties of nuclear structure*
- *properties of neutron star*
- *properties of heavy ion reaction mechanism*



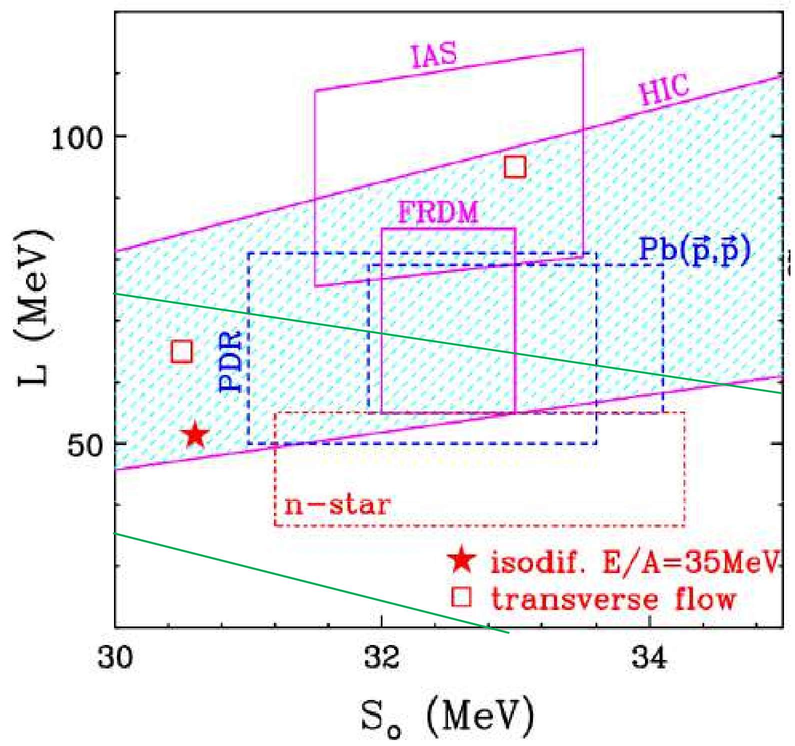

Density dependent of symmetry energy

$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_o}{\rho_o}\right) + \frac{K_{\mathrm{sym}}}{18}\left(\frac{\rho-\rho_o}{\rho_o}\right)^2 + \ldots,$$

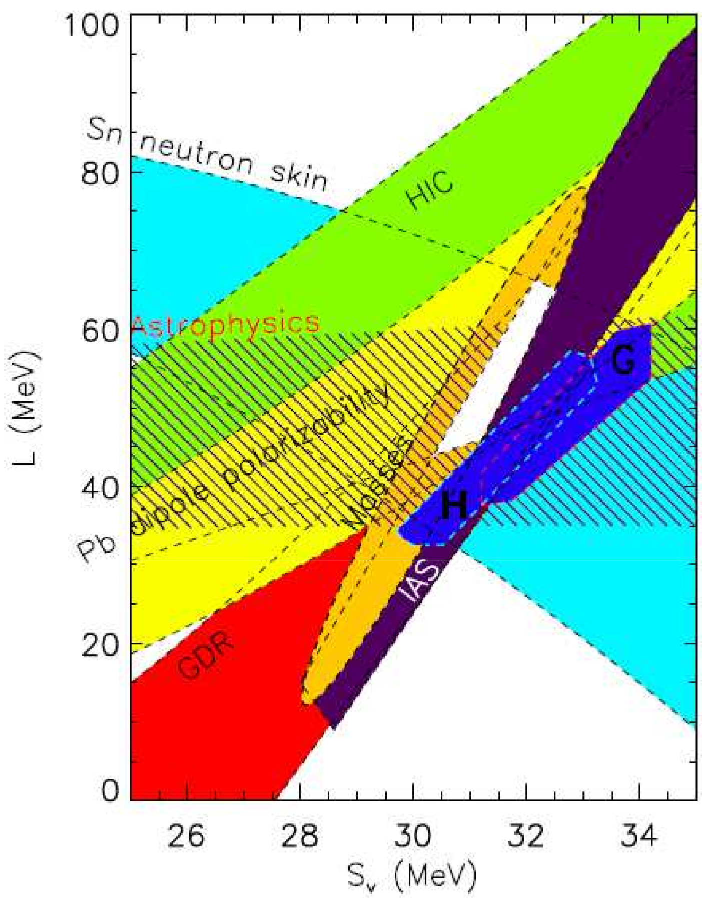
$S_0$: symemtry energy coefficient
L: slope of density dependent of symmetry energy
$K_{sym}$: curvature of density dependent of symmetry energy

*S(ρ)* is the density dependence of symmetry energy, it is a key ingredient of the isospin asymmetric *EOS. However, S(ρ)* uncertainty

