Investigations of nuclear equation of state in nucleus-nucleus collisions

Martin Veselsky

Institute of Physics, Slovak Academy of Sciences, Dubravska cesta 9, Bratislava, Slovakia

Yu-Gang Ma

Shanghai Institute of Applied Physics of CASc, Shanghai, China

Georgios A. Souliotis

Department of Chemistry, University of Athens, Athens, Greece

Boltzmann-Uehling-Uhlenbeck equation

The BUU equation reads

EoS

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_r f - \nabla_r U\cdot\nabla_p f = \frac{4}{(2\pi)^3}\int d^3p_2 d^3p_3 d\Omega$$
$$\frac{d\sigma_{NN}}{d\Omega} v_{12} \times [f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)]$$
$$\delta^3(p+p_2-p_3-p_4), \qquad (1)$$

where $f = f(r, p, t)$ is the phase-space distribution function. It is solved with the test particle method of Wong [15], with the collision term as introduced by Cugnon, Mizutani and Vandermeulen [16]. In Eq.(1), $\frac{d\sigma_{NN}}{d\Omega}$ and v_{12} are in-medium nucleon-nucleon cross section and relative velocity for the colliding nucleons, respectively, and U is the single-particle mean field potential with the addition of the isospin-dependent symmetry energy

In-medium nucleon-nucleon cross section in the nuclear matter are important component in the nuclear implementations of the Boltzmann equations, nevertheless they are practically unknown.

Typically, free nucleon-nucleon cross sections are used in simulations of nucleus-nucleus collisions, occasionally scaled down by some factor.

Density-dependence of nucleon-nucleon cross sections is approximated only empirically.

True in-medium nucleon-nucleon cross sections must depend on equation of state of the nuclear matter and thus the dependence of collision term on EoS needs to be implemented !!!

Estimation of in-medium cross sections by inversion of the Van der Waals-like equation of state

By formal transformation of any EoS into and subsequent inversion of the resulting Van der Waals-like equation of state it is possible to extract a parameter, called “proper volume” or “excluded volume”, which describes the volume per constituent particle of the non-ideal gas and its apparent area (geometric cross section) can be used for estimation of in-medium cross section.

Dominant attractive (repulsive) interaction increases (decreases) geometric cross section due to focusing (defocusing) effect.

Starting from EoS:

$$U = a\rho + b\rho^{\kappa} + 2a_s(\frac{\rho}{\rho_0})^{\gamma}\tau_z I,$$

$$p = \langle\frac{f_{5/2}(z)}{f_{3/2}(z)}\rangle\rho T + a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s\rho_0(\frac{\rho}{\rho_0})^{1+\gamma}\tau_z I.$$

Fermionic effects taken into account -
Two effects which cancel out mutually

Fermi statistics, which fermions like nucleons obey. To achieve this, one needs to multiply the classical temperature T_{Boltz}, corresponding to the Boltzmann statistics, by a factor $\langle\frac{f_{5/2}(z)}{f_{3/2}(z)}\rangle^{-1}$ and thus the formula (8) will turn into

$$b' = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s(\frac{\rho}{\rho_0})^{\gamma}\tau_z I}{\rho T_{Boltz} + b\kappa\rho^{1+\kappa} + 2\gamma\rho_0 a_s(\frac{\rho}{\rho_0})^{1+\gamma}\tau_z I}, \tag{11}$$

which corresponds to classical case of Boltzmann statistics. Thus, remarkably, this

Calculation of in-medium nucleon-nucleon cross sections in the nuclear matter presents a considerable challenge to the nuclear theory.

G-matrix theory was used to estimate in-medium nucleon-nucleon cross sections by Cassing et al. (W. Cassing, U. Mosel, Prog. Part. Nucl. Phys. 25, 235 (1990)).

Density-dependence of in-medium nucleon-nucleon cross section was studied for symmetric nuclear matter (G. Q. Li, and R. Machleidt, Phys. Rev. C 48, 1702 (1993); Phys. Rev. C 49, 566 (1994); T. Alm et al., Phys. Rev. C 50, 31 (1994); Nucl. Phys. A 587, 815 (1995)), and significant influence of nuclear density on resulting in-medium cross sections was observed in their density, angular and energy dependencies.

Using momentum-dependent interaction, ratios of in-medium to free nucleon-nucleon cross sections were evaluated at zero temperature via reduced effective nucleonic masses (B. A. Li, and L. W. Chen, Phys. Rev. C 72, 064611 (2005)) and used for transport simulations.

Still, transport simulation are mostly performed using parametrizations of the free nucleon-nucleon cross sections, eventually scaling them down empirically or using simple prescriptions for density-dependence of the scaling factor (D. Klakow et al., Phys. Rev. C 48, 1982 (1993)).

