

*The role of the symmetry energy
in the evolution of a proto-neutron star*

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Introduction

Motivation

- There is a direct connection between nuclear symmetry energy and nuclear astrophysics.
- The estimation of the symmetry energy from experiment and finding out the way it is related to the structure of neutron stars is one of the most important problem of both nuclear physics and astrophysics.
- The equation of state (EoS) of isospin asymmetric nuclear matter is a fundamental quantity that determines the properties of both an atomic nucleus, and a neutron star.
- Observations of the binary millisecond pulsars J1614-2230^a and J0348+0432^b have led to the precise estimation of neutron star masses: $(1.97 \pm 0.04)M_{\odot}$ and $(2.01 \pm 0.04)M_{\odot}$ - appearance of hyperons questionable.

^aP. Demorest et al., Nature **467**, 1081 (2010)

^bJ. Antoniadis et al., Science **340**, 6131 (2013)

Nuclear matter equation of state - general remarks

Characteristics of nuclear matter

The energy of uniform nuclear matter in its ground state is a function of baryon density, temperature and composition (proton fraction $x = n_p/n_B$), for $T = 0$:

$$\frac{E(n_B, x)}{n_B} = E(n, 1/2) + E_{sym}(n_B)(1 - 2x)^2 + Q(n_B)(1 - 2x)^4 + \dots$$

The nuclear symmetry energy can be defined as the difference in energy per nucleon between the pure neutron matter and the symmetric matter.

Symmetry energy parameters

Taylor expansion near n_0 :

$$E_{sym}(n_B) \simeq S_v + \frac{L}{3n_0}(n_B - n_0) + \frac{K_s}{18n_0^2}(n_B - n_0)^2 + \dots$$

Nuclear matter equation of state - general remarks

Density dependence of the symmetry energy

The most important factors that characterize the density dependence of the nuclear symmetry energy:

- The slope parameter

$$L = 3n_0 \frac{\partial E_{sym}(n_B)}{\partial n_B} \Big|_{n_0}$$

- The curvature parameter

$$K_{sym} = 9n_0^2 \frac{\partial^2 E_{sym}(n_B)}{\partial n_B^2} \Big|_{n_0}$$

The incompressibility of asymmetric nuclear matter includes also the isospin dependent part.

Nuclear matter equation of state - general remarks

Density dependence of the symmetry energy

- The EoS of asymmetric nuclear matter - the key ingredient is the density dependence of the symmetry energy.
- Different forms of the density dependence of the symmetry energy predicted by different theoretical studies are based on microscopic many-body calculations and phenomenological approaches.
- Two different forms have been identified:
 - the symmetry energy increases monotonically with increasing density - stiff dependence,
 - the symmetry energy increases initially up to saturation density and then decreases at higher densities - soft dependence.

The Walecka-type models

The EoS of asymmetric nuclear matter constructed on the basis of Walecka model - gives very "stiff" form of the symmetry energy. To provide additional freedom in varying the density dependence of the symmetry energy the model is supplemented by the term:

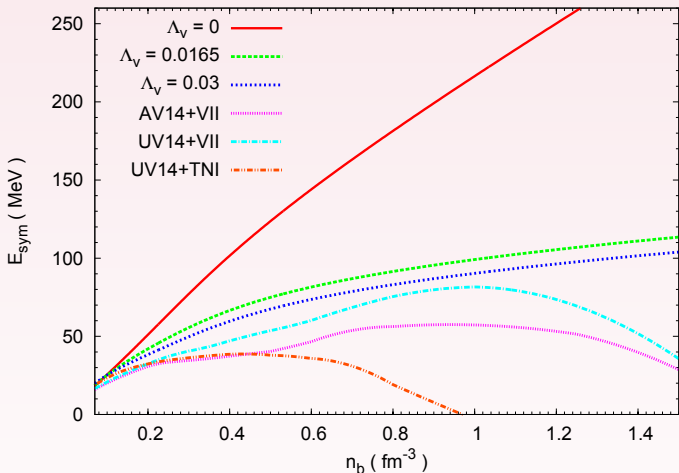
$$\Lambda_V(g_\omega\omega)(g_\rho\rho)$$

The density dependence of the symmetry energy

$$E_{sym}(n_B) = \frac{k_F^2}{6\sqrt{(k_F^2 + M_{eff}^2)}} + \frac{k_F^3}{12(m_\rho^2/g_\rho^2 + 2\Lambda_V(g_\omega\omega)^2)}$$

for $\Lambda_V = 0$ the symmetry energy varies linearly with the density.