M.B.Tsang, Yingxun Zhang, et.al., PRL2009



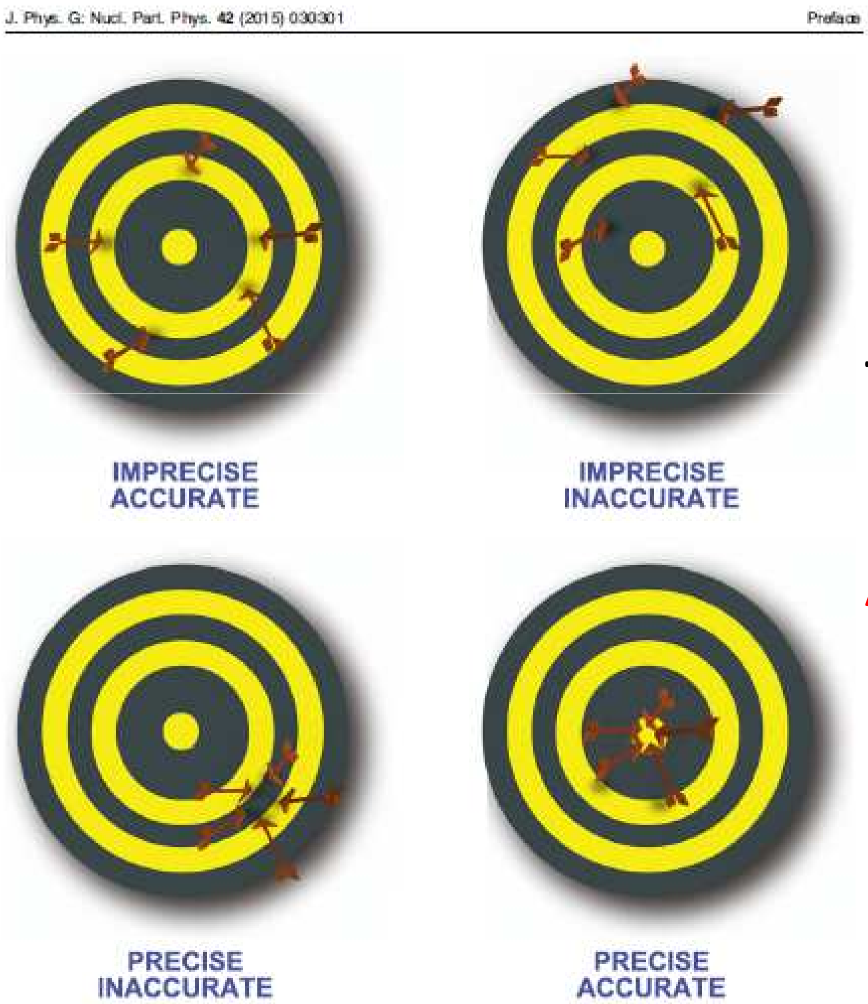

Lattimer, EPJA 50 (2014) 40




$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho - \rho_o}{\rho_o}\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho - \rho_o}{\rho_o}\right)^2 + \ldots,$$

Consensus on symmetry energy have been obtained at subsaturation density. Model uncertainties are left!

## How to reduce the uncertainty of symmetry energy constraints? and achieve ‘precise-accurate’ goals?




Precise:
how near the model is to reality?

Accurate:
how well known are model para?

D.G.Ireland, W.Nazarewicz, JPG:Nucl.Part.Phys.42(2015)030301..

- **Roadmap for achieving 'precise-accurate' constraints**

1) ***What uncertainties are in the density dependent of symmetry energy? (√)***
   - density dependence, momentum dependence, tensor force, correlation between force parameters, …
2) ***What observables are best for constraining the interested physical quantities? (√)***

3) ***Chi-square analysis, Bayesian analysis, …. to find best para. sets***
   - How can we estimate the uncertainties from statistical and systematic?
     - Statistical uncertainties need lot of data and calculations (*HIC data not enough in current*)
     - Systematical uncertainties are hard to deal *(need to know 'reality' in theory , need model comparisons,)*
4) How to verify the model extrapolations ?
5) ***……***

## 1) Uncertainties in the density dependent of symmetry energy and its correlations

Density dependent of symmetry energy from SHF

$$S(\rho) = \frac{1}{3}\frac{\hbar^2}{2m}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1} - \frac{1}{24}(\frac{3\pi^2}{2})^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3} \quad \text{(C8)}$$

$$S(\rho) = \frac{1}{3}\epsilon_F\rho^{2/3} + A_{sym}\rho + B_{sym}\rho^{\sigma+1} + C_{sym}(m_s^*, m_v^*)\rho^{5/3} \quad \text{(C11)}$$

**Density dependent of symmetry depends not only on density, effective mass splitting but also isoscalar effective mass**

$$f_I = \frac{1}{2\delta}(\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{k}\frac{\partial((U_n - U_p)/2\delta)}{\partial k} = \frac{\partial U_{sym}}{\partial E_k}.$$

$$f_I = \frac{m}{8\hbar^2}[t_2(2x_2+1) - t_1(2x_1+1)]\frac{\rho}{2}$$
$$= m/8\hbar^2(\Theta_s - 2\Theta_v)\rho$$
$$= (m/m_s^* - m/m_v^*)$$

(S0, L, ms*, mv*) or (S0, L, ms*, mn*-mp*)

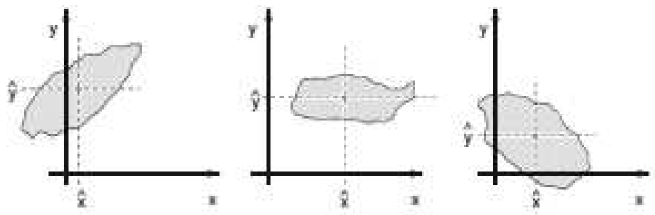
• Correlations between force parameters

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)} \quad (1)$$

$$cov(A,B) = \frac{1}{N-1}\sum_i (A_i - <A>)(B_i - <B>) \quad (2)$$

$$\sigma(X) = \sqrt{\frac{1}{N-1}\sum_i (X_i - <X>)^2}, X = A, B \quad (3)$$

$$<X> = \frac{1}{N}\sum_i X_i, i = 1, N. \quad (4)$$



C_AB=1 0 -1

C_AB between pairs of variables from 120 Skyrme sets

| $C_{AB}$ | $K_0$ | $S_0$ | L | Ms* | Mv* |
|---|---|---|---|---|---|
| $K_0$ | 1 | 0.003 | 0.161 | 0.131 | 0.295 |
| $S_0$ | 0.003 | 1 | **0.764** | 0.397 | 0.228 |
| L | 0.161 | **0.764** | 1 | **0.460** | 0.212 |
| Ms* | 0.131 | 0.397 | **0.460** | 1 | **0.715** |
| Mv* | 0.295 | 0.228 | 0.212 | **0.715** | 1 |

• Uncertainties in Effective mass and effective mass splitting

ms*/m~0.65-0.9,

F.Chappert, PLB668(2008)402,

**Table 2**
Infinite and semi-infinite nuclear matter properties of the D1S and D1N interactions compared to empirical values

| | D1S | D1N | Emp. values |
|---|---|---|---|
| $\rho_0$ (fm$^{-3}$) | 0.16 | 0.16 | $0.17 \pm 0.02$ |
| $E_0/A$ (MeV) | −15.9 | −16.0 | $-16 \pm 1$ |
| $K_\infty$ (MeV) | 210 | 230 | $220 \pm 10$ |
| $m^*/m$ | 0.70 | 0.75 | $0.70 \pm 0.05$ |
| $E_{sym}$ (MeV) | 32.0 | 29.3 | $30 \pm 2$ |
| $E_{surf}$ (MeV) | 20.0 | 19.3 | $21 \pm 2$ |