Implementation into the BUU equation

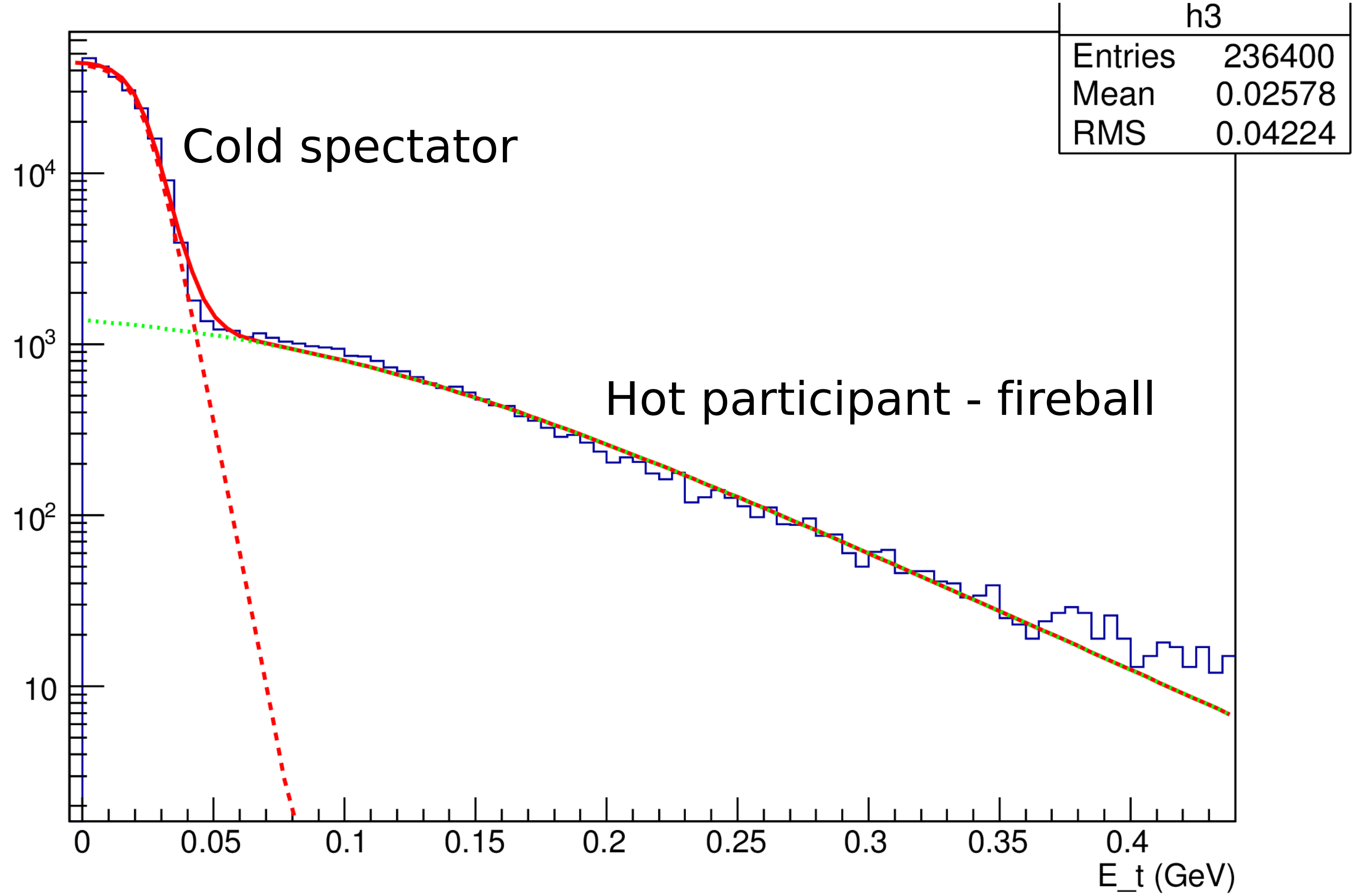
While BUU equation is formulated in terms of density, it does not consider temperature directly. Therefore temperature T must be estimated. It is possible to estimate temperature using the Maxwellian momentum distribution of nucleons

$$f(\vec{p}) = \frac{1}{(2\pi m T)^{3/2}} \mathrm{e}^{-\frac{p_x^2+p_y^2+p_z^2}{2mT}} \quad (9)$$

where m is the nucleon mass. Using this formula, local temperature can be estimated from momentum distribution in the c.m. frame by evaluating the momentum variance. This can be done for transverse momentum since it provides better measure of mutual thermalization of particles from the projectile and target, since it proceeds by distant elastic collisions generating the transverse momentum. More violent collisions would lead to emission of a given nucleon. This temperature estimate can be done without requiring stopping and formation of the source equilibrated in all three dimensions, better analogue would be the friction of two dilute gas clouds passing through each other.

Existing belief: BUU is "dynamical" and there is no temperature

However, already at 15 fm/c two distinct Fermi distributions for spectator and participant (fireball). Momentum distribution smoothed over all test particles used, in the same way as the density is treated.



Temperature is intrinsic to Boltzman equation (BUU). Furthermore, fast thermalization via quantum entanglement (arxiv:1506.03696) ?

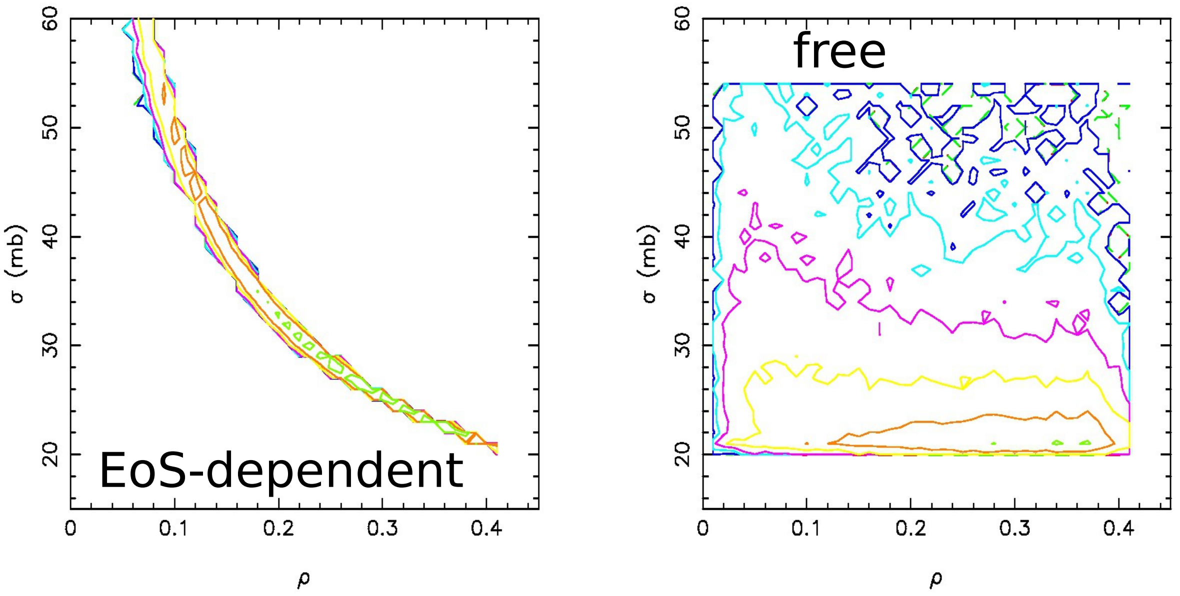


Figure 5: Comparison of the nucleon-nucleon cross sections in two variants of the BUU calculations. On the left panel are the isospin-dependent nucleon-nucleon cross sections, obtained as the proper volume of the Van der Waals form of the equation of state, as a function of density, while on the right panel are shown the corresponding nucleon-nucleon cross sections, obtained using standard energy dependent parametrization, used in BUU calculation. The results were obtained using the BUU in reaction ^{48}Ca+^{48}Ca at 400 AMeV using the impact parameter 3 fm.

