Symmetry and congruence energies in different macroscopic models

Krzysztof Pomorski and Bożena Nerlo-Pomorska

Maria Curie Skłodowska University, Lublin, Poland

NUSYM15, Kraków, June 29 – July 2, 2015

Outline:

- Historical remarks,
- Leptodermous expansion of the density functional,
- Nuclear masses in different macroscopic-microscopic models,
- Symmetry and Wigner energies in selected LD models,
- Systematics of the fission barrier heights for long chains of isotopes as a measure of the surface energy symmetry term,
- Systematics of the spontaneous fission half-lives and the LD symmetry terms,
- Influence of deformation on Q_β value of heavy nuclei,
- Conclusions

80^{th} Anniversary of the Liquid Drop Model

Zeitschrift fur Physik 96 (1935) 431

Zur Theorie der Kernmassen.

Von **C. F. v. Weizsäcker** in Leipzig.

Mit 5 Abbildungen. (Eingegangen am 6. Juli 1935.)

§ 1. Problemstellung. § 2. Erweiterung der Thomas-Fermi-Methode. § 3. Numerische Auswertung (gemeinsam mit F. S. Wang). § 4. Die Auszeichnung gerader Teilchenzahlen. § 5. Halbempirische Darstellung der Massendefekte. § 6. Zusammenfassung.

found by Carl Friedrich von Weizsäcker (1912 – 2007).

Leptodermous expansion of the energy functional

The one-body density of the nucleus is given by the integral:

$$\rho = A \int \int \dots \int \Psi^\star \Psi \, d^3 r_2 \cdots d^3 r_A \ ,$$

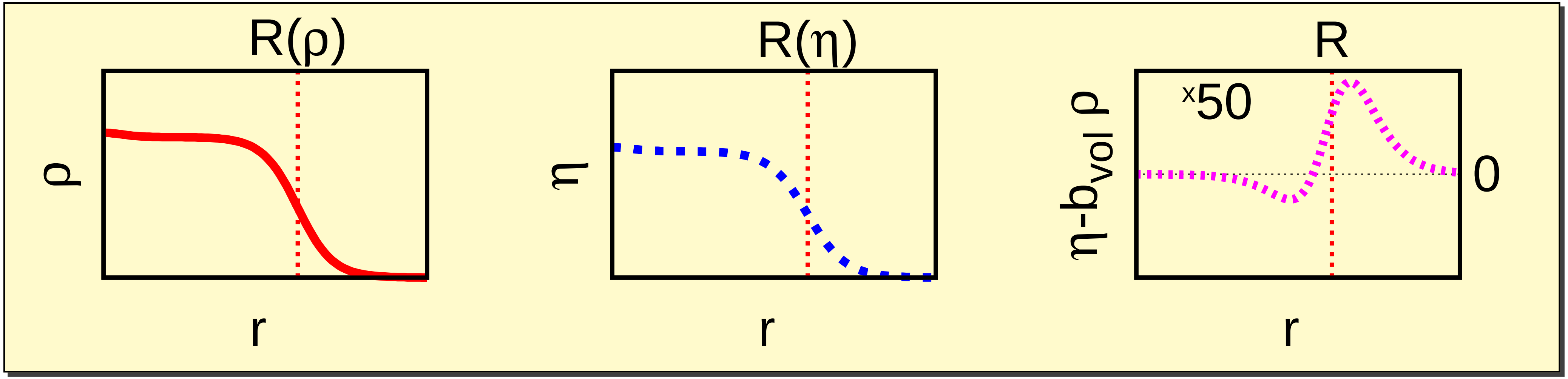
where $\Psi(\xi_1, \xi_2, \dots, \xi_A)$ is the many-body wave function and $A = \int \rho d^3 r$.