TM1 nonlinear (isovector sector)					
Λ_V	0	0.014	0.015	0.016	0.0165
g_ρ	9.264	9.872	9.937	10.003	10.037
L (MeV)	108.58	77.52	75.81	74.16	73.36



The density dependence of symmetry energy calculated for different values of parameter Λ_V and compared with the results obtained for the AV14+VII, UV14+VII and UV14+TNI models¹. The inclusion of $\omega - \rho$ coupling softens the symmetry energy and its density dependence resembles that obtained for the realistic nuclear models.

¹R. B. Wiringa and V. Fiks, Phys. Rev. C **38**, 1010 (1988)

Phases of neutron star evolution

The process of the gravitational collapse of the core of a massive star leads to the formation of a proto-neutron star. Models of neutron stars which describe subsequent phases of their evolution depend sensitively on the assumptions appropriate for the given evolutionary stage. In the simplified scenario it is possible to distinguish three different cases and each of them is represented by relevant physical conditions of a proto-neutron and neutron star matter.

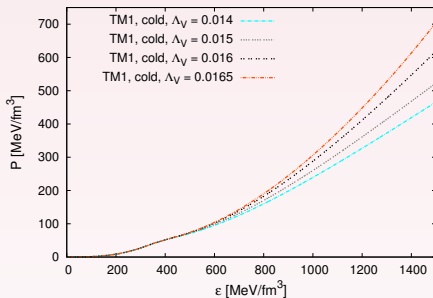
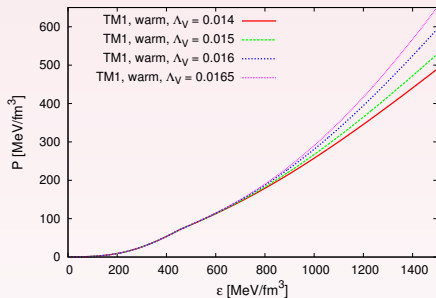
Simplified scheme of the evolutionary track of a neutron star

- The post bounce phase - the unshocked inner core settles into a hydrostatic equilibrium. Model that describes this phase of evolution is constructed on the basis of the following assumptions: the low-entropy core with trapped neutrinos is surrounded by a high entropy ($s = 2 - 5$) envelope. The neutrino trapping leads to initially high value of the lepton number $Y_{l_e} = (n_e + n_{\nu_e})/n_b \simeq 0.4$.
- The deleptonization of the core after which the matter of the core is neutrino-free $Y_{\nu_e} = 0$ with the entropy $s = 2$. Thermally produced neutrino pairs of all flavours are abundant. The cooling of the hot neutron star takes place.
- Cold, catalysed object.

Lagrangian of the model

$$\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (\gamma^\mu i D_\mu - m_{eff,B}) \psi_B + \sum_{l=e,\mu,\nu} \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l) \psi_l \\
& + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_3 \sigma^3 - \frac{1}{4} g_4 \sigma^4 + \frac{1}{2} \partial^\mu \sigma^* \partial_\mu \sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} \\
& + \frac{1}{2} m_\omega^2 (\omega^\mu \omega_\mu) + \frac{1}{2} m_\rho^2 (\rho^{\mu a} \rho_\mu^a) + \frac{1}{2} m_\phi^2 (\phi^\mu \phi_\mu) \\
& - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} - \frac{1}{4} R^{\mu\nu a} R_{\mu\nu}^a - \frac{1}{4} \Phi^{\mu\nu} \Phi_{\mu\nu} + \sum_{i,j,k} C_{ijk} \omega^i \rho^j \phi^k,
\end{aligned}$$

where the covariant derivative equals $D_\mu = \partial_\mu + ig_{B\omega}\omega_\mu + ig_{B\phi}\phi_\mu + ig_{B\rho}\mathbf{I}_B\rho_\mu$, \mathbf{I}_B denotes isospin of baryon B , $m_{eff,B} = m_B - g_{B\sigma}\sigma - g_{B\sigma^*}\sigma^*$ is the baryon effective mass, while $\Omega_{\mu\nu}$, $R_{\mu\nu}$, and $\Phi_{\mu\nu}$ are the field tensors of the ω , ρ , and ϕ mesons, C_{ijk} denotes coupling constants of different vector mesons.