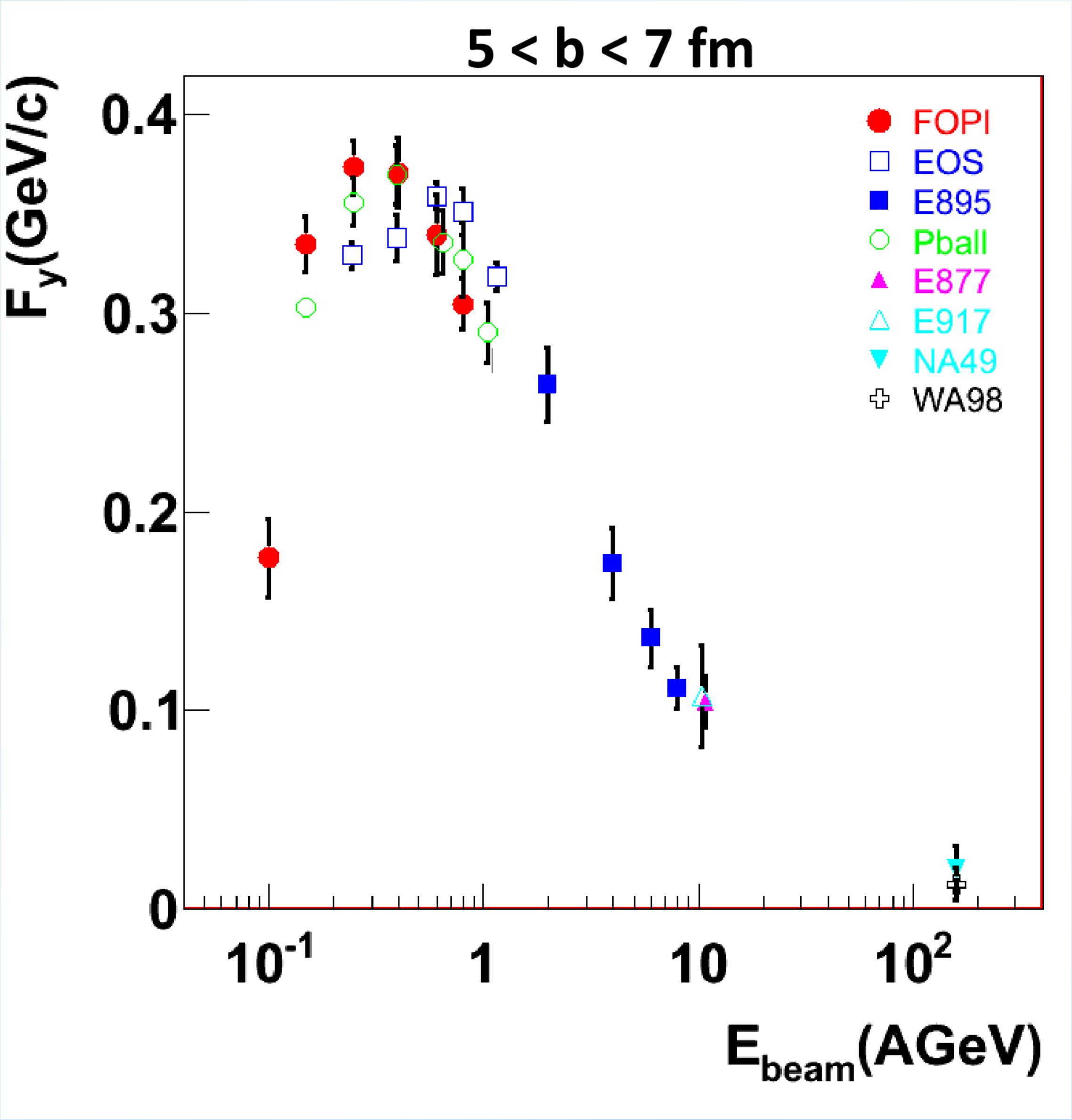
Global $1/\rho^{2/3}$ dependence of nucleon-nucleon cross sections from EoS

It is apparent that while the isospin-dependent nucleon-nucleon cross sections essentially follow the $1/\rho^{2/3}$-dependence, the nucleon-nucleon cross section parametrization of Cugnon et al. (J. Cugnon, T. Mizutani, and J. Vandermeulen, Nucl. Phys. A 352, 505 (1981)) leads to much larger spread, mostly due to its explicit energy dependence. Nevertheless, one observes that both parametrization cover essentially the same range of values of the nucleon-nucleon cross sections.

Furthermore, from the comparison of the parametrization of Cugnon et al. to in-medium cross sections at saturation density, calculated using the G-matrix theory by Cassing et al. (W. Cassing, and U. Mosel, Prog. Part. Nucl. Phys. 25, 235 (1990)), it can be judged that the in-medium cross sections, obtained using the proper volume of the Van der Waals-like equation of state, are in better agreement with somewhat higher values of G-matrix in-medium cross sections of Cassing et al., which reflect properly the Fermionic nature of nucleons.