Similarly one defines the binding energy density:

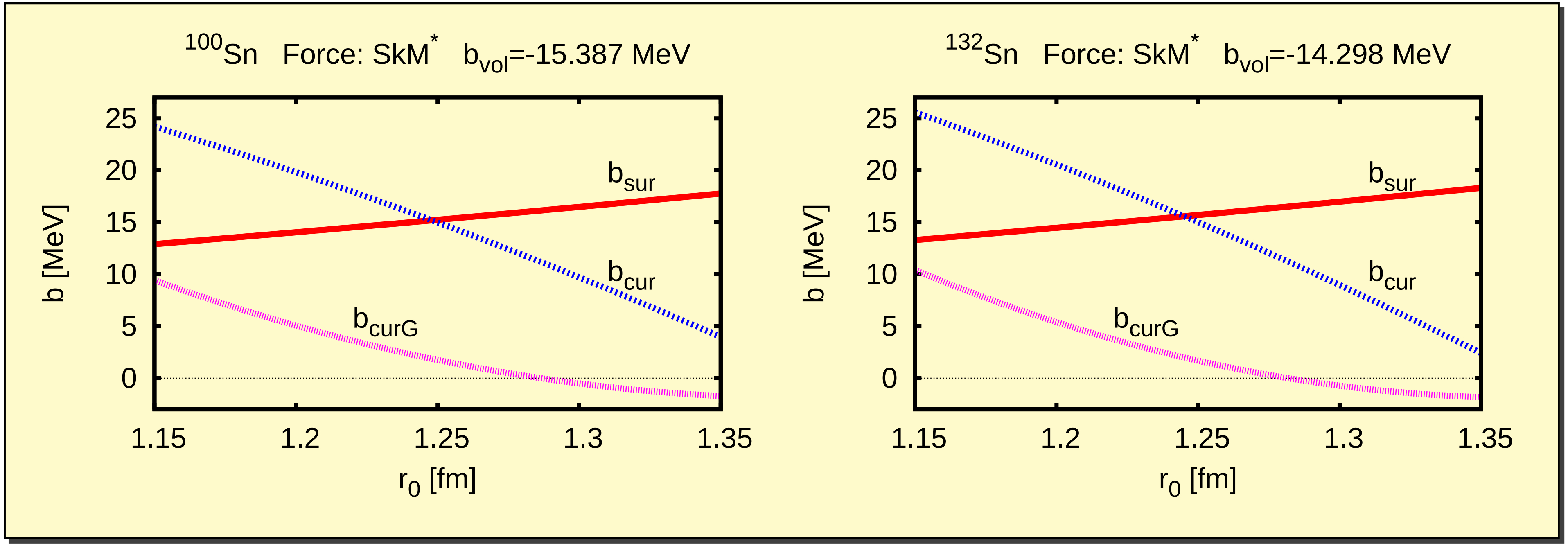
$$\eta = - \int \int \dots \int \Psi^\star \hat{H} \Psi \, d^3 r_2 \cdots d^3 r_A \ .$$

The total binding energy of the nucleus is:

$$B = \int_V \eta \, dV = \int_V [\eta - b_{\rm vol}(\rho - \rho)] \, dV = b_{\rm vol} A + \int_V [\eta - b_{\rm vol} \rho] \, dV$$



Leptodermous expansion of the ETF-Skyrme energy:



$$B = \int_{\mathrm{V}} \eta \, d^3\mathrm{r} = b_{\mathrm{vol}} A + b_{\mathrm{surf}} A^{2/3} + b_{\mathrm{curv}} A^{1/3} + b_{\mathrm{curG}} A^0 + \ldots$$

Note! The magnitudes of the surface and curvature terms depend on the choice of the expansion radius $R = r_0 A^{1/3}$.