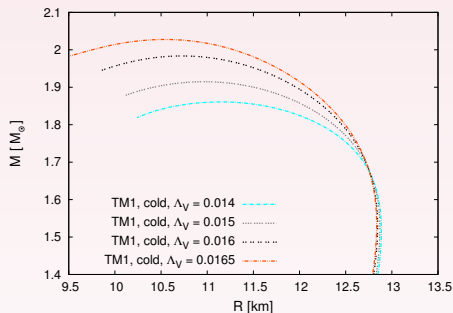
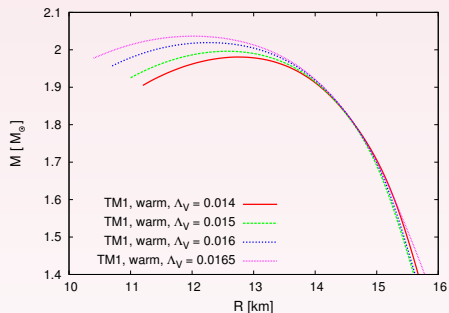
The EoSs calculated for models with different vector meson coupling terms.

Results have been obtained for different values of Λ_V parameter.

Left panel: warm proto-neutron star matter with trapped neutrinos.

Right panel: cold, deleptonized neutron star matter.

In both cases the increase of parameter Λ_V leads to the stiffening of the EoS.

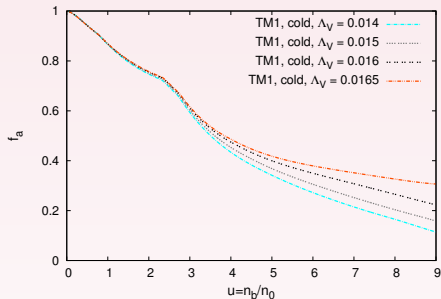
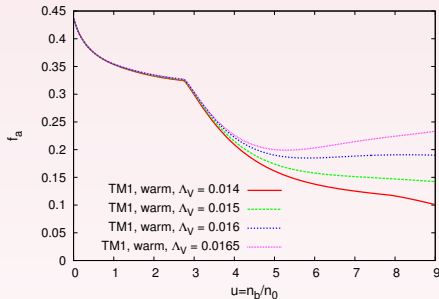


The mass–radius relations calculated for the nonlinear model for different values of parameter Λ_V .

Left panel: warm proto-neutron star matter with trapped neutrinos.

Right panel: cold, deleptonized neutron star matter.

The increase of parameter Λ_V gives higher values of maximum mass even close to $2M_{\odot}$.

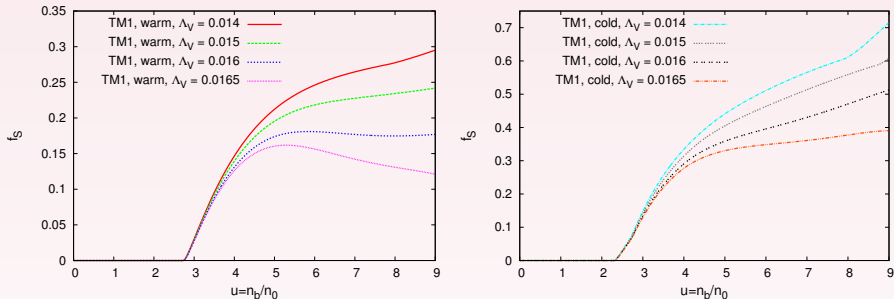


The asymmetry parameter $f_a = \frac{n_n - n_p}{n_b}$ as a function of baryon number density calculated for different values of Λ_V .

Left panel: warm proto-neutron star matter with trapped neutrinos.

Right panel: cold, deleptonized neutron star matter.

The differences in the asymmetry parameter between the proto-neutron and neutron star matter is connected with the fixed high value of the electron lepton number for warm, neutrino-trapped matter.