Systematics of proton directed flow in Au+Au, b=5-7fm

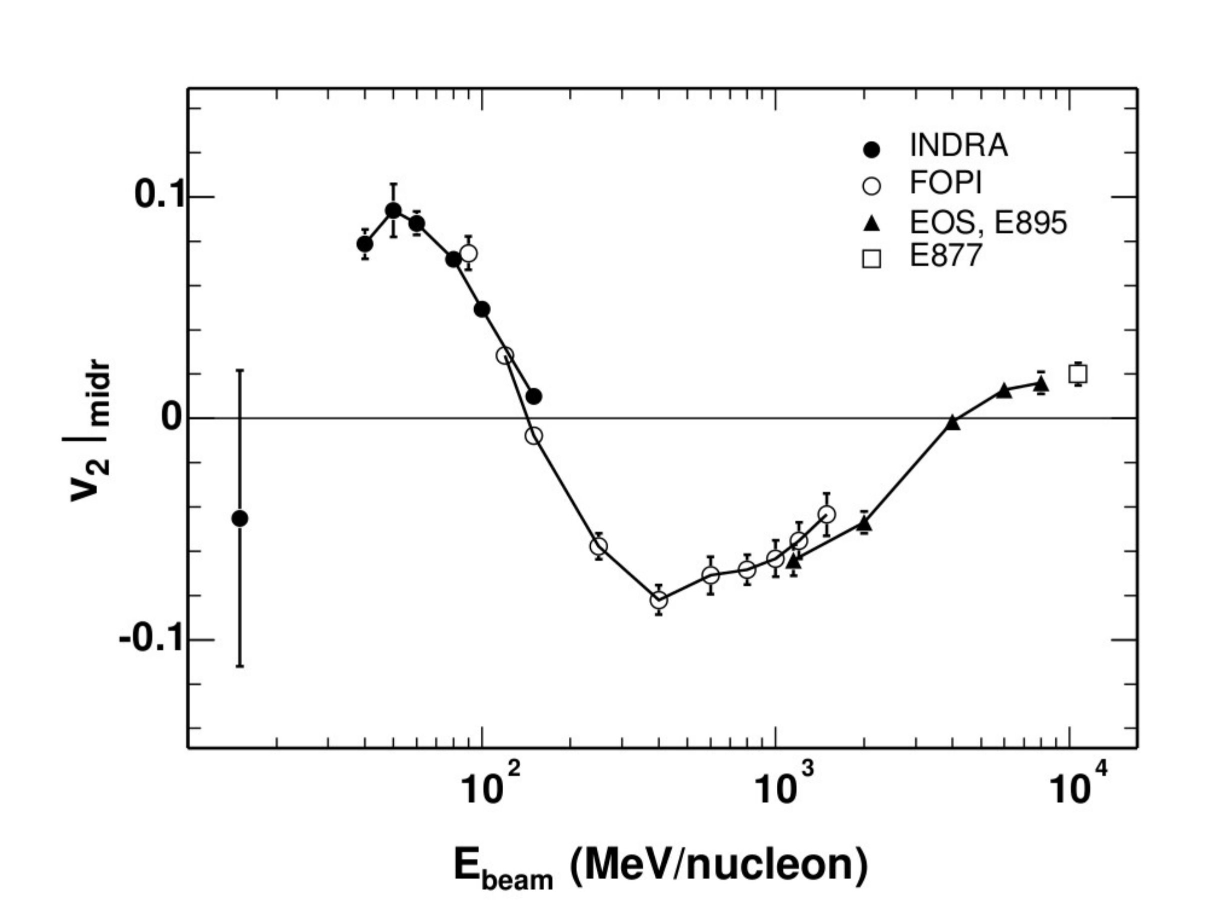
Directed flow is a 1st Fourier coefficient of the angular distribution in azimuthal (transverse) plane.



N. Herrmann, J. P. Wessels, and T. Wienold, Ann. Rev. Nucl. Part. Sci. 49 (1999) 581.

Systematics of proton elliptic flow in Au+Au, b=5-7fm

Elliptic flow is a 2nd Fourier coefficient of the angular distribution in azimuthal (transverse) plane.



A. Andronic, J. Lukasik, W. Reisdorf, and W. Trautmann, Eur. Phys. J. 30 (2006) 31.

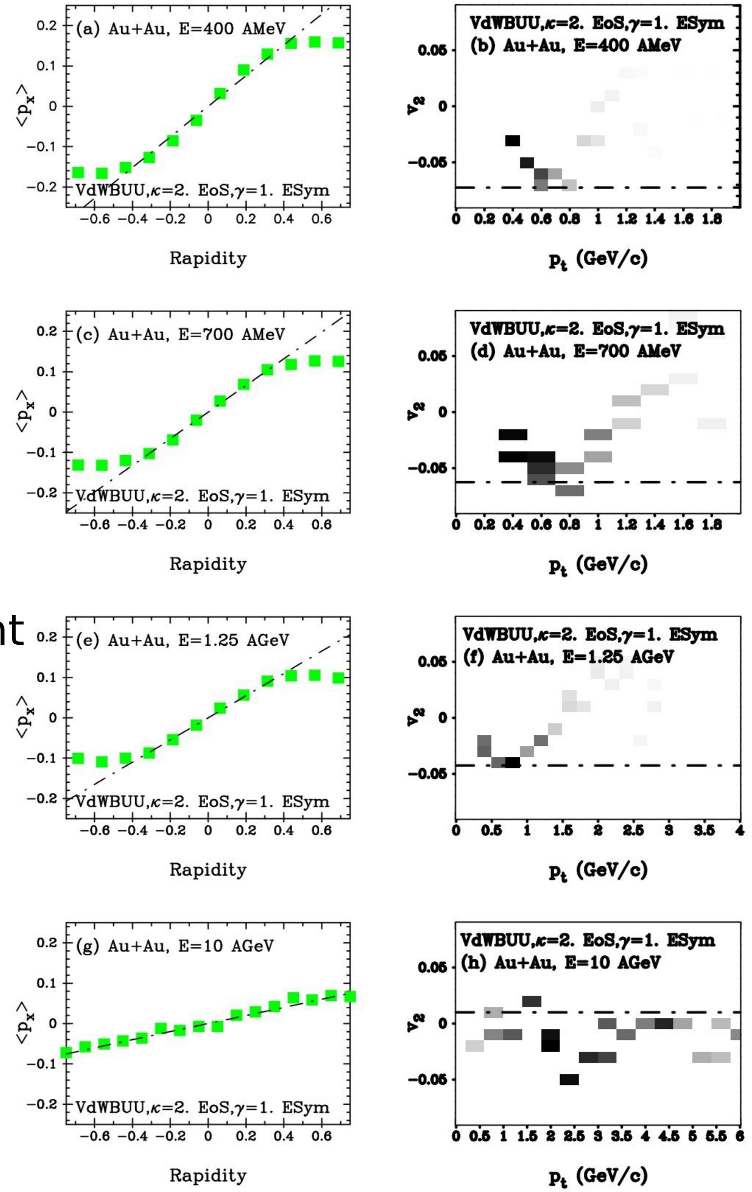
Stiff EoS (K_0=380 MeV)

Semi-stiff Esym (γ=1.)