P. Klupfel, et.al., PRC79, 0343310(2009)

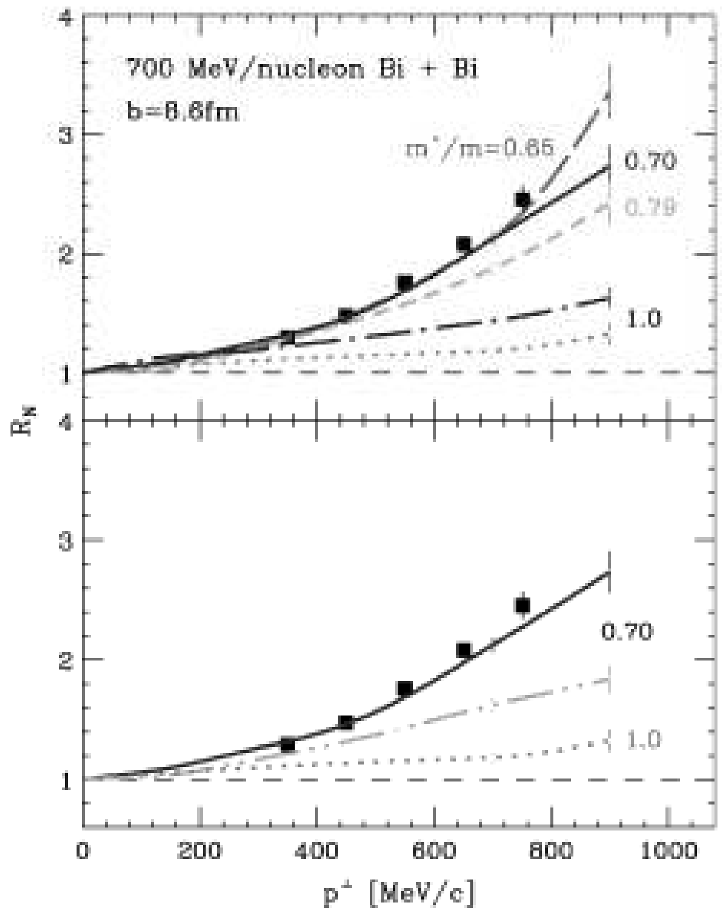
GQR Pb208, ms*/m=0.9

X.H.Li, et.al., PLB743(2015)408

**Table 4**
Nucleon isoscalar effective mass $m_0^*/m$ and the neutron–
ting $m_{n-p}^*$ from the three cases studied in this work.

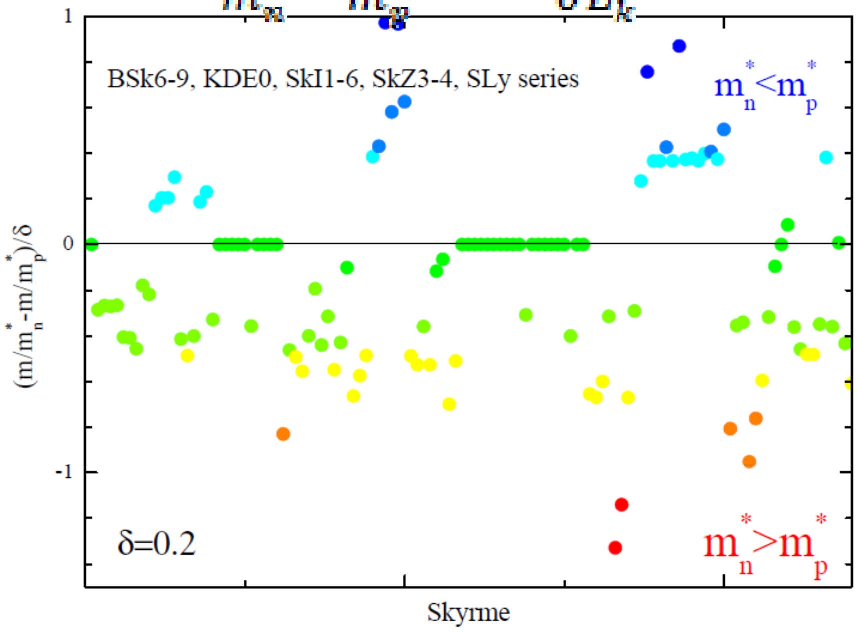
| Case | $m_0^*/m$ |
|---|---|
| I | $0.65 \pm 0.05$ |
| II | $0.67 \pm 0.06$ |
| III | $0.65 \pm 0.06$ |

P. Danielewicz / Nuclear Physics A 673 (2000) 375–410

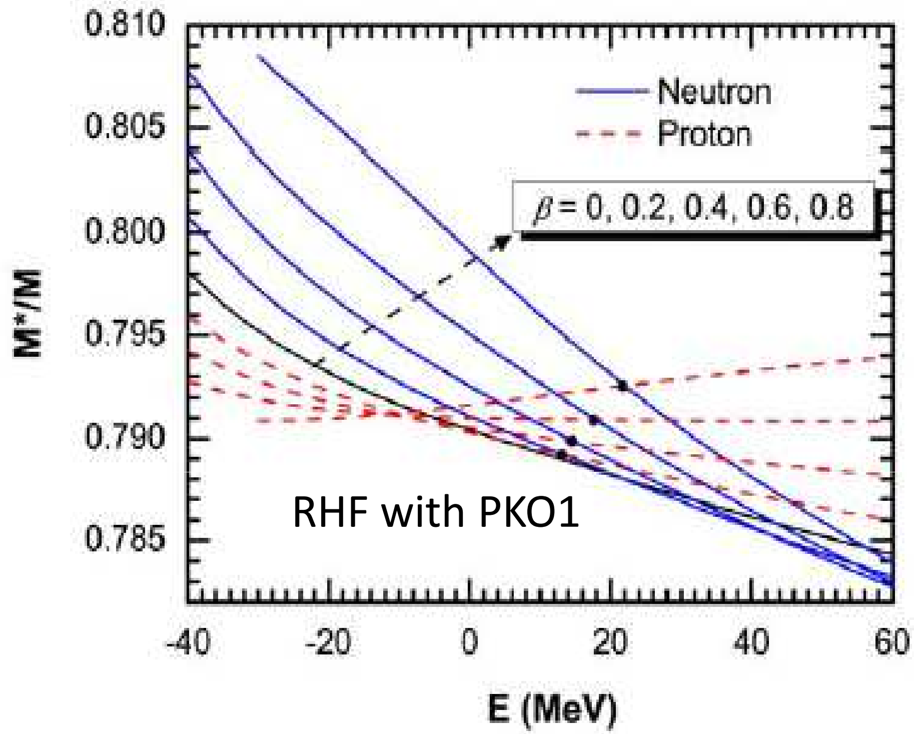


# Effective mass splitting $(m_n^*-m_p^*)/m$

$$\frac{m}{m_n^*}-\frac{m}{m_p^*}=\frac{\partial(U_n-U_p)}{\partial E_k}$$




WHLong, N Van Giai, J.Meng, PLB640(2008)150




B.A.Li, C. Xu, et.al., PRC2006,2010,
X.H.Li, PLB743(2015)408)