Macroscopic – microscopic model*:

$$
\begin{aligned}
M(Z,N;\mathrm{def}) \;&= Z M_{\mathrm{H}} + N M_{\mathrm{n}} - b_{\mathrm{elec}}\, Z^{2.39} \\
\text{volume} \qquad &+ b_{\mathrm{vol}}\,(1 - \kappa_{\mathrm{vol}}\, I^2)\, A \\
\text{surface} \qquad &+ b_{\mathrm{surf}}\,(1 - \kappa_{\mathrm{surf}} I^2)\, A^{2/3} B_{\mathrm{surf}}(\mathrm{def}) \\
\text{curvature} \qquad &+ b_{\mathrm{cur}}\,(1 - \kappa_{\mathrm{cur}}\, I^2)\, A^{1/3} B_{\mathrm{cur}}(\mathrm{def}) \\
\text{Coulomb} \qquad &+ \frac{3}{5}\frac{e^2 Z^2}{r_0^{ch} A^{1/3}} B_{\mathrm{Coul}}(\mathrm{def}) - C_4 \frac{Z^2}{A} \\
&+ E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)
\end{aligned}
$$

Here $I = (N-Z)/A$ is the reduced isospin and

$$E_{\mathrm{micr}}(Z,N;\mathrm{def}) = E_{\mathrm{shell}}(Z,N;\mathrm{def}) + E_{\mathrm{pair}}(Z,N;\mathrm{def})$$

while $E_{\mathrm{cong}} = -10\exp(-4.2|I|)$ MeV is the Wigner energy.

*W.D. Myers and W.J. Świątecki, Nucl. Phys. **81**, 1 1966,

LSD → K. Pomorski and J. Dudek, Phys. Rev. C **67** (2003) 044316.

Lublin Strasbourg Drop *

Fit to the 2766 experimental masses† with $Z \geq 8$ and $N \geq 8$:

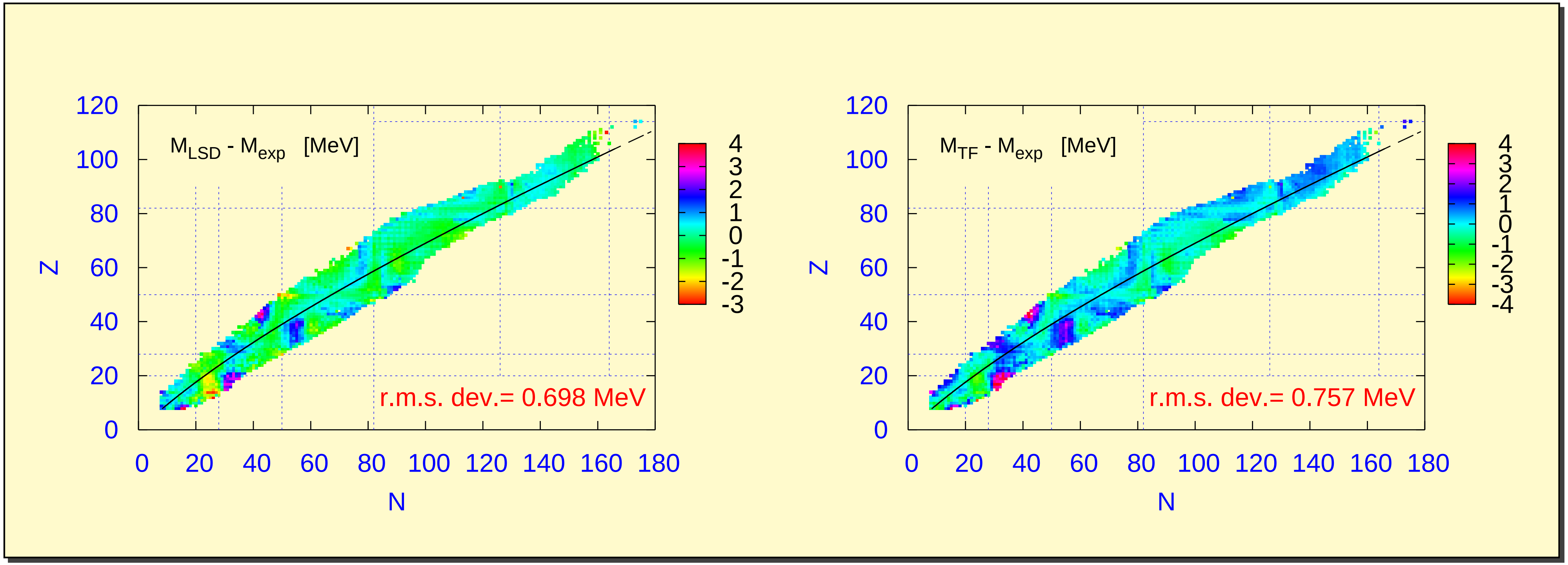
Term	Units	LDM	LSD
b_{vol}	MeV	-15.8484	-15.4920
κ_{vol}	-	1.8475	1.8601
b_{surf}	MeV	19.3859	16.9707
κ_{surf}	-	1.9830	2.2938
b_{cur}	MeV	0	3.8602
κ_{cur}	-	0	-2.3764
r_0	fm	1.18995	1.21725
C_4	MeV	1.1995	0.9181
δM	MeV	0.732	0.698
$\delta V_{B_{Z>70}}$	MeV	5.58	0.88