Directed flow:

symbols- calc.

line - experiment



Elliptic flow:

Symbols - calc.

line - experiment

ΔxΔp>2h

FIG. 1: Systematics of the proton directed flow (left panels, lines indicate experimentally observed slopes) and the transverse momentum dependence of the calculated proton elliptic flow at mid-rapidity versus the experimental value (boxes and the dash-dotted lines in right panels, respectively) in the collisions of Au+Au at beam energies ranging from 400 AMeV to 10 AGeV. Results were obtained using the VdWBUU simulation using the stiff EoS with $\kappa = 2$ (K_0=380 MeV) and the symmetry energy density dependence with $\gamma = 1$.

EoS-dependent collision term (with in-medium cross sections) leads to correct (positive) directed flow, while free cross sections lead to incorrect (negative) directed flow !!!

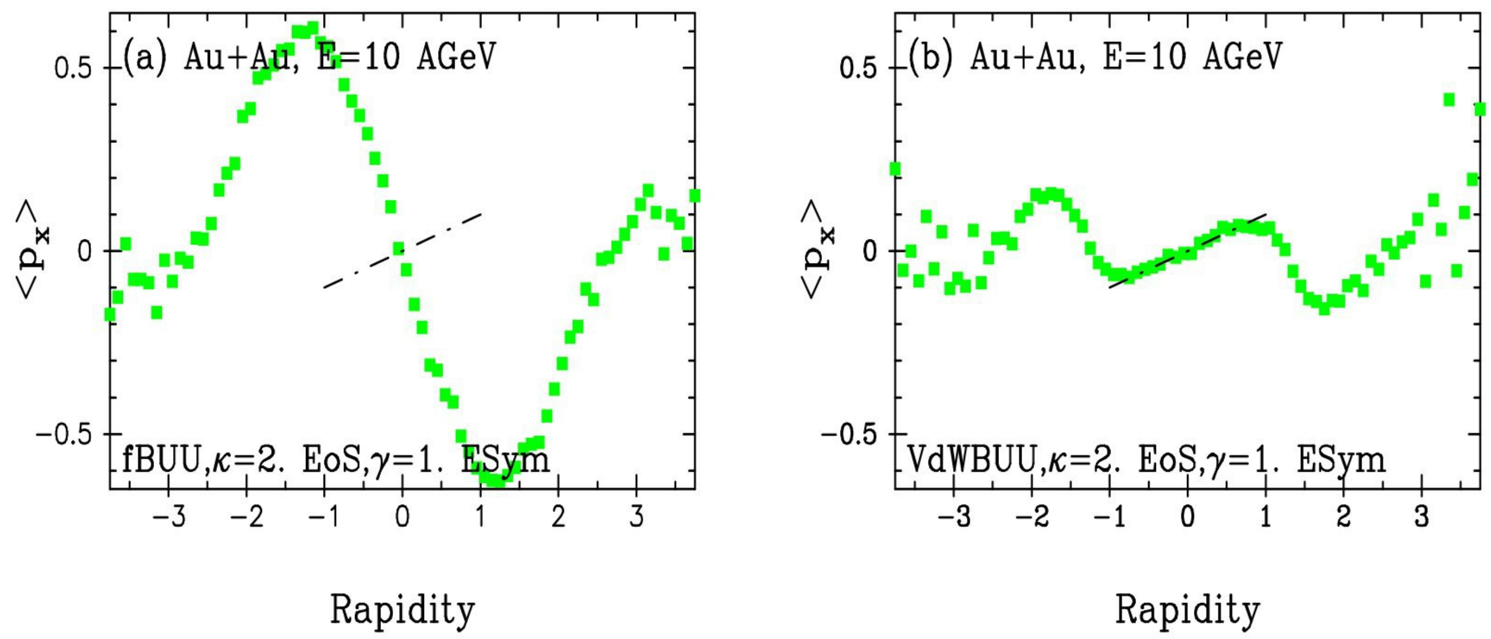


FIG. 2: Proton directed flow in the collisions of Au+Au at beam energy of 10 AGeV. Results were obtained using the simulation with and without EoS-dependent in-medium nucleon-nucleon cross sections (left and right panels, respectively) using the stiff EoS with $\boldsymbol{\kappa = 2}$ ($\boldsymbol{K_0}$=380 MeV) and the symmetry energy density dependence with $\boldsymbol{\gamma = 1}$. Lines indicate experimentally observed slope.