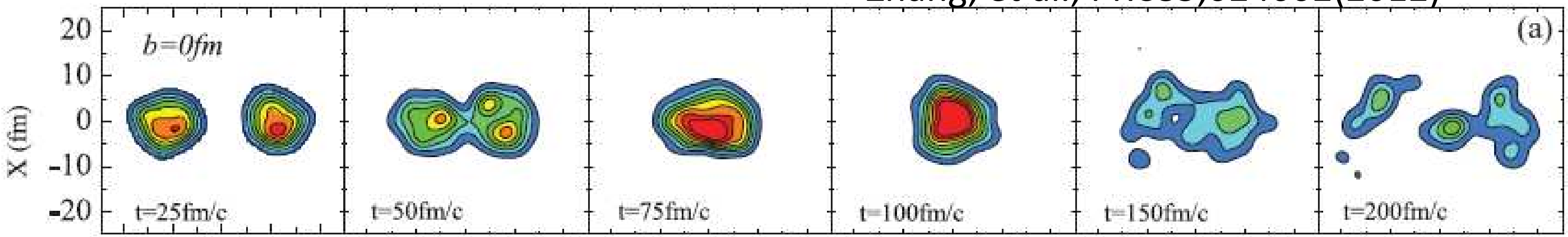
**At normal density,**

**$(m_n^*-m_p^*)/m=0.32\delta$,**
**$(m_n^*-m_p^*)/m=(0.41+-0.15)\delta$**

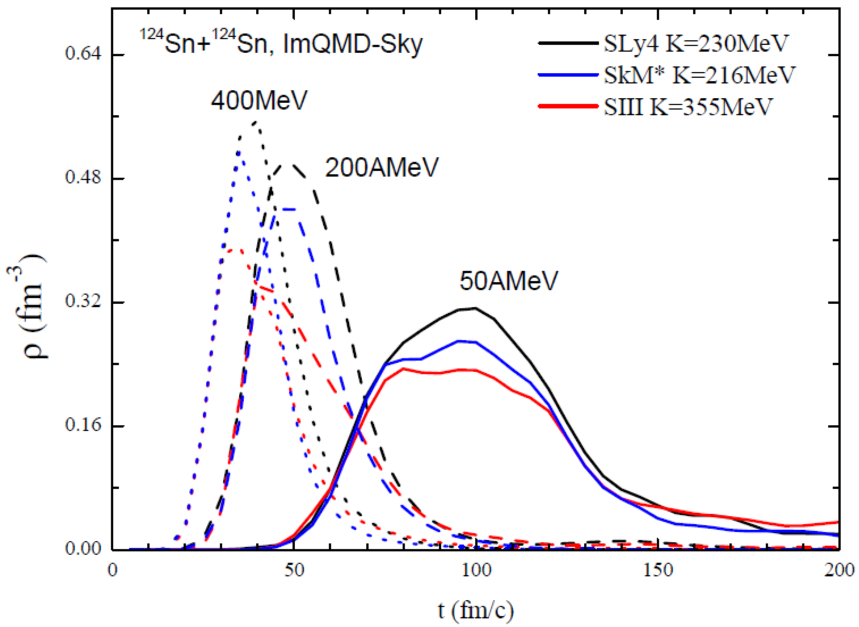
Effective mass splitting depends not only on the in-medium density but also on momentum or energy ??

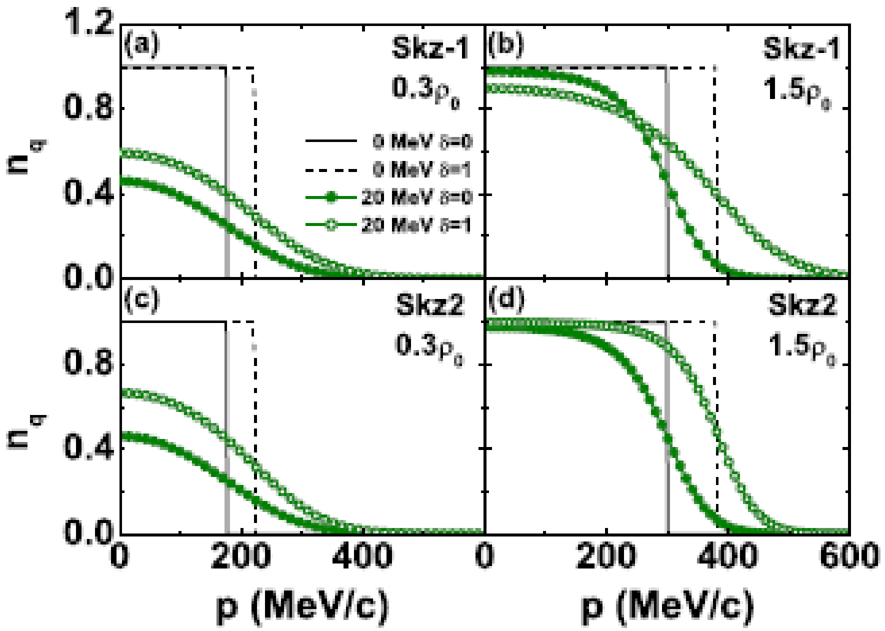
# • Probing the SE and effective mass splitting with HICs

Zhang, et al., PRC85,024602(2012)

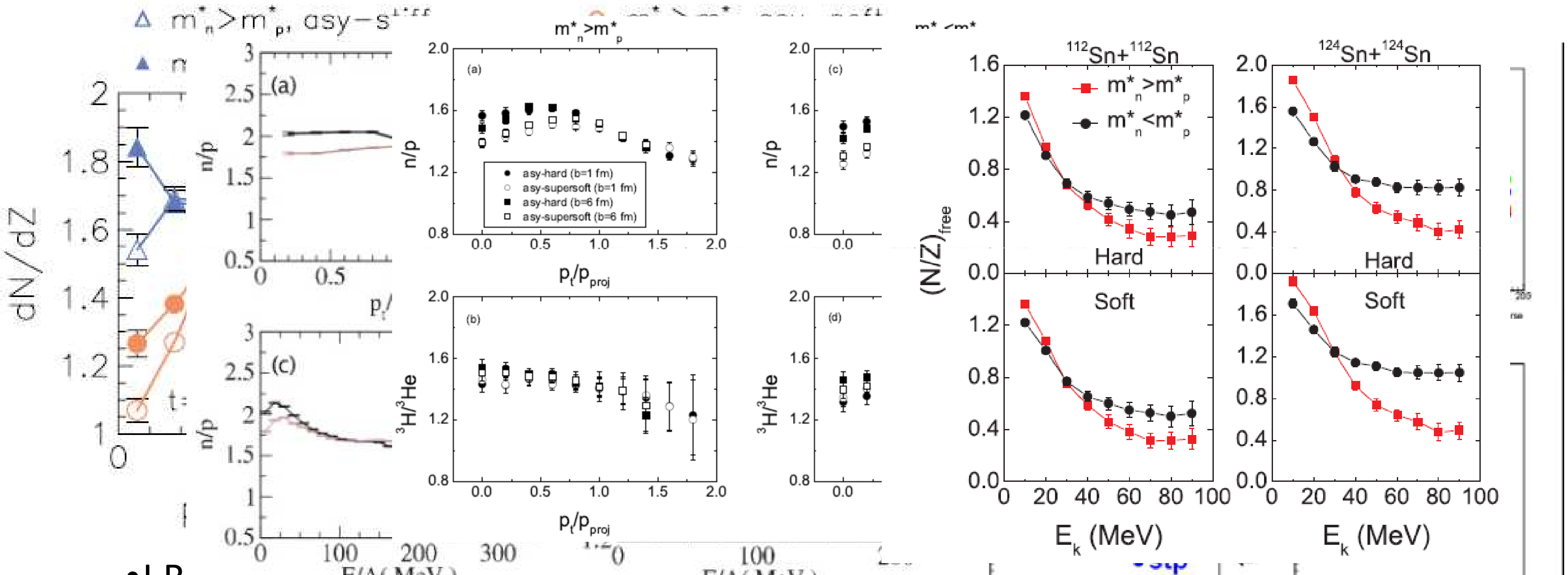



L.Ou, Z.X.Li, Y.X.Zhang, M.Liu, PLB697(2011)