M_{H}=7.289034 MeV; M_{n}=8.071431 MeV; b_{elec}=1.433 eV

*K. Pomorski , J. Dudek, Phys. Rev. **C67**, 044316 (2003).

†Chart of Nuclides by M.S. Antony, Strasbourg, 2002.

Masses of isotopes:



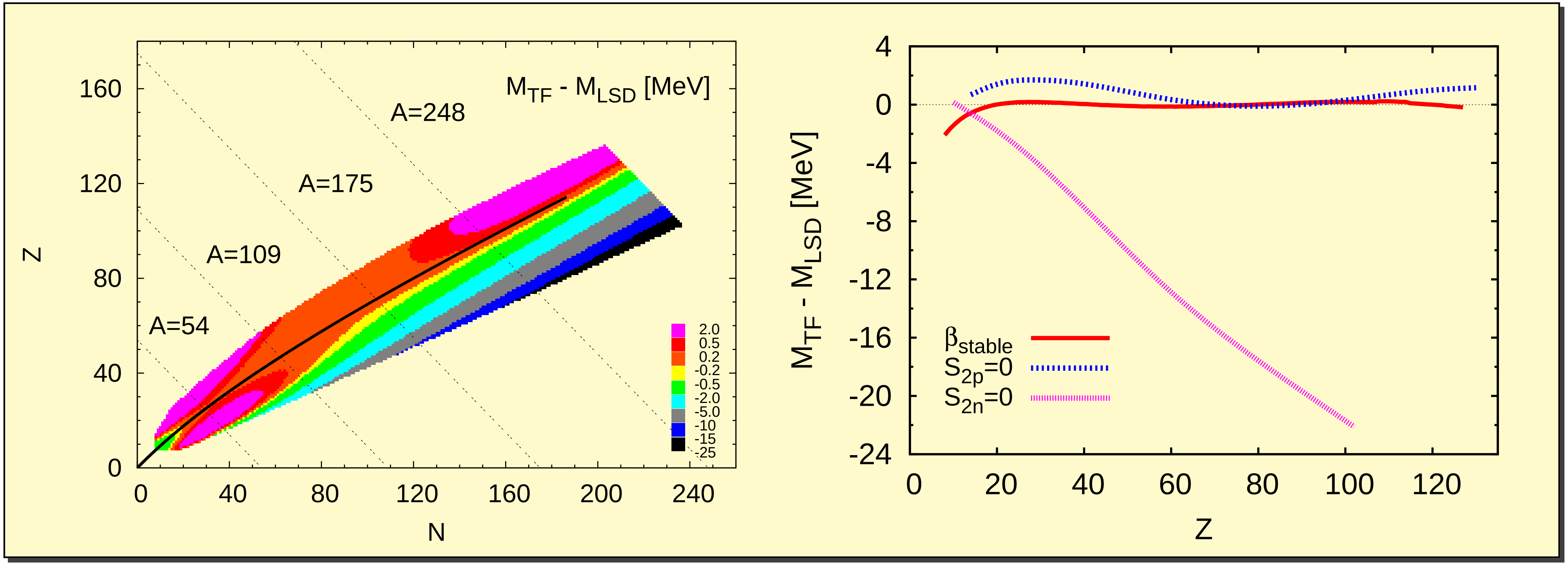
exp: The mass deviations are plotted for 2766 isotopes taken from the

. Chart of Nuclides by M.S. Antony, Strasbourg, 2002.