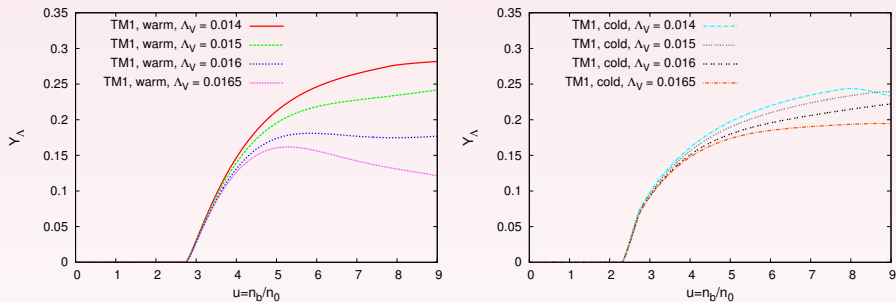


The strangeness content of the system $f_S = \sum_B \frac{S \cdot n_B}{n_b}$ as a function of baryon number density calculated for different values of Λ_V .

Left panel: warm proto-neutron star matter with trapped neutrinos.

Right panel: cold, deleptonized neutron star matter.

In the case of warm, neutrino-trapped matter the hyperons are less abundant and their onset point is shifted to higher density. Increase in the value of the parameter Λ_V leads to the matter with considerably reduced strangeness content.



The relative concentration of Λ hyperons and as a function of baryon number density calculated for different values of Λ_V .

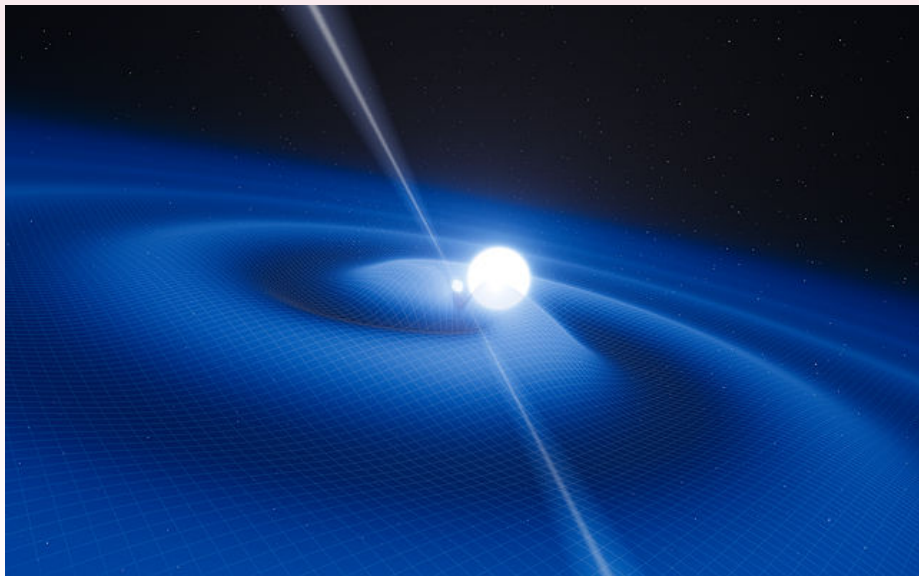
Left panel: warm proto-neutron star matter with trapped neutrinos.

Right panel: cold, deleptonized neutron star matter.

In both cases it is evident that the increase of Λ_V parameter leads to reduced concentration of Λ hyperons.

Conclusions

- Neutron star matter that includes only nucleons - modification of the density dependence of the symmetry energy by the $\omega - \rho$ coupling.
- Neutron star matter with hyperons - density dependence of the symmetry energy is modified, especially when the model includes coupling of the hidden strangeness ϕ meson with the ρ meson.
- Modification of neutron star evolution.
- Modification of neutron stars parameters and properties: masses, radii, chemical composition.



Artist's impression of the pulsar PSR J0348+0432 and its white dwarf companion.
<http://www.eso.org/public/images/eso1319c/>

Baryon–vector meson coupling constants, $g_{B\omega} = (1 - x_B)g_{N\omega}$ and x_B counts the contribution of strange quarks, $g_{N\omega} \equiv g_\omega$ and $g_{N\rho} \equiv g_\rho$.

