Relationship between the Symmetry Energy and Neutron-Proton Effective Mass Splitting

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- The symmetry energy (Esym)
- The symmetry potential (Usym) and Esym
- n-p effective mass splitting (m^*_{n-p})
- Constraints on m^*_{n-p}
- Summary

Refs: B.A. Li/LWC, MPLA30, 1530010 (2015); Z. Zhang/LWC, to be submitted

"5th International Symposium on Nuclear Symmetry Energy – NuSYM15",
June 29 – July 2, 2015, Krakow, Poland

Outline

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The Nuclear Matter Symmetry Energy

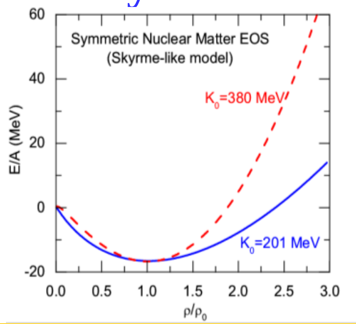
EOS of Isospin Asymmetric Nuclear Matter (Parabolic law)

$$E(\rho,\delta) = E(\rho,0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter (relatively well-determined)

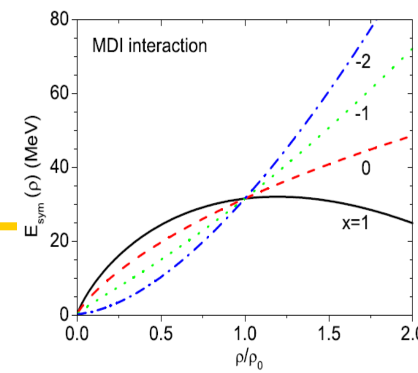
Isospin asymmetry

Symmetry energy term (poorly known)



Nuclear Matter Symmetry Energy

$$\boxed{E_{\text{sym}}(\rho) \equiv \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}}$$



$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{\text{sym}}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots\cdots, \quad (\rho \sim \rho_0)$$

$E_{\text{sym}}(\rho_0) \approx 30$ MeV (LD mass formula: *Myers & Swiatecki, NPA81; Pomorski & Dudek, PRC67*)

$$L \equiv 3\rho_0 \left.\frac{\partial E_{\text{sym}}(\rho)}{\partial \rho}\right|_{\rho=\rho_0}$$ (Many-Body Theory: L : –50 ~ 200 MeV; Exp: ???)

$$K_{\text{sym}} \equiv 9\rho_0^2 \left.\frac{\partial^2 E_{\text{sym}}(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0}$$ (Many-Body Theory: K_{sym} : –700 ~ 466 MeV; Exp: ???)

Liquid-Drop Model

$$a_{\rm v}A - a_{\rm s}A^{2/3} - a_4\frac{(N-Z)^2}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}$$

Symmetry energy term

$$S_{\rm v}\frac{(N_{\rm v}-Z_{\rm v})^2}{A} + S_{\rm s}\frac{(N_{\rm s}-Z_{\rm s})^2}{A^{2/3}}$$

$$a_{\rm v}A - a_{\rm s}A^{2/3} - \frac{S_{\rm v}}{1+y_{\rm s}A^{-1/3}}\frac{(N-Z)^2}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}$$

Symmetry energy including surface diffusion effects ($y_s=S_v/S_s$)

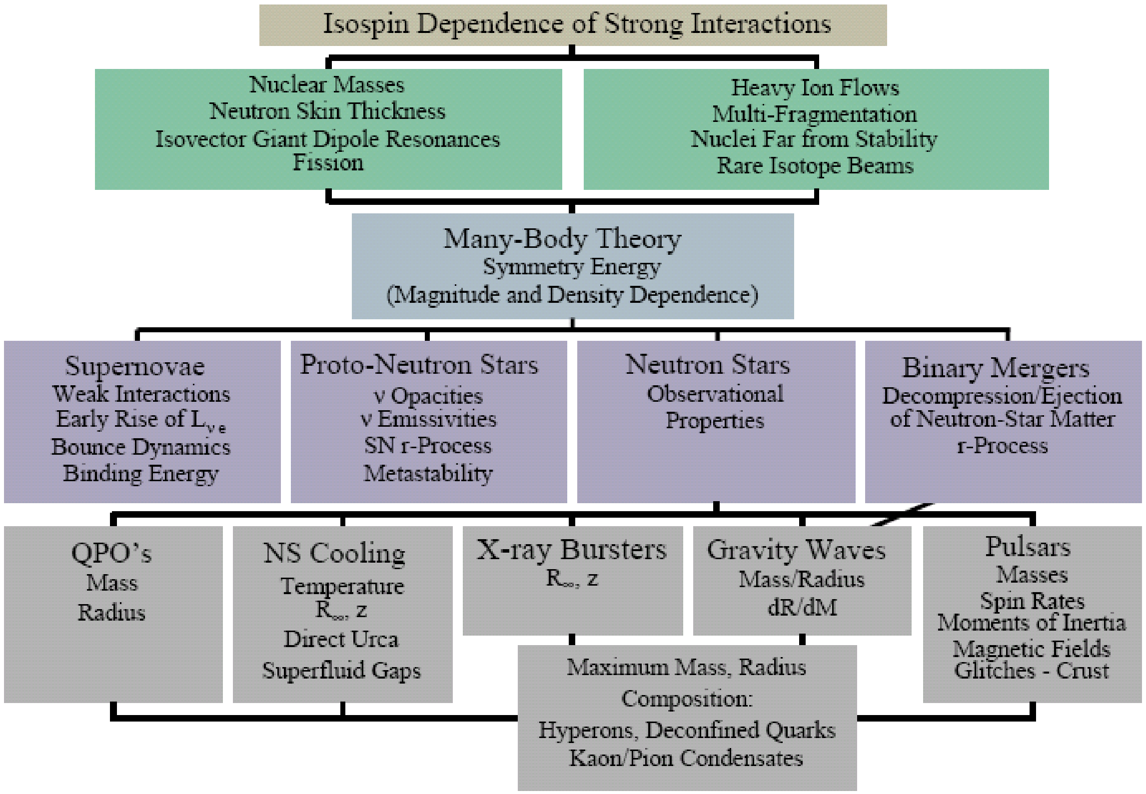
W. D. Myers, W.J. Swiatecki, P. Danielewicz, P. Van Isacker, A. E. L. Dieperink,……

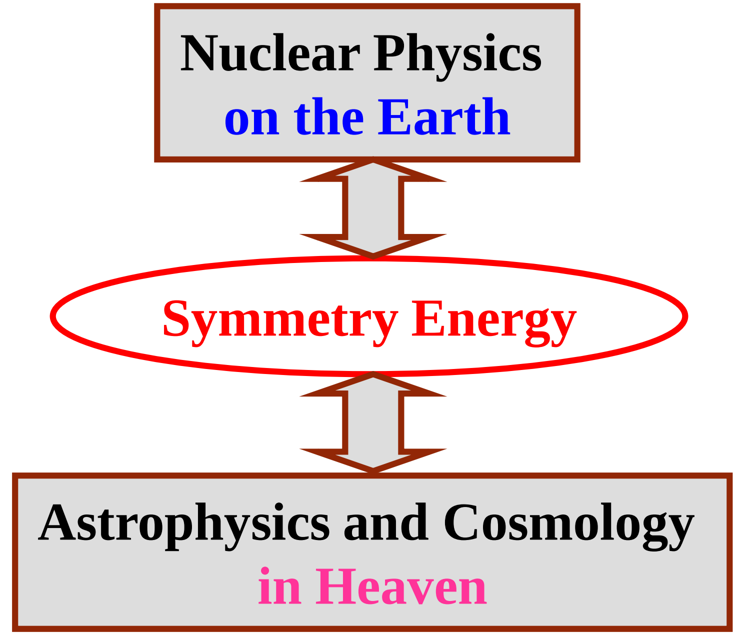
Why Symmetry Energy?