LSD: K. Pomorski, J. Dudek, Phys. Rev. **C67**, 044316 (2003).

TF: P. Möller, J.R. Nix, W.D. Myers, W.J. Świątecki, ADNDT **59**, 185 (1995).

Masses of nuclei far from stability:



LSD: K. Pomorski, J. Dudek, Phys. Rev. **C67**, 044316 (2003).

TF: P. Möller, J.R. Nix, W.D. Myers, W.J. Świątecki, ADNDT **59**, 185 (1995).

Isospin square dependent LD formula *

$$E_B = \left[b_{\rm vol}A + a_{\rm surf}A^{2/3} + a_{\rm cur}A^{1/3}\right]\left[1 - \kappa\frac{|N-Z|(|N-Z|+2)}{A^2}\right] + a_{\rm Coul}\frac{Z(Z-1)}{A^{1/3}} \pm \frac{\delta}{\sqrt{A}}$$

It is to be noted that volume, surface and curvature terms carry the same isospin parameter k. The last term decribes the odd-even mass difference.

Note that the **Wigner term** is already contained in the LD energy.

* L.G. Moretto, P.T. Lake, L. Phair, J.B. Elliott, Phys. Rev. **C86** (2012) 021303(R).

Four different parameter sets fitted by Moretto et al.*

	a_{vol} [MeV]	a_{surf} [MeV]	a_{cur} [MeV]	κ	a_{Coul} [MeV]	δ [MeV]	r.m.s. [MeV]
M_{i}	-15.597	17.32	0.0	1.8048	0.7060	11.4	0.76
M_{ii}	-14.843	14.843	0.0	1.7196	0.6585	10.1	2.06
M_{iii}	-15.25	15.17	3.8	1.779	0.6932	11.3	0.73
M_{iv}	-15.264	15.264	3.6	1.7805	0.6938	11.3	0.73