• efforts on probing the momentum dependence of symmetry potential(or n/p effective mass splitting) by HICs




•J.R

ZQFeng, et al, PLB(2012)

WJXie, et al., PRC88,061601

MDI, Log form

V.Giordano, et al., PRC2010

H.H.Wolter

In the new version of ImQMD code, nucleons are represented by Gaussian wavepackets

- ***Developments of ImQMD for Covariance analysis***

the potential energy $U$

term:

Version of ImQMD with standard Skyrme int.

$$U = U_\rho + U_{md} + U_{coul} \quad (2)$$

and $U_{coul}$ is the Coulomb energy. The nuclear contributions are represented in a local form with

$$U_{\rho,md} = \int u_{\rho,md} d^3 r$$

and

$$u_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{sym}\rho^2\delta^2 + B_{sym}\rho^{\eta+1}\delta^2$$

Y.X. Zhang, M.B.Tsang, Z.X. Li, HLiu, PLB(2014)

$$u_{\mathrm{md}} = \frac{1}{2\rho_0}\sum_{N_1,N_2=\mathrm{n,p}}\frac{1}{16\pi^6}\int \mathrm{d}^3 p_1 \mathrm{d}^3 p_2 f_{N_1}(\mathbf{p}_1) f_{N_2}(\mathbf{p}_2) \times 1.57\,[\ln(1.+5.\times 10^{-4}(\Delta p)^2)]^2, \quad (4)$$

$$\delta(r_1 - r_2)(p_1 - p_2)^2$$

$$\begin{aligned} u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \quad (5)\\ &= C_0\int d\vec{p}\, d\vec{p}'\, f(\vec{r},\vec{p}) f(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 \\ &+ D_0\left[\int d\vec{p}\, d\vec{p}'\, f_n(\vec{r},\vec{p}) f_n(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 + \int d\vec{p}\, d\vec{p}'\, f_p(\vec{r},\vec{p}) f_p(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2\right] \end{aligned}$$

**Skyrme type MDI**

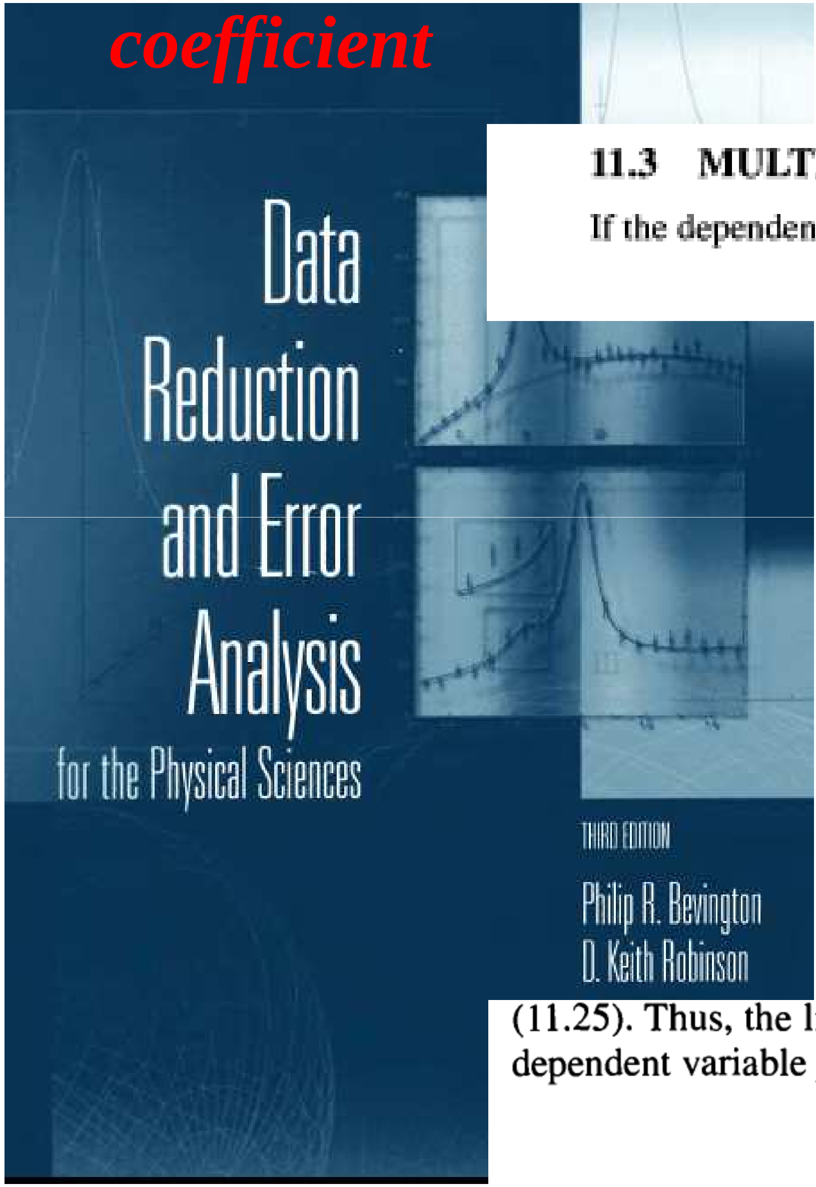
1. Simple and contain sufficient physics, widely used in nuclear structure, reaction and astrophysics.

2. In Skyrme EDF, one can easily choose different values L, m*_v for similar K_0 and $S_0$, m*_s from lot of sets.

3. $\{\alpha,\ \beta,\ \eta,\ A_{sym},\ B_{sym},\ C_0,\ D_0\} \Longleftrightarrow \{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$

4. One could get the constraints on Skyrme parameters, also on symmetry energy and n/p effective mass splitting from reaction data simultaneously.