21.65.Ef

The multifaceted influence of the nuclear symmetry energy

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, ***Phys. Rep.* 411, 325 (2005).**





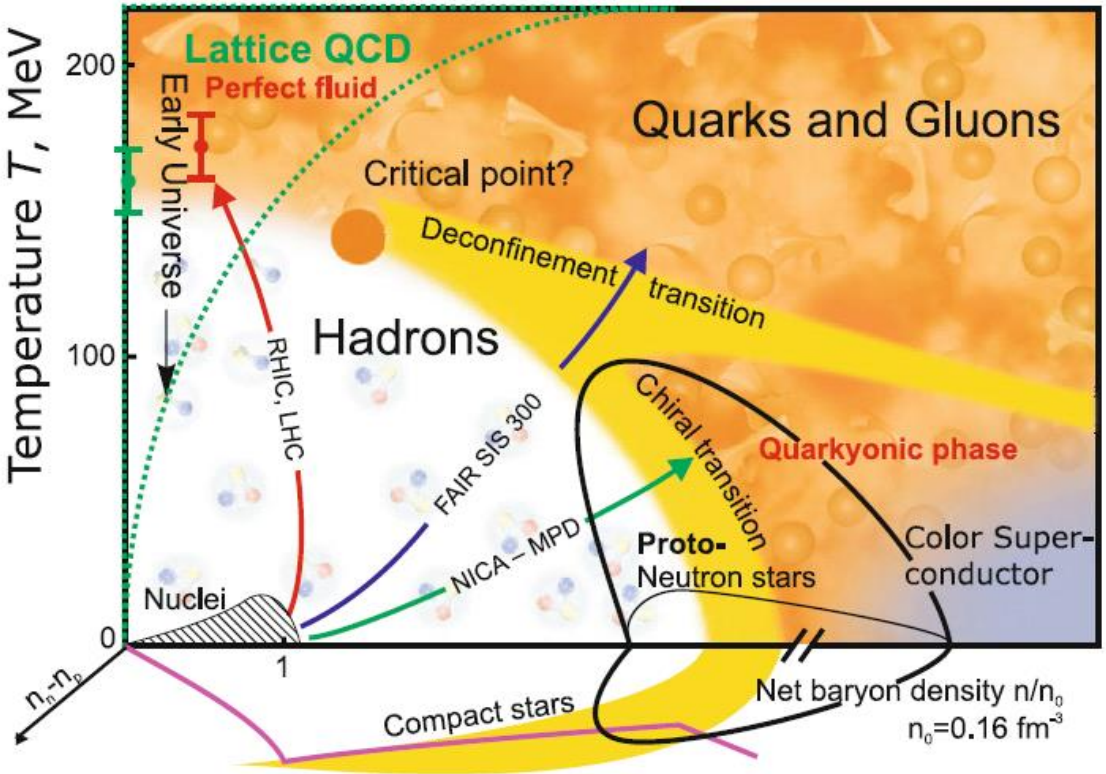
The symmetry energy is also related to some issues of fundamental physics:

1. **The precision tests of the SM through atomic parity violation observables (Sil et al., PRC2005)**
2. **Possible time variation of the gravitational constant (Jofre et al. PRL2006; Krastev/Li, PRC2007)**
3. **Non-Newtonian gravity proposed in the grand unified theories (Wen/Li/Chen, PRL2009)**
4. **Dark Matter (Zheng/Zhang/Chen, JCAP2014; Zheng/Sun/Chen, ApJ2015)**

Phase Diagram of Strong Interaction Matter

QCD Phase Diagram in 3D: density, temperature, and isospin

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag Berlin Heidelberg 2011



Esym: Important for understanding the EOS of strong interaction matter and QCD phase transitions at extreme isospin conditions

1. **Heavy Ion Collisions (Terrestrial Lab);**
2. **Compact Stars(In Heaven); …**

Quark Matter
Symmetry Energy ?

M. Di Toro et al. NPA775, 102(2006); Pagliara/Schaffner-Bielich, PRD81, 094024(2010); Shao et al., PRD85, 114017(2012);Chu/Chen, ApJ780, 135 (2014)

At extremely high baryon density, the main degree of freedom could be the deconfined quark matter rather than confined baryon matter, and there we should consider quark matter symmetry energy (isospin symmetry is still satisfied). The isopsin asymmetric quark matter could be produced/exist in HIC/Compact Stars

Esym: Experimental Probes

Promising Probes of the $E_{sym}(\rho)$

(an incomplete list !)

At sub-saturation densities (亚饱和密度行为)

- Sizes of n-skins of unstable nuclei from total reaction cross sections
- Proton-nucleus elastic scattering in inverse kinematics
- Parity violating electron scattering studies of the n-skin in ^{208}Pb
- n/p ratio of FAST, pre-equilibrium nucleons
- Isospin fractionation and isoscaling in nuclear multifragmentation
- Isospin diffusion/transport
- Neutron-proton differential flow
- Neutron-proton correlation functions at low relative momenta
- t/^{3}He ratio
- Hard photon production
- Pigmy/Giant resonances
- Nucleon optical potential

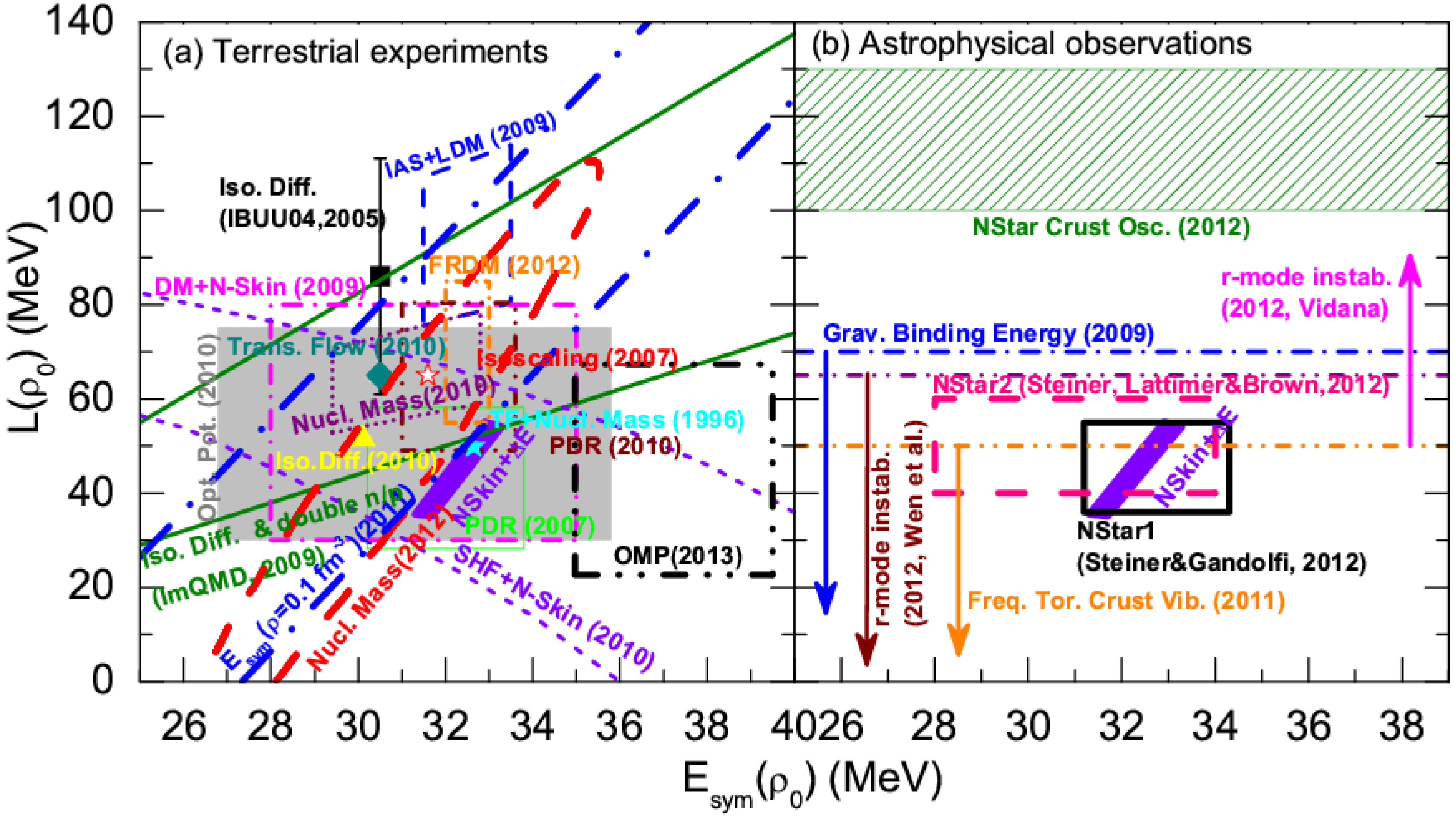
Towards high densities reachable at CSR/Lanzhou, FAIR/GSI, RIKEN, GANIL and, FRIB/MSU (高密度行为)

- π^-/π^+ ratio, K^+/K^0 ratio?
- Neutron-proton differential transverse flow
- n/p ratio at mid-rapidity
- Nucleon elliptical flow at high transverse momenta
- n/p ratio of squeeze-out emission

B.A. Li, L.W. Chen, C.M. Ko
Phys. Rep. 464, 113(2008)

E_{sym}: Around saturation density

Current constraints (An incomplete list) on $E_{sym}(\rho_0)$ and L from terrestrial experiments and astrophysical observations



$E_{sym}(\rho_0) = 32.5 \pm 2.5$ MeV, $L = 55 \pm 25$ MeV

L.W. Chen, Nucl. Phys. Rev. 31, 273 (2014) [arXiv:1212.0284]
B.A. Li, L.W. Chen, F.J. Fattoyev, W.G. Newton, and C. Xu, arXiv:1212.1178