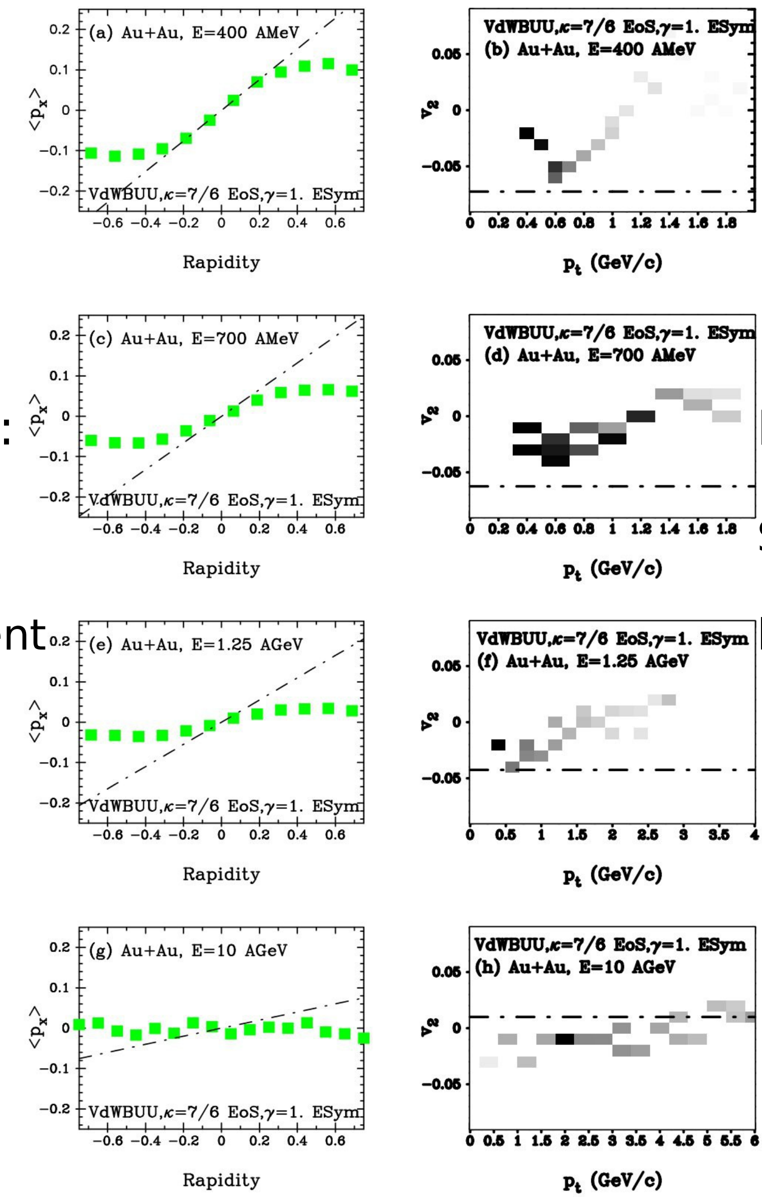
Soft EoS (K_0=200 MeV)

Semi-stiff Esym (γ=1.)

Directed flow:

symbols- calc.

line - experiment



Elliptic flow:

Symbols - calc.

line - experiment

FIG. 3: Systematics of the proton directed flow (left panels, lines indicate experimentally observed slopes) and the transverse momentum dependence of the calculated proton elliptic flow at mid-rapidity versus the experimental value (boxes and the dash-dotted lines in right panels, respectively) in the collisions of Au+Au at beam energies ranging from 400 AMeV to 10 AGeV. Results were obtained using the VdWBUU simulation using the soft EoS with $\boldsymbol{\kappa = 7/6}$ ($\boldsymbol{K_0}$=200 MeV) and the symmetry energy density dependence with $\boldsymbol{\gamma = 1}$.

EoS vs incompressibility

Starting from potential

$$U = a(\frac{\rho}{\rho_0}) + b(\frac{\rho}{\rho_0})^{\kappa}$$

After applying standard conditions for saturation we arrive to solution

$$a = \frac{2(-\kappa B - \frac{3}{5}(\kappa - \frac{2}{3})\epsilon_F(\rho_0))}{\kappa - 1}$$

$$b = \frac{\kappa + 1}{\kappa - 1}(B + (\frac{1}{5})\epsilon_F(\rho_0))$$

And get incompressibility (linear to κ !)

$$K_0 = 9\kappa(B + \frac{\epsilon_F(\rho_0)}{5}) - \frac{6}{5}\epsilon_F(\rho_0)$$

Semi-stiff EoS (K_0=270 MeV)

Semi-stiff Esym (γ=1.)

Directed flow:

symbols- calc.

line - experiment

Elliptic flow:

Symbols - calc.