- ***What observables are best for constraining the interested physical quantities in HICs? (√) correlation coefficient***




**11.3 MULTIVARIABLE CORRELATIONS**

If the dependent variable $y_i$ is a function of more than one variable,

$$y_i = a + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3} + \cdots \tag{11.20}$$

$$s_{jk}^2 \equiv \frac{1}{N-1} \sum \left[ (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) \right]$$

$$s_j^2 \equiv s_{jj}^2 = \frac{1}{N-1} \sum (x_{ij} - \bar{x}_j)^2$$

(11.25). Thus, the linear-correlation coefficient between the $j$th variable $x_j$ and the dependent variable $y$ is given by

$$r_{jy} = \frac{s_{jy}^2}{s_j s_y} \tag{11.27}$$

# Covariance analysis for correlation coefficient

input variables for ImQMD, similar relation as in MSL

L.W.Chen, et al., PRC82, 024321(2010)

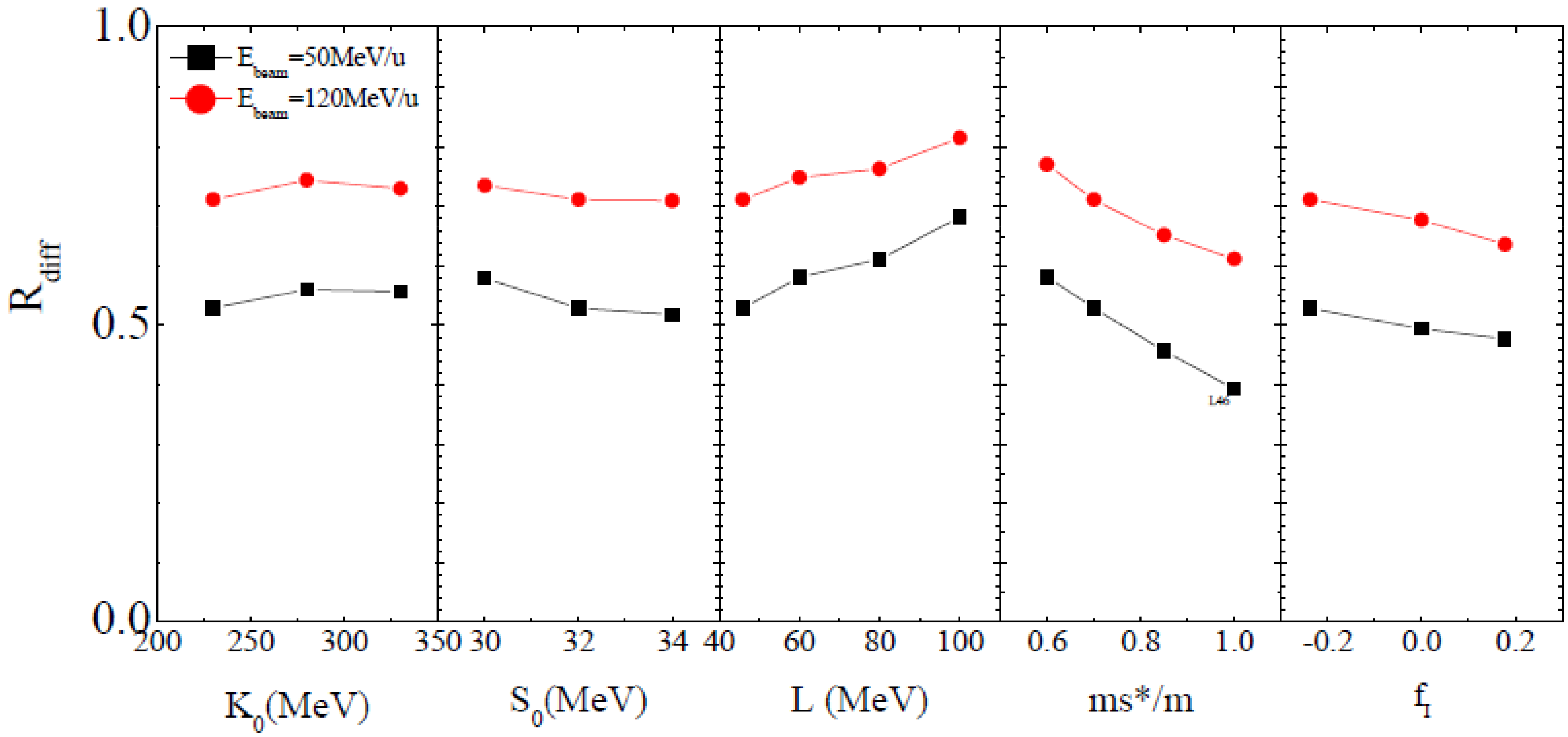
$$\{\alpha,\ \beta,\ \eta,\ A_{sym},\ B_{sym},\ C_0,\ D_0\} \longleftarrow \{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$$

Table 1: List of twelve parameters used in the ImQMD calculations. $\rho_0 = 0.16 fm^{-3}$, $E_0 = -16MeV$, and $g_{sur} = 24.5MeV fm^2$, $g_{sur,iso} = -4.99MeV fm^2$

| Para. | $K_0$ (MeV) | $S_0$(MeV) | $L$ (MeV) | $m_s^*/m$ | $f_I$ |
|---|---|---|---|---|---|
| 1 | **230** | **32** | **46** | **0.7** | **-0.238** |
| 2 | **280** | 32 | 46 | 0.7 | -0.238 |
| 3 | **330** | 32 | 46 | 0.7 | -0.238 |
| 4 | 230 | **30** | 46 | 0.7 | -0.238 |
| 5 | 230 | **34** | 46 | 0.7 | -0.238 |
| 6 | 230 | 32 | **60** | 0.7 | -0.238 |
| 7 | 230 | 32 | **80** | 0.7 | -0.238 |
| 8 | 230 | 32 | **100** | 0.7 | -0.238 |
| 9 | 230 | 32 | 46 | **0.85** | -0.238 |
| 10 | 230 | 32 | 46 | **1.00** | -0.238 |
| 11 | 230 | 32 | 46 | 0.7 | **0.0** |
| 12(SLy4) | 230 | 32 | 46 | 0.7 | **0.178** |