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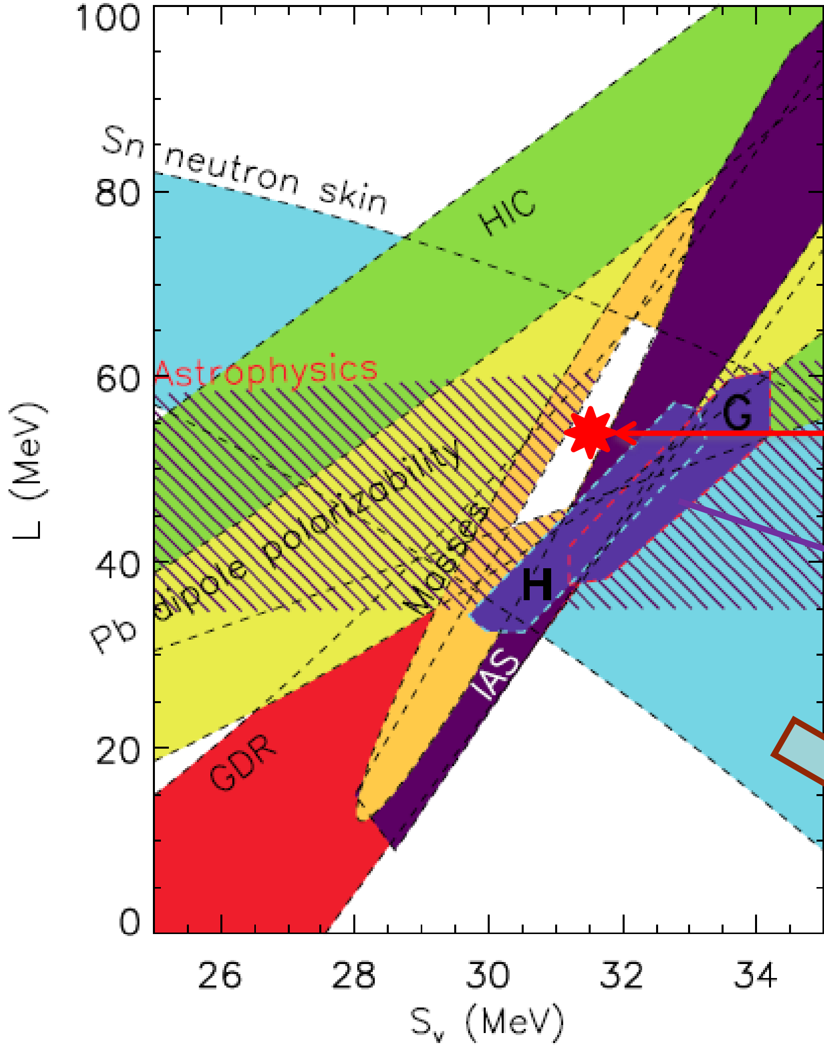
E_{sym}: Around saturation density

Jim Lattimer and Andrew Steiner using 6 out of approximately 30 available constraints



The centroid is around S_v=31 MeV and L=50 MeV

Microscopic calculations do not include higher-order contribution? EOS of SNM?

Chen/Ko/Li/Xu, PRC82, 024321 (2010)

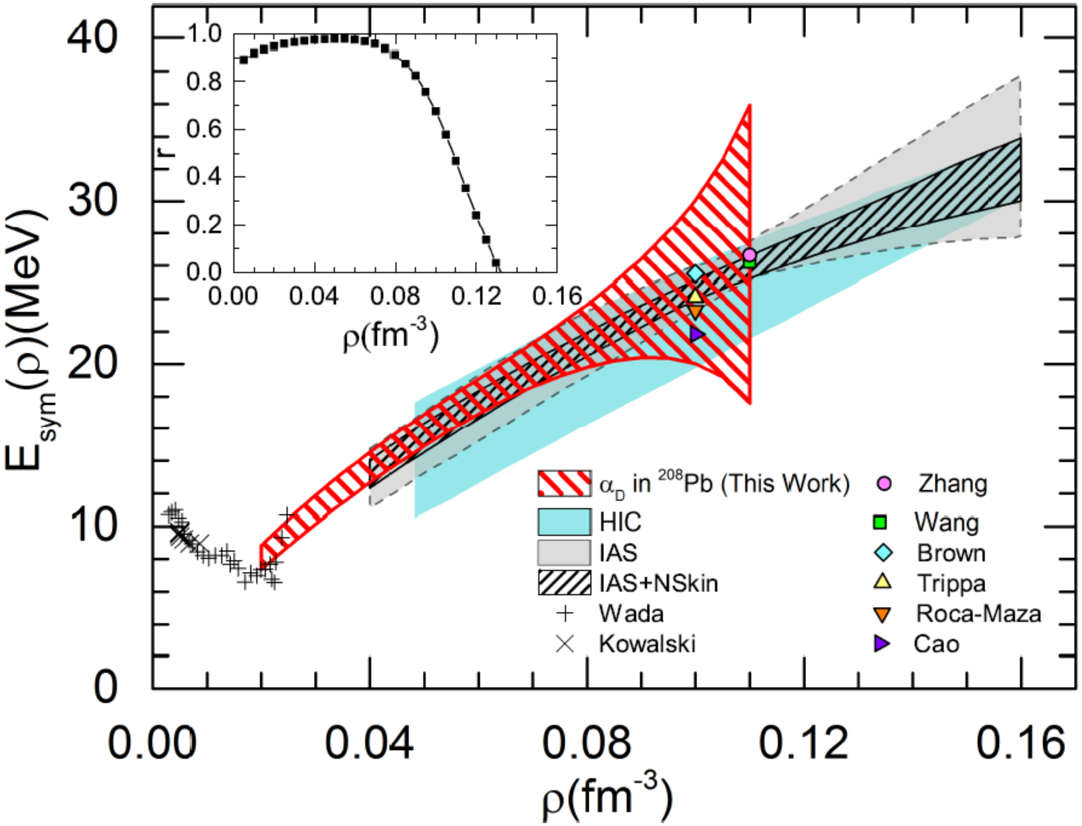
Why? Zhang/Chen, PLB726, 234 (2013)

Neutron skin is actually determined by L(0.11 fm^{-3}) rather than L(0.16 fm^{-3})

J.M. Lattimer and A.W. Steiner, EPJA50, (2014) 40

E_{sym}: Subsaturation densities

Z. Zhang and L.W. Chen, arXiv:1504.01077



- HIC: Sn+Sn
  M.B. Tsang *et al.*, Phys. Rev. Lett.**102**, 122701(2009)
- IAS and IAS+NSkin
  P. Danielewicz and J. Lee, Nucl. Phys. **A922**, 1 (2014)
- Zhang: Isotope binding energy difference
  Z. Zhang and L.W. Chen, Phys. Lett. **B726**, 234 (2013)
- Wang: Fermi energy difference
  N. Wang *et al.*, Phys. Rev. C **87**, 034327 (2013)
- Brown: Doubly magic nuclei
  B.A. Brown, Phys. Rev. Lett. **11**, 232502 (2013)
- Trippa: Giant dipole resonance
  L. Trippa *et al.*, Phys. Rev. C **77**
- Roca-Maza: Giant quadrupole resonance
  X. Roca-Maza *et al.*, Phys. Rev. C **87**, 034301 (2013)
- Cao: Pygmy dipole resonance
  L.G. Cao and Z.Y. Ma, Chin. Phys. Lett. **25**, 1625 (2008)

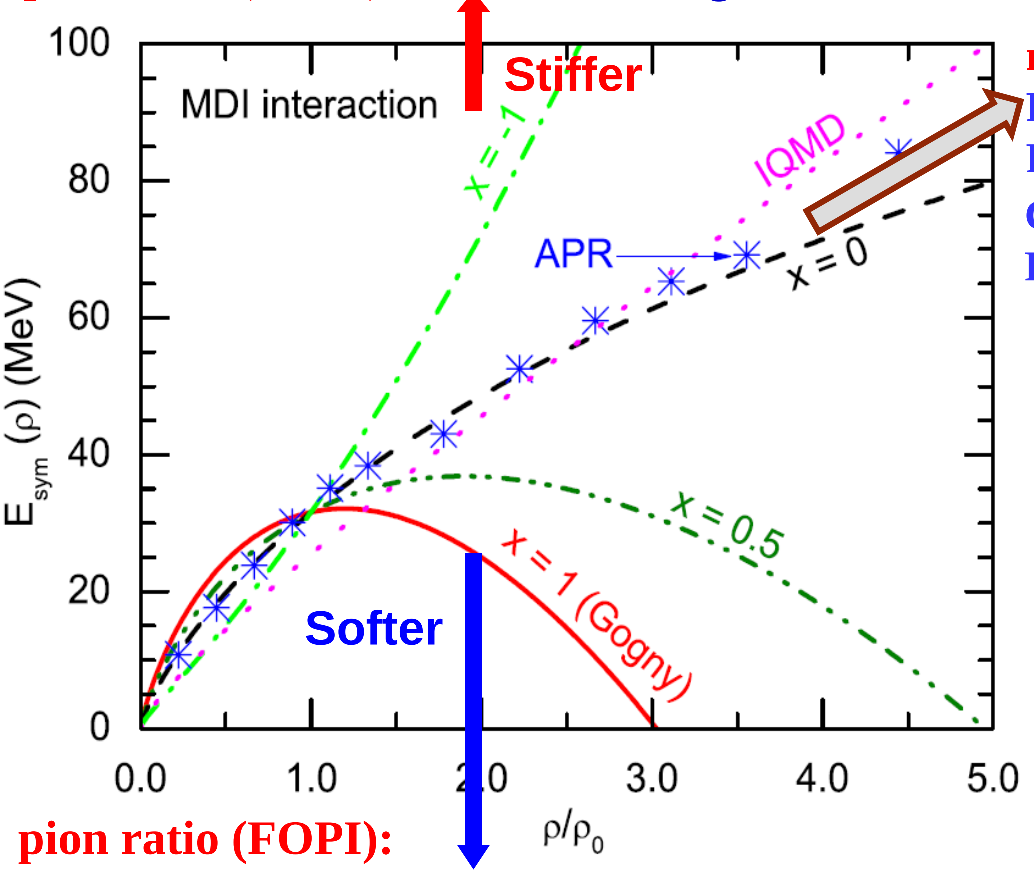
Wada and Kowalski: experimental results of the symmetry energies at densities below $0.2\rho_0$ and temperatures in the range 3 ~11 MeV from the analysis of cluster formation in heavy ion collisions.