Baryon (B)	x_B	$g_{B\omega}$	$g_{B\phi} = x_B g_{N\omega}$	$g_{B\rho}$
n	0	g_ω	0	g_ρ
p	0	g_ω	0	g_ρ
Λ	$\frac{1}{3}$	$\frac{2}{3}g_\omega$	$-\frac{\sqrt{2}}{3}g_\omega$	0
Σ	$\frac{1}{3}$	$\frac{2}{3}g_\omega$	$-\frac{\sqrt{2}}{3}g_\omega$	$2g_\rho$
Ξ	$\frac{2}{3}$	$\frac{1}{3}g_\omega$	$-2\frac{\sqrt{2}}{3}g_\omega$	g_ρ

The critical value of the parameter $\Lambda_{V,cr}$ calculated for the selected parameterisations and the incompressibility K_0 taken at the saturation density.

Parameter set	c_3	$\Lambda_{V,cr}$	K_0 (MeV)
NL3 ²	0	-	271
FSUGold ³	418.39	0.0517	230
TMA ⁴	151.59	0.0318	318
TM1* ⁵	134.624	0.0215	281.1
TM1 ⁶	71.3	0.0156	281.1
TM2 ⁷	84.5318	0.0186	343.8

²G. A. Lalazissis, J. Koenig, and P. Ring, Phys. Rev. C **55**, 540 (1997)

³B. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. **95**, 122501 (2005)

⁴H. Toki, D. Hirata, Y. Sugahara, K. Sumiyoshi, and I. Tanihata, Nucl. Phys. A **588**, c357(1995)

⁵M. Del Estal, M. Centelles, X. Vinas, and S. K. Patra, Phys. Rev. C **63**, 024314 (2001)

⁶Y. Sugahara and H. Toki, Prog. Theor. Phys. **92**, 803 (1994)

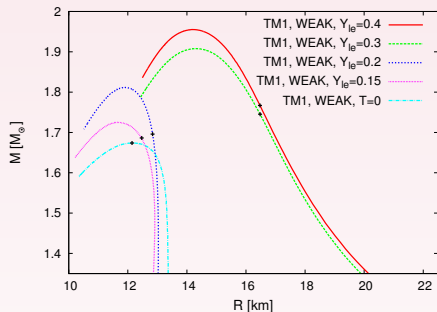
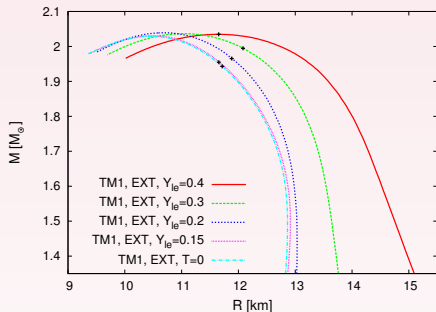
⁷Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994)

TM1 parameter set⁸ with the extended isovector sector

TM1		
$m_\sigma = 511.2 \text{ MeV}$	$g_\sigma = 10.029$	$g_3 = 7.2325 \text{ fm}^{-1}$
$m_\omega = 783 \text{ MeV}$	$g_\omega = 12.614$	$g_4 = 0.6183$
$m_\rho = 770 \text{ MeV}$	$g_\rho = 9.264$	$c_3 = 71.0375$

TM1 nonlinear (isovector sector)						
Λ_V	0	0.014	0.015	0.016	0.0165	0.017
g_ρ	9.264	9.872	9.937	10.003	10.037	10.071
$L \text{ (MeV)}$	108.58	77.52	75.81	74.16	73.36	72.56

⁸Y. Sugahara and H. Toki, Prog. Theor. Phys. **92**, 803 (1994)



The M-R relations calculated for different stages of neutron star evolution for the chosen value of parameter $\Lambda_V = 0.0165$. In the case of extended nonlinear model (*left panel*) black points illustrate the evolutionary path of proto-neutron star starting with the phase when $Y_{le} = 0.4$. Calculations have been done for the maximum mass configuration. The proto-neutron star evolves into stable cold depleted neutron star. The same evolutionary sequence is presented in the right panel for the TM1-weak model, which leads to the maximum mass configuration of cold neutron star ($T = 0$). In this case there exist non-stable configurations.