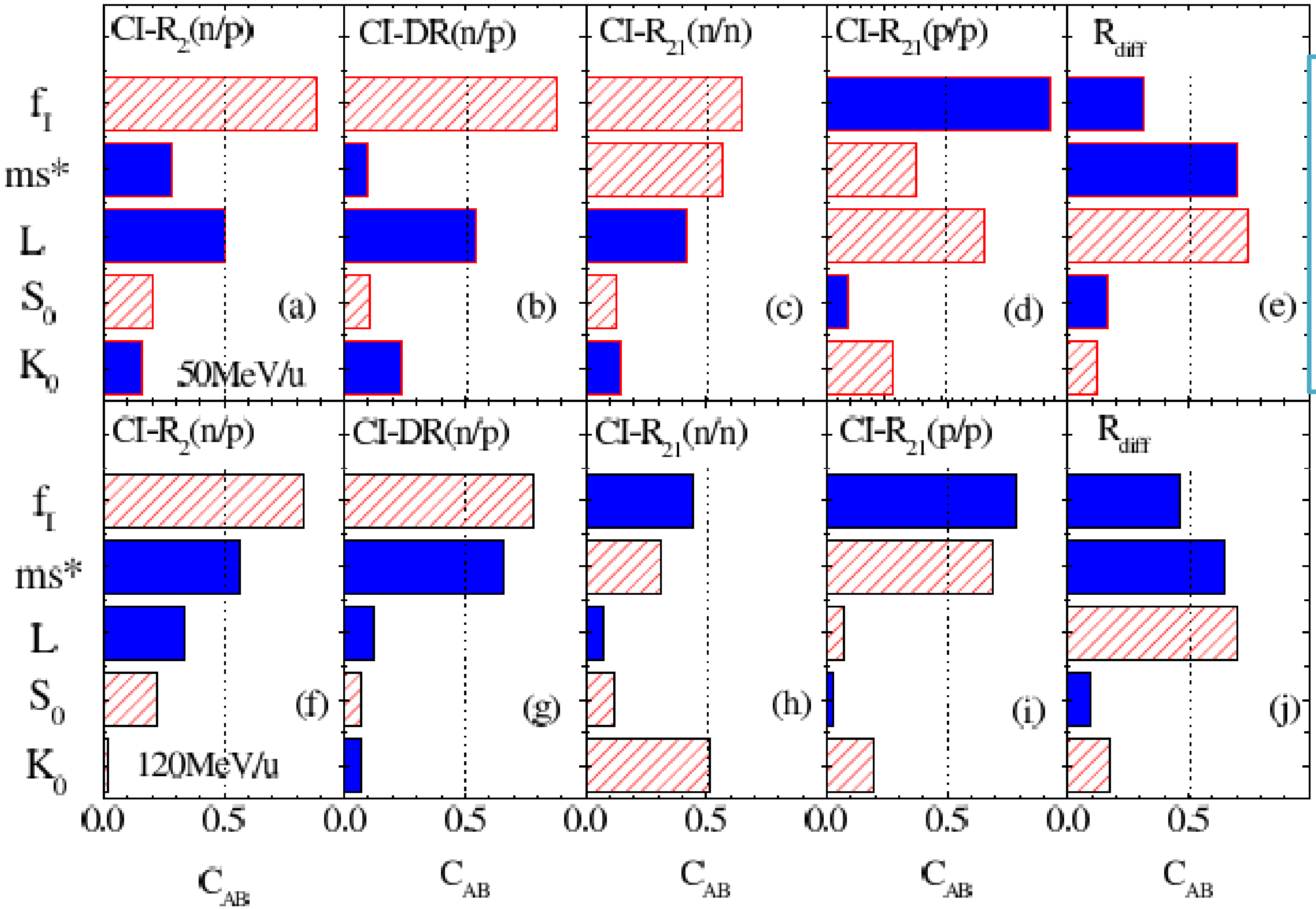
# Isospin diffusion at 120MeV/u and 50MeV/u

Sn+Sn, at b=6fm



# Covariance analysis from 12 Parameter sets

Y.X.Zhang,M.B.Tsang, Z.X.Li, submitted




**Blue:**
**negative correlation**

**Red:**
**Positive correlation**

The ratios are constructed with Ek>40MeV

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u.**

## •Comparison with data

A=124Sn, B=112Sn

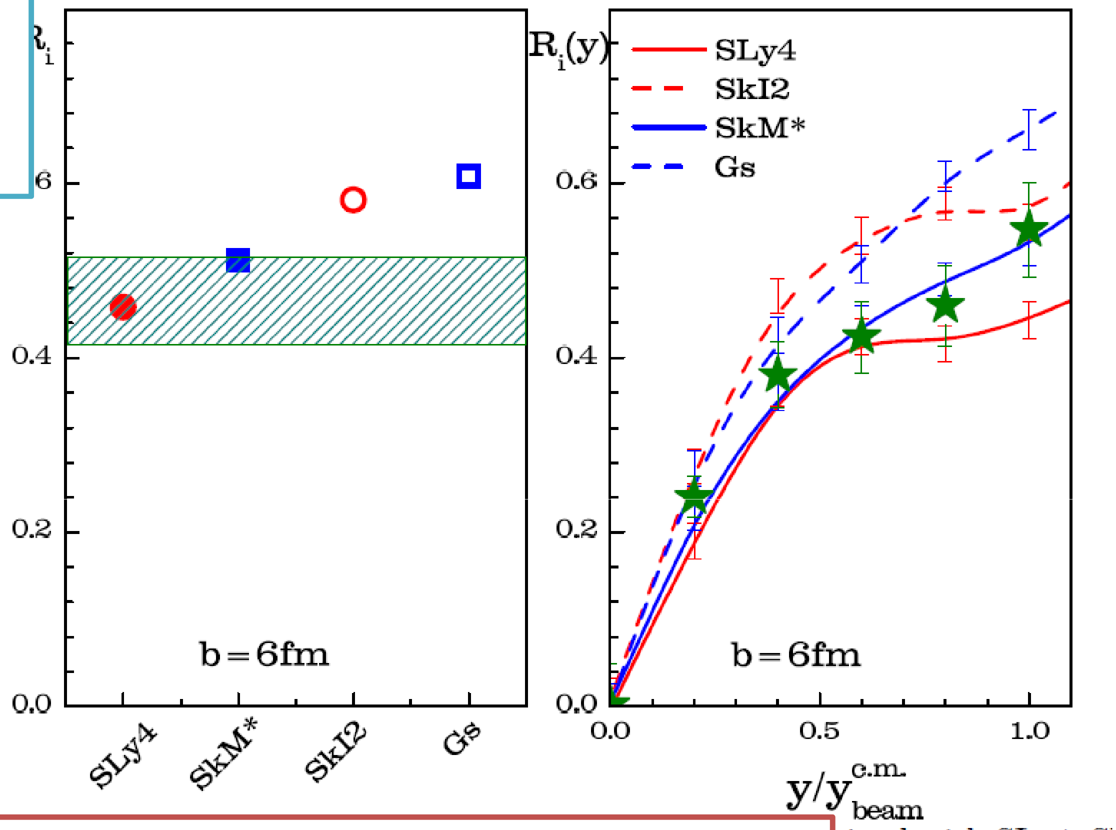
Isospin diffusion occurs only in asymmetric systems A+B, and diffusion ability depends on the symmetry energy and n/p effective mass splitting.