Wada et al., Phys. Rev. C85, (2012) 064618; Kowlski et al., Phys. Rev. C75, (2007) 014601.
Natowitz et al. Phys. Rev. Lett. 104, (2010) 202501.

E_{sym}: Supra-saturation density

A Soft or Stiff Esym at supra-saturation densities ???

pion ratio (FOPI): ImIQMD, Feng/Jin, PLB683, 140(2010)



$(\rho/\rho_0)^\gamma$ with $\gamma = 0.9 \pm 0.4$

n/p v2 (FOPI):
Russotto/Trauntmann/Li et al., PLB697, 471(2011) (UrQMD)
Cozma/Trauntmann/Li et al., PRC88, 044912 (2013) (Tubingen QMD - MDI)

Pion Medium Effects?
Threshold effects?
Δ resonances?
……
Xu/Ko/Oh
PRC81, 024910(2010)
Xu/Chen/Ko/Li/Ma
PRC87, 067601(2013)
Hong/Danielewicz,
PRC90, 024605 (2014)
Song/Ko,
PRC91, 014901 (2015)

pion ratio (FOPI):
IBUU04, Xiao/Li/Chen/Yong/Zhang, PRL102,062502(2009)
ImIBLE, Xie/Su/Zhu/Zhang, PLB718,1510(2013)

E_{sym}: Current Status

● Cannot be that all the constraints on E_{sym} (ρ_0) and L are equivalently reliable since some of them don't have any overlap. However, essentially all the constraints seem to agree with:

$$E_{sym}(\rho_0) = 32.5 \pm 2.5 \text{ MeV}$$
$$L = 55 \pm 25 \text{ MeV}$$

● The symmetry energy at subsaturation densities have been relatively well-constrained

● All the constraints on the high density Esym come from HIC's (FOPI), and all of them are based on transport models. The constraints on the high density Esym are elusive and controversial for the moment !!!

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Potential decomposition of E_{sym}

The Esym constraints are strongly model dependent, even around saturation density!!!

- **Is there a general principle at some level, independent of the interaction and many-body theory, telling us what determines the $E_{sym}(\rho_0)$ and L?**

- **If possible, how to constrain separately each component of $E_{sym}(\rho_0)$ and L?**

$E_{sym}(\rho)$ and $L(\rho)$ can be decomposed in terms of nucleon potential in asymmetric nuclear matter !

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012).

The symmetry potential

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012).

$$\boxed{\begin{aligned} U_\tau(\rho,\delta,k) &= U_0(\rho,k) + \sum_{i=1,2,\cdots} U_{\mathrm{sym,i}}(\rho,k)(\tau\delta)^i \\ &= U_0(\rho,k) + U_{\mathrm{sym},1}(\rho,k)(\tau\delta) \\ &\quad + U_{\mathrm{sym},2}(\rho,k)(\tau\delta)^2 + \cdots, \end{aligned}}$$

$$\boxed{\begin{aligned} U_{\mathrm{sym,i}}(\rho,k) &\equiv \frac{1}{i!}\frac{\partial^i U_n(\rho,\delta,k)}{\partial\delta^i}|_{\delta=0} \\ &= \frac{(-1)^i}{i!}\frac{\partial^i U_p(\rho,\delta,k)}{\partial\delta^i}|_{\delta=0} \end{aligned}}$$

$\tau = 1$ for neutrons and -1 for protons

The Lane potential: $U_\tau(\rho,\delta,k) \approx U_0(\rho,k) + U_{\mathrm{sym},1}(\rho,k)(\tau\delta)$
A.M. Lane, NP35, 676 (1962)

(First-order) symmetry potential: $U_{\mathrm{sym},1}(\rho,k)$ **(conventionally denoted as** U_{sym} **)**

Second-order symmetry potential: $U_{\mathrm{sym},2}(\rho,k)$

上海交通大学 SHANGHAI JIAO TONG UNIVERSITY

Decomposition of the Esym and L according to the Hugenholtz-Van Hove (HVH) theorem

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)
C. Xu, B.A. Li, and L.W. Chen, PRC82, 054607 (2010)
R. Chen, B.J. Cai. L.W. Chen, B.A. Li, X.H. Li, and C. Xu, PRC85, 024305 (2012).

$$t(k_{F_n}) + U_n(\rho, \delta, k_{F_n}) = \frac{\partial \varepsilon(\rho, \delta)}{\partial \rho_n}$$

$$t(k_{F_p}) + U_p(\rho, \delta, k_{F_p}) = \frac{\partial \varepsilon(\rho, \delta)}{\partial \rho_p}$$

Hugenholtz-Van Hove theorem

N. M. Hugenholtz, L. Van Hove, Physica **24**, 363 (1958)

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho, k_F),$$

Brueckner/Dabrowski, Phys. Rev. 134 (1964) B722

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho, k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F} \cdot k_F + 3U_{sym,2}(\rho, k_F)$$

$$m_0^*(\rho, k) = \frac{m}{1 + \frac{m}{\hbar^2 k}\frac{\partial U_0(\rho, k)}{\partial k}},$$

$E_{sym}(\rho)$ and $L(\rho)$ can be decomposed in terms of nucleon potential in asymmetric nuclear matter !

Lorentz Covariant Self-energy Decomposition of the E_{sym} and L

B.J. Cai and L.W. Chen, PLB711, 104 (2012)

$$\begin{aligned}\Sigma^J(\rho,\alpha,|\mathbf{k}|) &= \Sigma_S^J(\rho,\alpha,|\mathbf{k}|) - \gamma_\mu \Sigma^{\mu,J}(\rho,\alpha,|\mathbf{k}|)\\ &= \Sigma_S^J(\rho,\alpha,|\mathbf{k}|) + \gamma^0 \Sigma_V^J(\rho,\alpha,|\mathbf{k}|)\\ &\quad + \boldsymbol{\gamma}\cdot\mathbf{k}^0 \Sigma_K^J(\rho,\alpha,|\mathbf{k}|),\end{aligned}$$

$$E_{\mathrm{sym}}(\rho) = \frac{|\mathbf{k}|^2}{6M^*_{0,\mathrm{Lan}}(\rho,|\mathbf{k}|)}\bigg|_{|\mathbf{k}|=k_F} + E_{\mathrm{sym}}^{1\mathrm{st,K}}(\rho) + E_{\mathrm{sym}}^{1\mathrm{st,S}}(\rho) + E_{\mathrm{sym}}^{1\mathrm{st,V}}(\rho)$$

$$L(\rho) = L^{\mathrm{kin}}(\rho) + L^{\mathrm{mom}}(\rho) + L^{1\mathrm{st}}(\rho) + L^{\mathrm{cross}}(\rho) + L^{2\mathrm{nd}}(\rho)$$

$$L^{\mathrm{kin}}(\rho) = \frac{k_F k_F^*}{6\mathcal{E}_F^*} + \frac{k_F^2 M_0^{*2}}{6\mathcal{E}_F^{*3}}$$