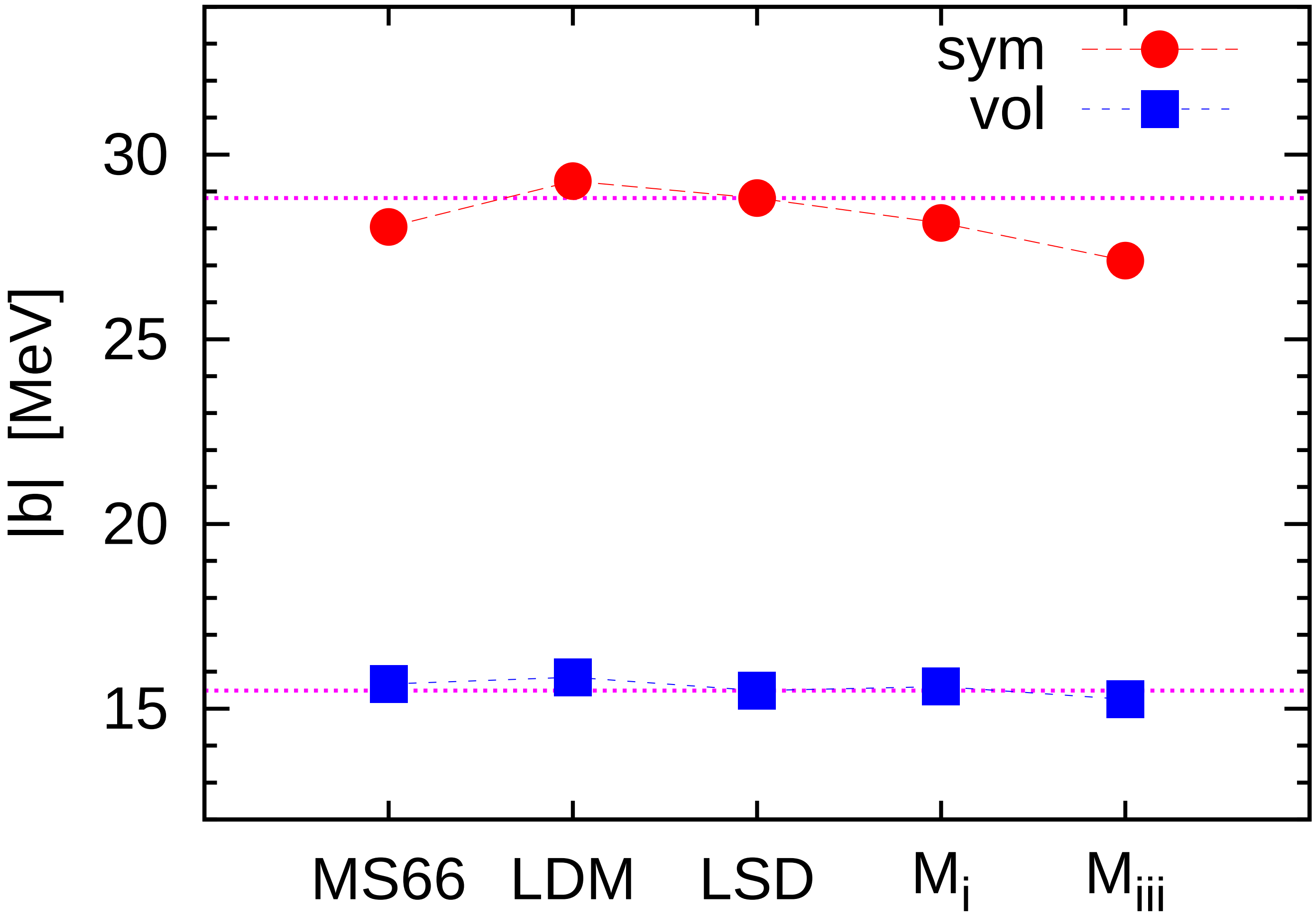
The fit was performed using a set of 2076 masses**, corrected for the shell and deformation effects according to Moller et al.***

* L.G. Moretto, P.T. Lake, L. Phair, J.B. Elliott, Phys. Rev. **C86** (2012) 021303(R).

** G. Audi, O. Bersillon, J. Blachot, A. H. Wapstra, Nucl. Phys. **A729** (2003) 3.

*** P. Moller, J. R. Nix, W. D. Myers, W. J. Swiatecki, ADNDT **59** (1995) 185.

Volume and symmetry energy in different LD models



MS66: W.D. Myers and W.J. Swiatecki, Nucl. Phys. **81** (1966) 1.

LDM,LSD: K. Pomorski and J. Dudek, Phys. Rev. C **67** (2003) 044316.

$\mathbf{M_i, M_{iii}}$: L.G. Moretto et al., Phys. Rev. **C86** (2012) 021303(R).

Wigner term in Swiatecki and Moretto approaches

It is interesting to compare the Wigner/congruence energy term in the models considered above:

MS66: $-7\exp(-6|I|)$ MeV $\longrightarrow \sim 42 \cdot |I|$ MeV

MNNS: $-10\exp(-4.2|I|)$ MeV $\longrightarrow \sim 42 \cdot |I|$ MeV

M-i: $-2\kappa\left(-15.594 + \frac{17.32}{A^{1/3}}\right)|I|$ MeV $\longrightarrow$ $40.7 \cdot |I|$ MeV for $A = 64$