line - experiment

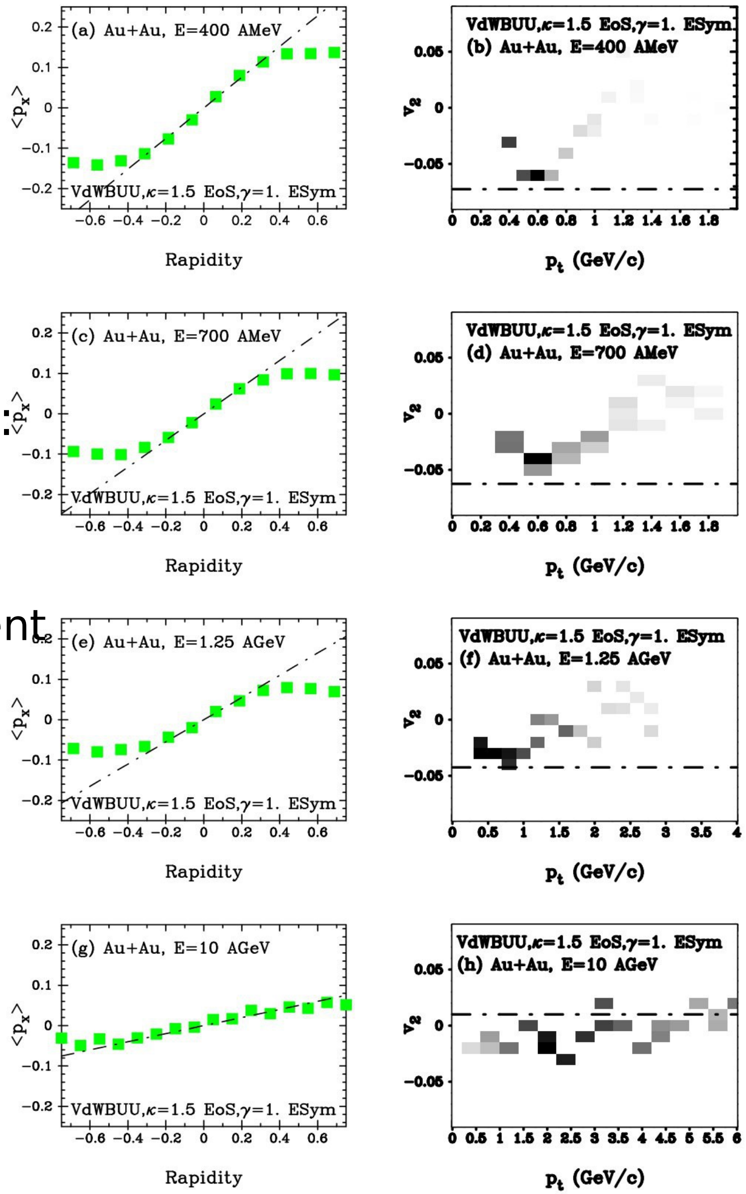


FIG. 4: Systematics of the proton directed flow (left panels, lines indicate experimentally observed slopes) and the momentum dependence of the calculated proton elliptic flow at mid-rapidity versus the experimental value (boxes and the dash-dotted line in right panels, respectively) in the collisions of Au+Au at beam energies ranging from 400 AMeV to 10 AGeV. Results were obtained using the VdWBUU simulation using the intermediate EoS with $\boldsymbol{\kappa = 3/2}$ ($\boldsymbol{K_0}$=272 MeV) and the symmetry energy density dependence with $\boldsymbol{\gamma = 1}$.

Simultaneous analysis of directed and elliptic flow in Au+Au semi-peripheral collisions (0.4-10 AGeV)

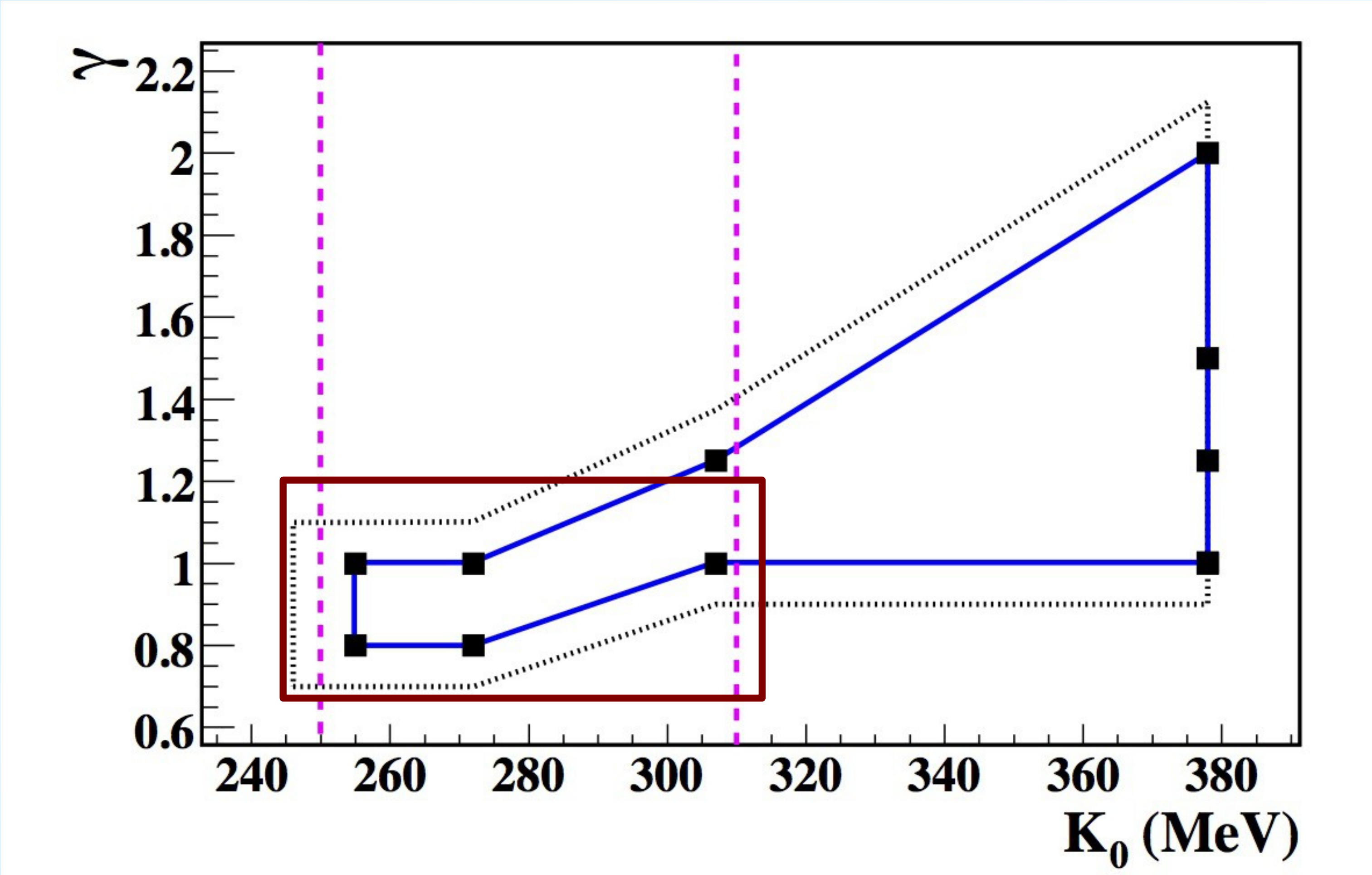


Fig. 5: The filled area in the γ vs K_0 plot shows the values of the EoS and symmetry energy parameters, constrained by the analysis using the VdWBUU simulations (squares show the values where calculation was performed and dashed line shows the uncertainty resulting from restricted set of calculated points). Dotted area shows the constrained values of K_0 from re-analysis of giant monopole resonance data [24].