$$\begin{aligned}L^{\mathrm{mom}}(\rho) &= \frac{k_F^2 M_0^{*2}}{3\mathcal{E}_F^{*2}}\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|}\bigg|_{|\mathbf{k}|=k_F}\\ &+ \frac{k_F^2}{6}\left[\frac{k_F^*}{\mathcal{E}_F^*}\frac{\partial^2\Sigma_K^0}{\partial|\mathbf{k}|^2} + \frac{M_0^*}{\mathcal{E}_F^*}\frac{\partial^2\Sigma_S^0}{\partial|\mathbf{k}|^2} + \frac{\partial^2\Sigma_V^0}{\partial|\mathbf{k}|^2}\right]_{|}\\ &+ \frac{k_F}{6}\left[\frac{k_F^*}{\mathcal{E}_F^*}\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_F^*}\frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|} + \frac{\partial\Sigma_V^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k}\\ &+ \frac{k_F^2}{6\mathcal{E}_F^{*3}}\left[M_0^{*2}\left(\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|}\right)^2 + k_F^{*2}\left(\frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|}\right)^2\right]_{|\mathbf{k}|=k_F}\\ &- \frac{k_F^2 k_F^* M_0^*}{3\mathcal{E}_F^{*3}}\left[\frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|}\left(1+\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|}\right)\right]_{|\mathbf{k}|=k_F},\end{aligned}$$

$$\begin{aligned}L^{1\mathrm{st}}(\rho) &= \frac{3}{2\mathcal{E}_F^{*3}}\left[M_0^*\Sigma_K^{\mathrm{sym},1} - k_F^*\Sigma_S^{\mathrm{sym},1}\right]^2\\ &+ \frac{3}{2}\left[\frac{k_F^*}{\mathcal{E}_F^*}\Sigma_K^{\mathrm{sym},1} + \frac{M_0^*}{\mathcal{E}_F^*}\Sigma_S^{\mathrm{sym},1} + \Sigma_V^{\mathrm{sy}}\right.\\ &+ \frac{k_F M_0^{*2}\Sigma_K^{\mathrm{sym},1}}{\mathcal{E}_F^{*3}} - \frac{k_F k_F^* M_0^*\Sigma_S^{\mathrm{sym},1}}{\mathcal{E}_F^{*3}},\end{aligned}$$

$$\begin{aligned}L^{\mathrm{cross}}(\rho) &= k_F\left[\frac{k_F^*}{\mathcal{E}_F^*}\frac{\partial\Sigma_K^{\mathrm{sym},1}}{\partial|\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_F^*}\frac{\partial\Sigma_S^{\mathrm{sym},1}}{\partial|\mathbf{k}|} + \frac{\partial\Sigma_V^{\mathrm{sym},1}}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_F}\\ &- \frac{k_F\Sigma_K^{\mathrm{sym},1}}{\mathcal{E}_F^*}\left[\frac{k_F^{*2}}{\mathcal{E}_F^{*2}}\left(\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|} + \frac{M_0^*}{k_F^*}\frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|}\right) - \frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_F}\\ &- \frac{k_F\Sigma_S^{\mathrm{sym},1}}{\mathcal{E}_F^*}\left[\frac{M_0^{*2}}{\mathcal{E}_F^{*2}}\left(\frac{k_F^*}{M_0^*}\frac{\partial\Sigma_K^0}{\partial|\mathbf{k}|} + \frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|}\right) - \frac{\partial\Sigma_S^0}{\partial|\mathbf{k}|}\right]_{|\mathbf{k}|=k_F}\end{aligned}$$

$$L^{2\mathrm{nd}}(\rho) = 3\left[\frac{k_F^*}{\mathcal{E}_F^*}\Sigma_K^{\mathrm{sym},2} + \frac{M_0^*}{\mathcal{E}_F^*}\Sigma_S^{\mathrm{sym},2} + \Sigma_V^{\mathrm{sym},2}\right].$$

Lorentz covariant nucleon self-energy can be obtained from Dirac phenomenology, QCD sum rules,……

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Nucleon Effective Mass

Nucleon effective mass plays an important role in various fields (see, e.g., B.A. Li, LWC, MPLA30, 1530010 (2015), but its definition is a little bit complicated!

J.P. Jeukenne, A. Lejeune, C. Mahaux, Phys. Rep. 25, 83 (1976)

- **In non-relativistic approaches, microscopic nuclear many-body theories indicate that the real part of the single-nucleon potential in nuclear matter of density ρ and isospin-asymmetry δ depends on not only the nucleon momentum k but also its energy E, reflecting the nonlocality in both space and time of nuclear interactions. These two kinds of nonlocality can be characterized by using the so-called nucleon effective k-mass and E-mass, respectively defined in terms of the partial derivative of U with respect to k and E.**

- **The effective mass is usually evaluated at the Fermi momentum or corresponding Fermi energy, yielding the so-called Landau mass that is related to the f_1 Landau parameter of a Fermi liquid.**

Nucleon Effective Mass

The definition of nucleon effective mass is a little bit complicated!

J.P. Jeukenne, A. Lejeune, C. Mahaux, Phys. Rep. 25, 83 (1976)

- **Once a dispersion relation $k(E)$ or $E(k)$ is known from the on-shell condition $E = k^2/2m + U(k, E, \rho, \delta)$, an equivalent potential either local in space or time, i.e. $U(k(E), E, \rho, \delta)$ or $U(k, E(k), \rho, \delta)$, can be obtained. Thus, the total nucleon effective mass can be calculated using either the first or second part of its defining equation depending on whether the E or k is selected as the explicit variable**

$$\boxed{\frac{m_\tau^*}{m_\tau} = 1 - \frac{dU_\tau(k(\mathcal{E}), \mathcal{E}, \rho, \delta)}{d\mathcal{E}} = \left[1 + \frac{m_\tau}{\hbar^2 k_F^\tau}\frac{dU_\tau(k, \mathcal{E}(k), \rho, \delta)}{dk}\bigg|_{k_F^\tau}\right]^{-1}}$$

- **In relativistic approaches, the definition of the effective mass is much more complicated: Dirac mass, Landau mass, Lorenz mass (NR effective mass), Optical Potential mass, …**

Jaminon/Mahaux, PRC40, 354 (1989); Chen/Ko/Li, PRC76, 054316 (2007)

Isospin splitting of nucleon effective mass

According to the definition of the nucleon effective mass, the n-p effective mass splitting $m^*_{n-p}(\rho, \delta)$ can be expressed as

$$m^*_{n-p} = \frac{\dfrac{m}{\hbar^2}\left(\dfrac{1}{k_F^p}\dfrac{dU_p}{dk}\Big|_{k_F^p} - \dfrac{1}{k_F^n}\dfrac{dU_n}{dk}\Big|_{k_F^n}\right)}{\left[1+\dfrac{m_p}{\hbar^2 k_F^p}\dfrac{dU_p}{dk}\Big|_{k_F^p}\right]\left[1+\dfrac{m_n}{\hbar^2 k_F^n}\dfrac{dU_n}{dk}\Big|_{k_F^n}\right]}$$

Up to **the first-order in isospin-asymmetry parameter δ**, the expression for $m^*_{n-p}(\rho, \delta)$ can be further simplified to

$$\boxed{m^*_{n-p} \approx 2\delta \frac{m}{\hbar^2 k_F}\left[-\frac{dU_{\rm sym,1}}{dk} - \frac{k_F}{3}\frac{d^2U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2}$$

This expression is valid at an arbitrary density ρ and it is seen that the $m^*_{n-p}(\rho, \delta)$ generally depends explicitly on the momentum dependence of both the isovector $U_{\rm sym,1}$ and isoscalar U_0 potentials

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Connections among Esym, Usym and m*$_{n\text{-}p}$

From the discusions above, one can see Esym, Usym and m*$_{n\text{-}p}$ are closely related to each other. In particular, Esym and m*$_{n\text{-}p}$ can be expressed explicitly in terms of U0, Usym,1, and Usym,2 and their momentum dependence at different densities.

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho, k_F),$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho, k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F} \cdot k_F + 3U_{sym,2}(\rho, k_F)$$

$$m_0^*(\rho, k) = \frac{m}{1 + \frac{m}{\hbar^2 k}\frac{\partial U_0(\rho,k)}{\partial k}},$$

$$\frac{m_\tau^*}{m_\tau} = 1 - \frac{dU_\tau(k(\mathcal{E}), \mathcal{E}, \rho, \delta)}{d\mathcal{E}} = \left[1 + \frac{m_\tau}{\hbar^2 k_F^\tau}\frac{dU_\tau(k, \mathcal{E}(k), \rho, \delta)}{dk}\Big|_{k_F^\tau}\right]^{-1}$$

$$m_{n-p}^* \approx 2\delta \frac{m}{\hbar^2 k_F}\left[-\frac{dU_{\text{sym},1}}{dk} - \frac{k_F}{3}\frac{d^2 U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2$$

Therefore, it is particularly important to determine the magnitude and momentum dependence of U0, Usym,1, and Usym,2 at different densities:
Heavy ion collisions, N-A elastic scattering, Nuclear structure properties, ……