For $m^*_n<m^*_p$, the isospin diffusion process is accelerated due to larger Lane potential at subsaturation density.

$R_i=(2X-X_{AA}-X_{BB})/(X_{AA}-X_{BB})$

*In absence of isospin diffusion R=1 or R=-1, R~0 for isospin equilibrium*

Zhang, Tsang, Li, Liu,, PLB732,186(2014)

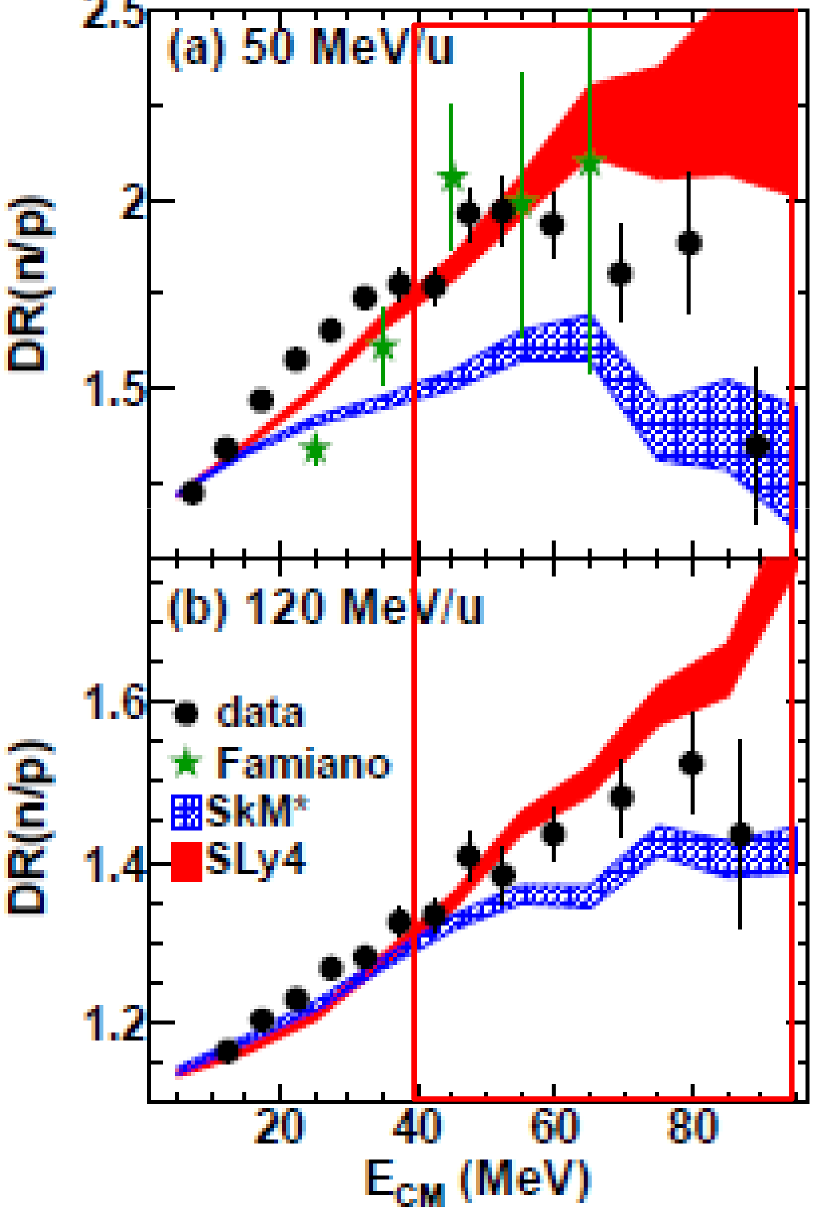



Ri(SLy4, L=46MeV, m*_n<m*_p)<Ri(SkM*, L=46MeV, m*_n>m*_p) < Ri(SkI2, L=104MeV, m*_n<m*_p)<Ri(Gs, L=93MeV, m*_n>m*_p)

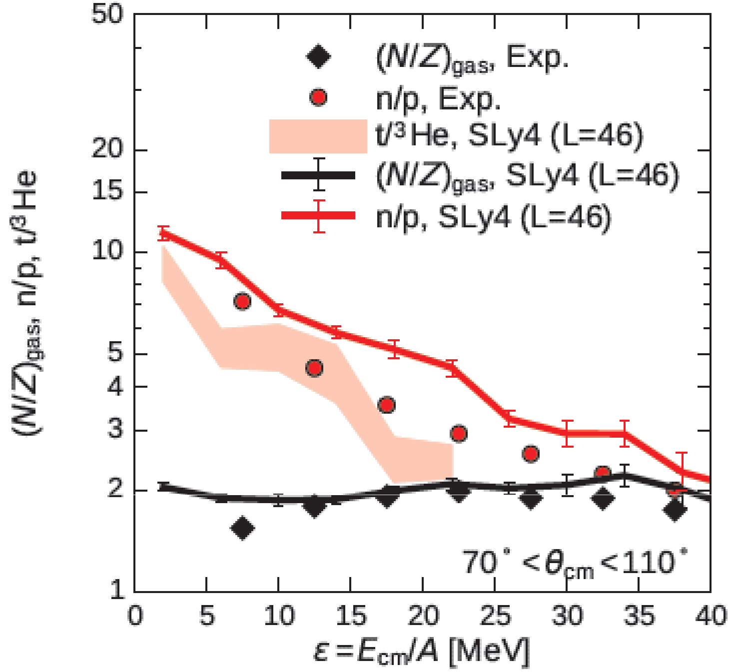
*$R_i$ is more sensitive to L than n/p effective mass splitting.*

# Theoretical predictions and New data on coalescence invariant DR(n/p)

D.D.S.Coupland, et al., arXiv:1406.4546




Ono, Talk at NN2015




- New data seems to favor small effective mass splitting at high momentum

- ***need more calculations with different effective mass splitting to understand this difference.***

# 4, Summary and outlook

1, Developed a new version of ImQMD which can accommodate the Standard Skyrme interaction in parameters. It can bridge the reaction and structure study by using same EDF.

2, The Ri and Ri(y) support the SLy4 and SkM* interactions, they have L=46MeV.

**3, high energy n/p yield ratio is sensitive to the effective mass splitting (momentum dependent of symmetry potential), and the data is between SLy4 and SkM*.**

***4, Covariance analysis suggest that there is lager influence of isoscalar effective mass on isospin sensitive observables, one should narrow its ranges in order to further improve the constraints on SE.***

5, combination analysis from R(n/p), R(p/p) and Rdiff may disentangle the effects of ms*, fi and L, and place constraints on that with reasonable uncertainties after best fit data.

Thanks for your attention!