[24] J.R. Stone, N.J. Stone and S.A. Moszkowski, PRC 89, 044316 (2014)

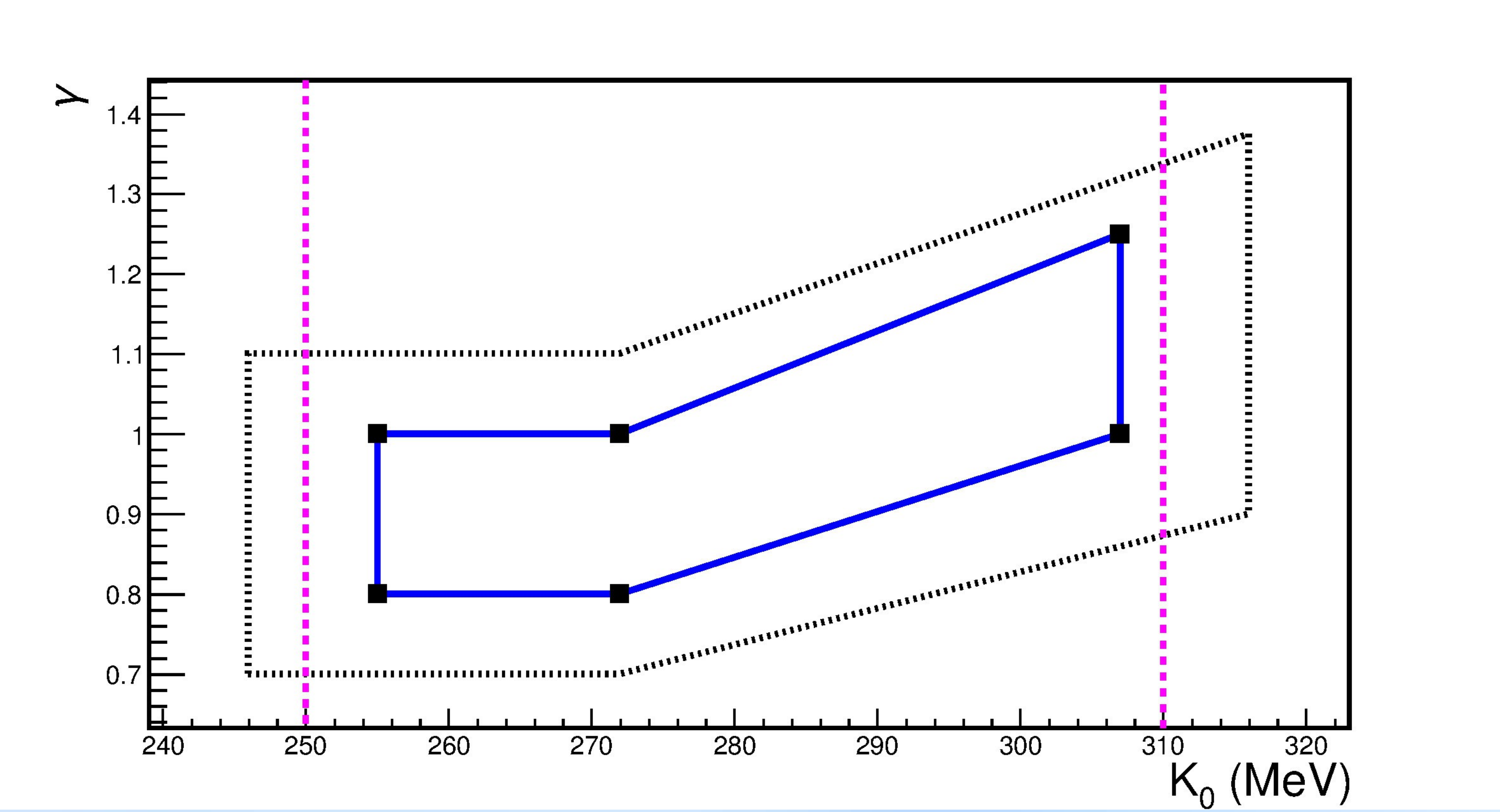
Some properties:

No apparent limit on K_0 from above, flow appears to saturate

Increase (up to 100 %) of production cross section of Δ-resonance has no effect on the directed flow => saturation of directed flow.

Addition of inelastic channels leading to Σ, Λ (strangeness production):

Overall agreement is preserved, upper limit of K_0 < 310 MeV is obtained



Summary

BUU simulations with nucleon-nucleon cross sections from inversion of VdW EoS reproduce the flow systematics

Constraint on K_0 = 245 – 315 MeV obtained

Corresponding constraint on symmetry energy stiffness γ = 0.7 – 1.25

Comparison with further observables necessary

Starting from EoS:

$$p = \rho T + a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s \rho_0 (\frac{\rho}{\rho_0})^{1+\gamma} \tau_z I$$

Method:

Formally transform EoS into Van der Waals form, then perform inversion and extract the proper volume.

M. Veselsky and Y.G. Ma, PRC 87 (2013) 034615

When looking for relation of equation of state and emission rates one can consider the van der Waals equation of state. It is written as

$$(p + a'\rho^2)(V - Nb') = NT \tag{4}$$

or

$$(p + a'\rho^2)(1 - \rho b') = \rho T \tag{5}$$

where the parameter a' is related to attractive interaction among particles and b' represents the proper volume of the constituent particles. In geometrical picture the volume of the particle can be directly related to its cross section for interaction with particles. It is possible to formally transform the equation of state of asymmetric nuclear matter (and practically any equation of state of any form) into the van der Waals equation. Then one obtains coefficients

$$a' = -a \tag{6}$$

and

$$b' = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s (\frac{\rho}{\rho_0})^{\gamma} \tau_z I}{p - a\rho^2} = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s (\frac{\rho}{\rho_0})^{\gamma} \tau_z I}{\rho T + b\kappa\rho^{1+\kappa} + 2\gamma\rho_0 a_s (\frac{\rho}{\rho_0})^{1+\gamma} \tau_z I} \tag{7}$$

where the latter provides a measure of the proper volume of the constituent of the gas, nucleon in this case, as a measure of deviation from its behavior of the ideal gas. The proper volume of nucleon can be used to estimate its cross section within the nucleonic medium

$$\sigma = 1.209\ b'^{2/3} \tag{8}$$

which can be implemented into the collision term of the BUU equation.