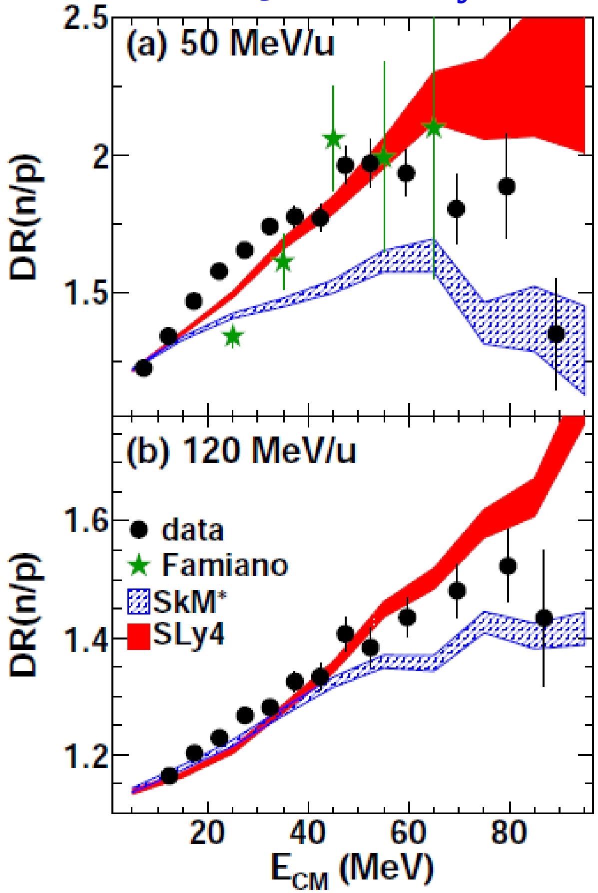
Constraint on m^*_{n-p}: heavy ion collisions

The magnitude and momentum dependence of U0, Usym,1, and Usym,2 … at different densities are basic input in transport mo-del simulations for heavy ion collisions, and so the heavy ion collisions provide an important tool to extract information on U0, Usym,1, and Usym,2 , and thus Esym and m^*_{n-p} …

Many probes for m^*_{n-p} in heavy ion collisions (see, e.g., Li/Chen, MPLA30,1530010(2015))

Double n/p ratio in Sn+Sn

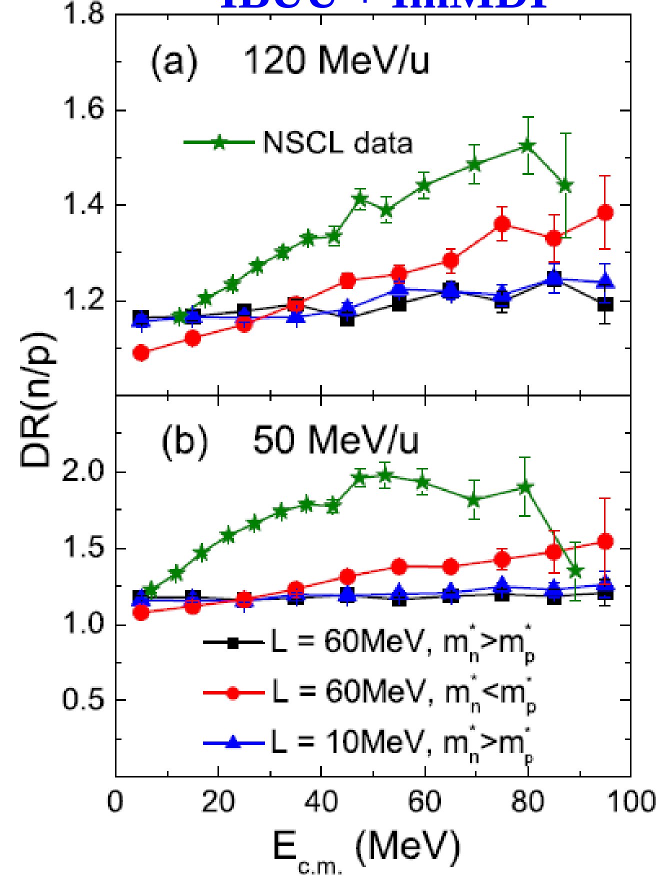
ImQMD + Skyrme



D.D.S. Coupland et al., arXiv:1406.4546

$m^*_n < m^*_p$ (SLy4)?

IBUU + ImMDI



H.Y. Kong et al., PRC91, 047601 (2015)

Underpredict the data for both $m^*_n < m^*_p$ and $m^*_n > m^*_p$?

Clustering? Short Range Correlation (O. Hen et al. PRC91, 025803 (2015); B.A. Li et al, PRC91, 044601 (2015))?

Constraint on m*$_{n\text{-}p}$: N-A scattering data

X.H. Li, B.J. Cai. L.W. Chen, R. Chen, B.A. Li, and C. Xu, PLB721, 101 (2013)

Both $U_{sym,1}$ (ρ,p) and $U_{sym,2}$ (ρ,p) at saturation density can be extracted from global neutron-nucleus scattering optical potentials

$$V(r,\mathcal{E}) = -V_{\rm v} f_{\rm r}(r) - iW_{\rm v} f_{\rm v}(r) + i4a_{\rm s}W_{\rm s}\frac{\mathrm{d}f_{\rm s}(r)}{\mathrm{d}r} + 2\lambda\!\!\!^{-}{}_{\pi}^{2}\frac{V_{\rm so}+iW_{\rm so}}{r}\frac{\mathrm{d}f_{\rm so}(r)}{\mathrm{d}r}\mathbf{S}\cdot\mathbf{L},$$

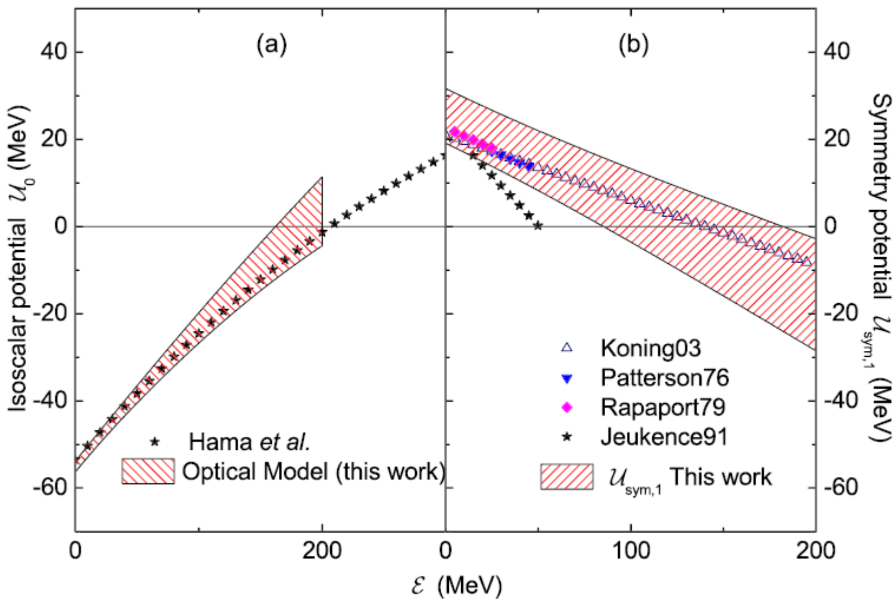
$$\begin{aligned} V_{\rm v} =& V_0 + V_1\mathcal{E} + V_2\mathcal{E}^2 + (V_3 + V_{3\rm L}\mathcal{E})\frac{N-Z}{A} \\ &+ (V_4 + V_{4\rm L}\mathcal{E})\frac{(N-Z)^2}{A^2}, \end{aligned} \tag{2}$$

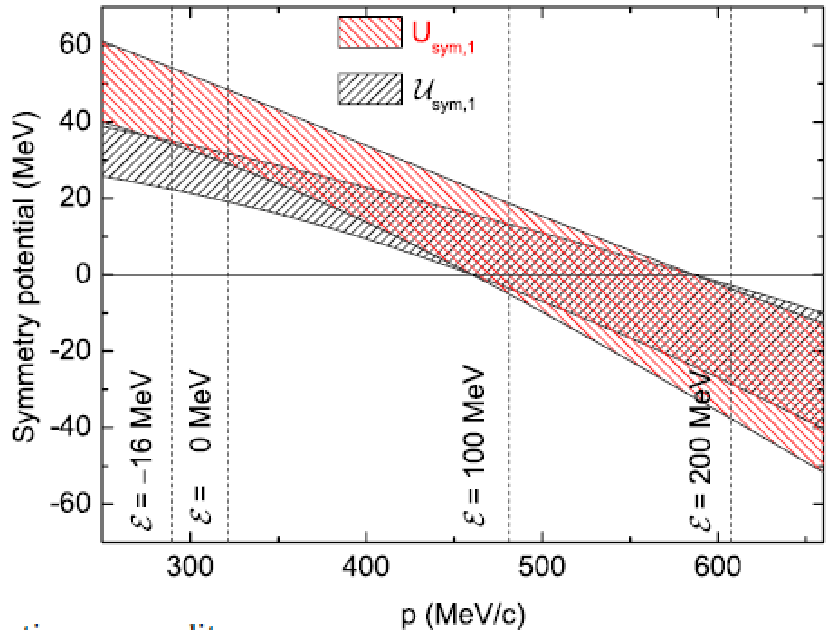
$$\begin{aligned} W_{\rm s} =& W_{\rm s0} + W_{\rm s1}\mathcal{E} + (W_{\rm s2} + W_{\rm s2L}\mathcal{E})\frac{N-Z}{A} \\ &+ (W_{\rm s3} + W_{\rm s3L}\mathcal{E})\frac{(N-Z)^2}{A^2}, \end{aligned} \tag{3}$$

$$\begin{aligned} W_{\rm v} =& W_{\rm v0} + W_{\rm v1}\mathcal{E} + W_{\rm v2}\mathcal{E}^2 + (W_{\rm v3} + W_{\rm v3L}\mathcal{E})\frac{N-Z}{A} \\ &+ (W_{\rm v4} + W_{\rm v4L}\mathcal{E})\frac{(N-Z)^2}{A^2}, \end{aligned} \tag{4}$$

Constraint on m^*_{n-p}: N-A scattering data

X.H. Li, W.J. Guo, B.A. Li, L.W. Chen, F.J. Fattoyev, and W.J. Newton, PLB743, 408 (2015)





Nucleon isoscalar effective mass m_0^*/m and the neutron–proton effective mass splitting m^*_{n-p} from the three cases studied in this work.

Case	m_0^*/m	$m^*_{n-p}(\delta)$
I	0.65 ± 0.05	0.41 ± 0.14
II	0.67 ± 0.06	0.44 ± 0.16
III	0.65 ± 0.06	0.41 ± 0.15

Case I: n-A
Case II: p-A
Case III: n/p-A

Extrapolation to negative energy (-16 MeV) from scattering state has been made, which may lead to some uncertainty (Dispersive OM may help?)

Constraint on $m^*_{n\text{-}p}$: Giant Resonances in ^{208}Pb

Z. Zhang and LWC, to be submitted

Centroid energy of IVGDR:

$$E_{-1} = \sqrt{m_1/m_{-1}}$$

$$\boxed{E_{-1} = 13.46\,\mathrm{MeV}}$$

B.L. Berman and S.C. Fultz, Rev. Mod. Phys. 47 (1975) 713;
S. S. Dietrich and B. L. Berman, At. Data Nucl. Data Tab. 38, 199 (1988)

Electric Dipole Polarizability α_{D}:

$$\alpha_D = \frac{\hbar c}{2\pi^2}\int \frac{\sigma_{\mathrm{abs}}}{\omega^2}d\omega = \frac{8\pi}{9}\int\frac{S_{E1}(\omega)}{\omega}\mathrm{d}\omega = \frac{8\pi}{9}e^2 m_{-1}$$

$$\boxed{\alpha_{\mathrm{D}} = 20.1 \pm 0.6\mathrm{MeV}}$$

$$m_k = \int_0^\infty S(E)E^k dE \qquad m_{-1} = \int S(E)E^{-1}dE$$

A. Tamii et al. (RCNP), PRL107, 062502 (2011)

$$\boxed{E_{-1} = 13.46\,\mathrm{MeV}} \quad \boxed{\alpha_{\mathrm{D}} = 20.1 \pm 0.6\mathrm{MeV}} \Rightarrow \boxed{m_1 = 905.9 \pm 27.2\ \mathrm{MeV}\cdot\mathrm{fm}^2}$$

Energy weighted (Thomas-Reiche-Kuhn) sum rule of IVGDR:

$$m_1 = \frac{9}{4\pi}\frac{\hbar^2}{2m}\frac{NZ}{A}(1+\kappa)$$

M.N. Harakeh, A. van der Woude, *Giant Resonances-Fundamental High-frequency Modes of Nuclear Excitation* (Clarendon, Oxford, 2001).

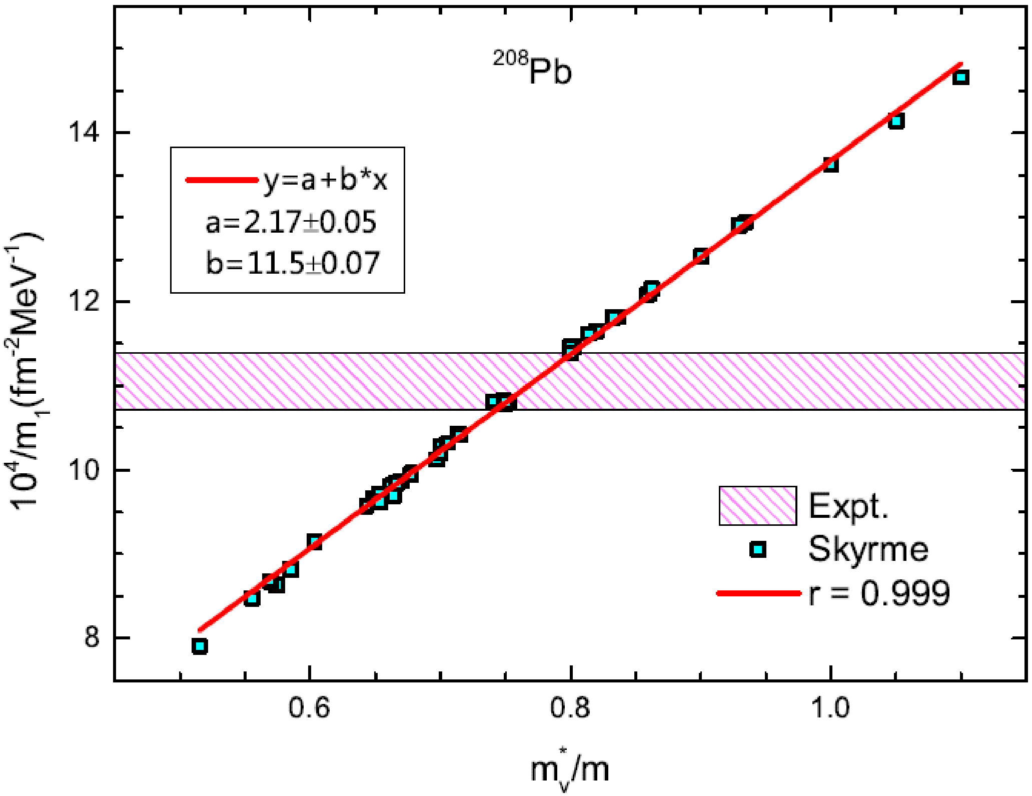
κ is the enhancement factor reflecting the deviation from the TRK sum rule due to the exchange and momentum dependent force, and it is related to the isovector effective mass by $m^*_{v,0}/m = 1/(1+\kappa)$

Constraint on $m^*_{n\text{-}p}$: Giant Resonances in ^{208}Pb

Z. Zhang and LWC, to be submitted

For the standard Skyrme interaction, the isovector effective mass is expressed as

$$\frac{\hbar^2}{2m_v^*} = \frac{\hbar^2}{2m} + \frac{1}{8}t_1(x_1+2)\rho + \frac{1}{8}t_2(x_2+2)\rho$$



Skyrme-RPA calculations (Colo et al., CPC184, 142 (2013)) with 50 representative Skyrme interactions

$$\boxed{\frac{m^*_{v,0}}{m} = 0.77 \pm 0.03}$$

$$m^*_{v,0}/m = 1/(1+\kappa)$$

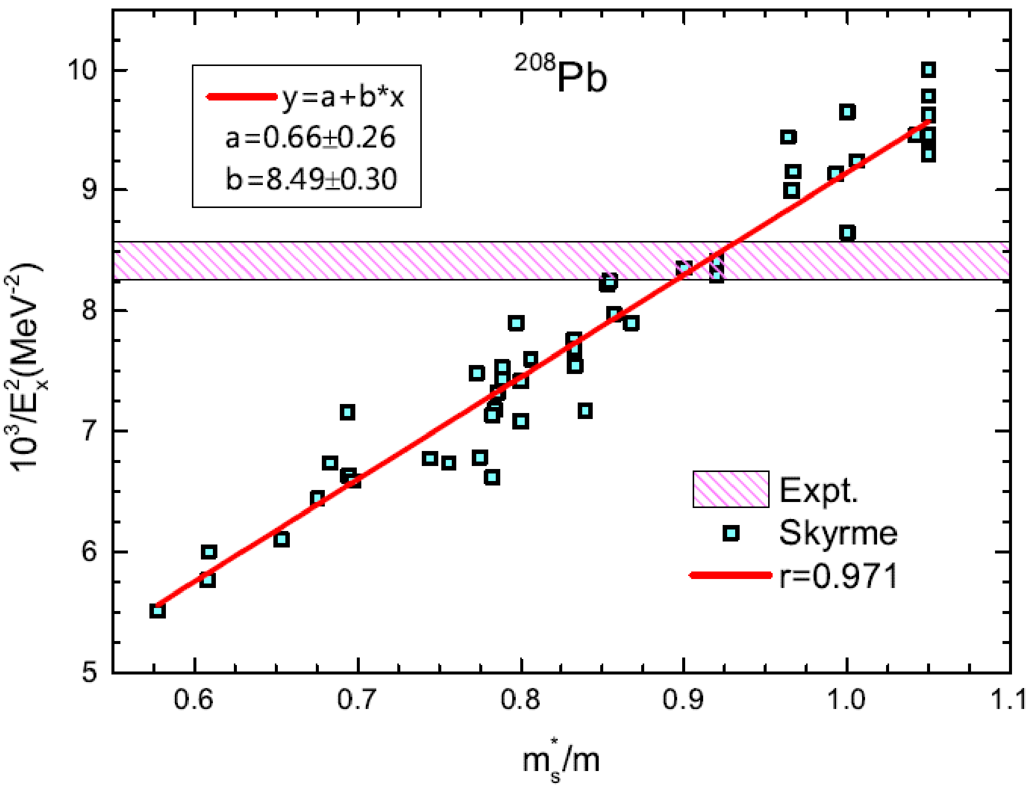
$$\boxed{\kappa = 0.33 \pm 0.05}$$

Constraint on m^*_{n-p}: Giant Resonances in ^{208}Pb

Z. Zhang and LWC, to be submitted

As well known, the isoscalar effective mass, m^*_s, is intimately related to the excitation energy of the ISGQR (see, e.g., J.-P. Blaizot, Phys. Rep. 64, 171 (1980); X. Roca-Maza et al., Phys. Rev. C 87, 034301 (2013)). In the harmonic oscillator model, the ISGQR energy is

$$\boxed{E_x = \sqrt{\frac{2m}{m^*_{s,0}}}\hbar\omega_0} \qquad \frac{\hbar^2}{2m^*_s} = \frac{\hbar^2}{2m} + \frac{3}{16}t_1\rho + \frac{1}{16}t_2(4x_2+5)\rho$$



Skyrme-RPA calculations (Colo et al., CPC184, 142 (2013)) with 50 representative Skyrme interactions

$$\boxed{\frac{m^*_{s,0}}{m} = 0.91 \pm 0.05}$$

Constraint on m^*_{n-p}: Giant Resonances in ^{208}Pb

Z. Zhang and LWC, to be submitted

For the standard Skyrme interaction, the isospin splitting of nucleon effective mass can be expressed as

$$\boxed{\Delta m^* = \frac{m_n^* - m_p^*}{m} = 2\frac{m_s^*}{m}\sum_{n=1}^{\infty}\left(\frac{m_s^* - m_v^*}{m_v^*}\delta\right)^{2n-1}}$$

Linear isospin splitting of nucleon effective mass at saturation density is

$$\boxed{\Delta m_0^* = (0.33 \pm 0.16)\delta} \quad \boxed{\frac{m_{s,0}^*}{m} = 0.91 \pm 0.05} \quad \boxed{\frac{m_{v,0}^*}{m} = 0.77 \pm 0.03}$$

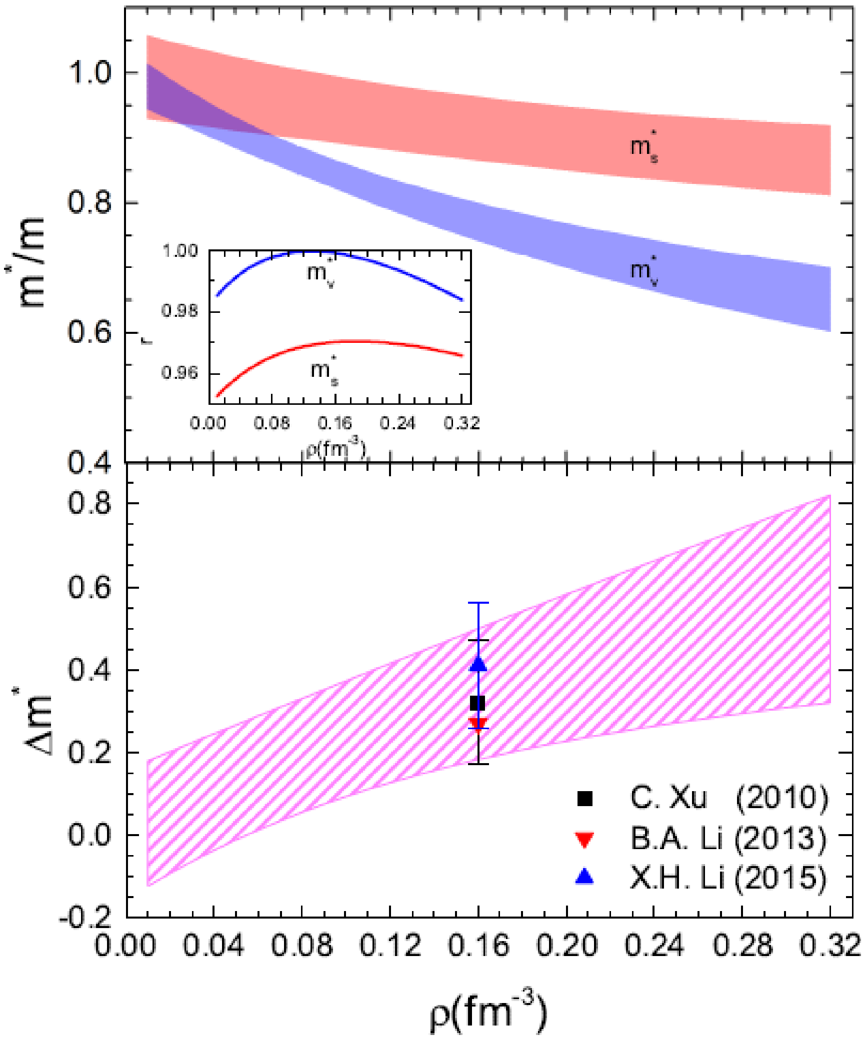
This value is in good agreement with result $\Delta m_0^* = (0.41 \pm 0.15)\delta$ extracted from N-A scattering (X.H. Li et al., PLB743, 408 (2015))

The higher-order isospin splitting of nucleon effective mass at saturation density is negligibly small, e.g., the coefficient of δ^3 term is only 0.01+/- 0.01.

Constraint on m^*_{n-p}: Giant Resonances in ^{208}Pb

Z. Zhang and LWC, to be submitted

Similarly, we can obtain the corresponding constraints at different densities



N-A scattering:

C. Xu (2010): (0.32+/- 0.15) δ

(C. Xu, B.A. Li, LWC, PRC82, 054607 (2010))

X.H. Li (2015): (0.41+/- 0.15) δ

(X.H. Li, W.J. Guo, B.A. Li, LWC, F.J. Fattoyev, and W.J. Newton, PLB743, 408 (2015))

Empirical constraints on Esym :

B.A. Li (2013): 0.27 δ

(B.A. Li, X. Han, PLB727, 276 (2013))

Linear isospin splitting of nucleon effective mass could become stronger at higher densities !

Outline

- The symmetry energy (Esym)
- The symmetry potential (Usym) and Esym
- n-p effective mass splitting (m^*_{n-p})
- Constraints on m^*_{n-p}
- Summary

Summary

- The isospin splitting of nucleon effective mass is closely related to the symmetry energy through the nucleon (isoscalar and isovector) potentials and their momentum dependence

- While the nuclear matter symmetry energy at sub- and saturation densities is relatively well constrained, its supra-saturation behaviors remain a big challenge

- While the analyses from N-A scattering within optical model and giant resonances in ^{208}Pb favor a positive neutron-proton effective mass splitting in neutron-rich matter at saturation density:

$$\boxed{\Delta m_0^* = (0.33 \pm 0.16)\delta}$$

  the constraints from double n/p ratio in heavy ion collisions (Sn+Sn) are still significantly model dependent.

- The isospin splitting of nucleon effective mass could become stronger at higher densities !

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Acknowledgements

Collaborators:

Wei-Zhou Jiang (SEU. Nanjing)
Che Ming Ko (TAMU. Texas)
Bao-An Li, Bao-Jun Cai (TAMU-Commerce, Texas)
Xiao-Hua Li (USC, Hengyang)
De-Hua Wen (SCUT, Guanzhou)
Zhi-Gang Xiao (Tsinghua, Beijing)
Chang Xu (NJU, Nanjing)
Jun Xu (SINAP, CAS, Shanghai)
Gao-Chan Yong (IMP, Lanzhou)
Xin Wang, Peng-Cheng Chu, Zhen Zhang , Kai-Jia Sun, Hao Zheng, Rui Wang, Ying Zhou (SJTU, Shanghai)

Funding:

National Natural Science Foundation of China
Major State Basic Research Development Program (973 Program) in China
Shanghai Rising-Stars Program
Shanghai “Shu-Guang” Project
Shanghai “Eastern Scholar”
Science and Technology Commission of Shanghai Municipality

谢 谢！
Thanks！