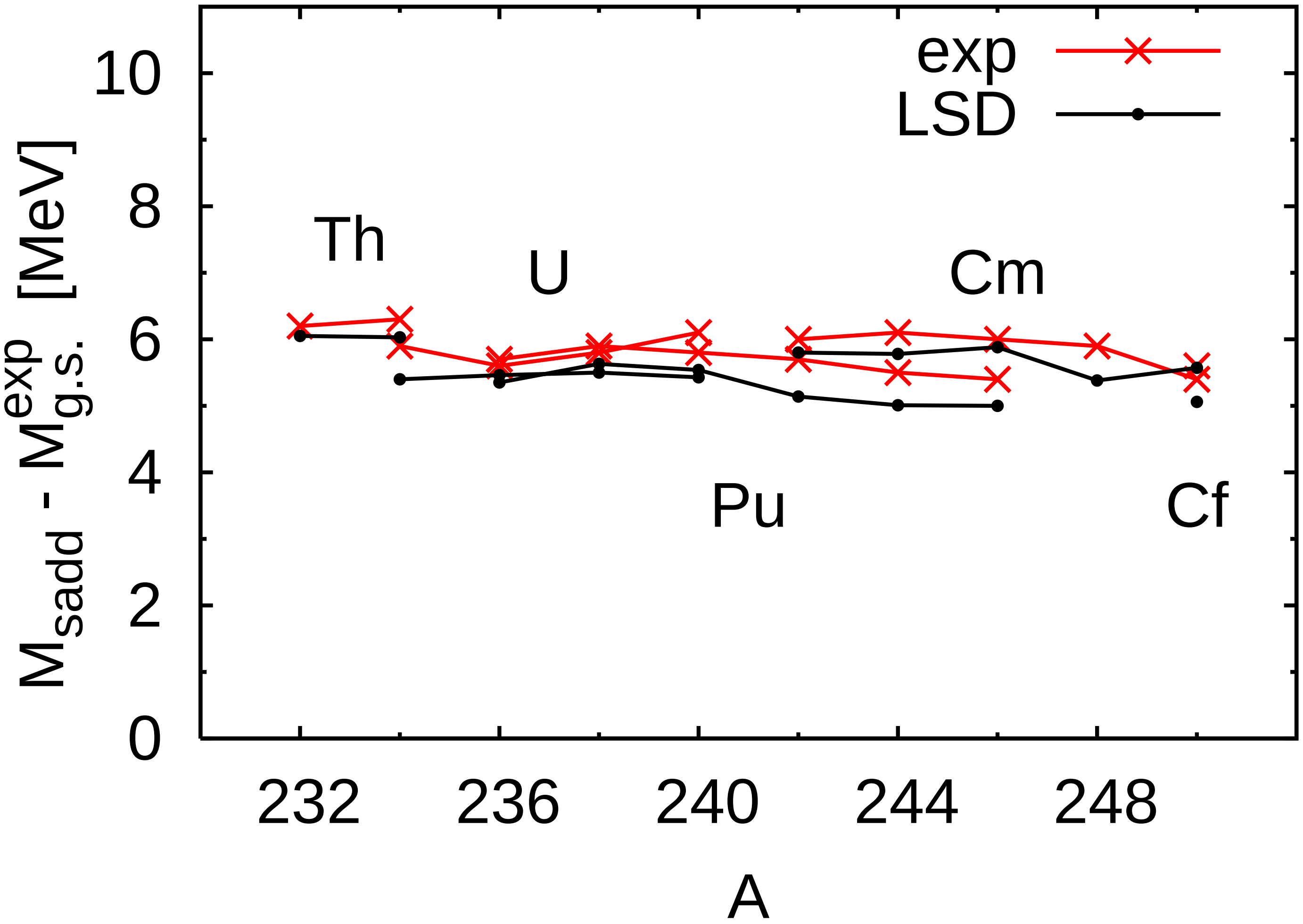
$\longrightarrow$ $45.9 \cdot |I|$ MeV for $A = 216$

Contrary to the exponential, the linear dependent on I Wigner term does not vanish for large values of isospin. It will influence significantly the binding energy predictions for the heavy neutron reach nuclei.

MS66: W.D. Myers and W.J. Swiatecki, Nucl. Phys. **81** (1966) 1.
MNMS: P. Moller, J. R. Nix, W. D. Myers, W. J. Swiatecki, ADNDT **59** (1995) 185.
M-i : L.G. Moretto, P.T. Lake, L. Phair, J.B. Elliott, Phys. Rev. **C86** (2012) 021303(R).


Topographical theorem of Swiatecki

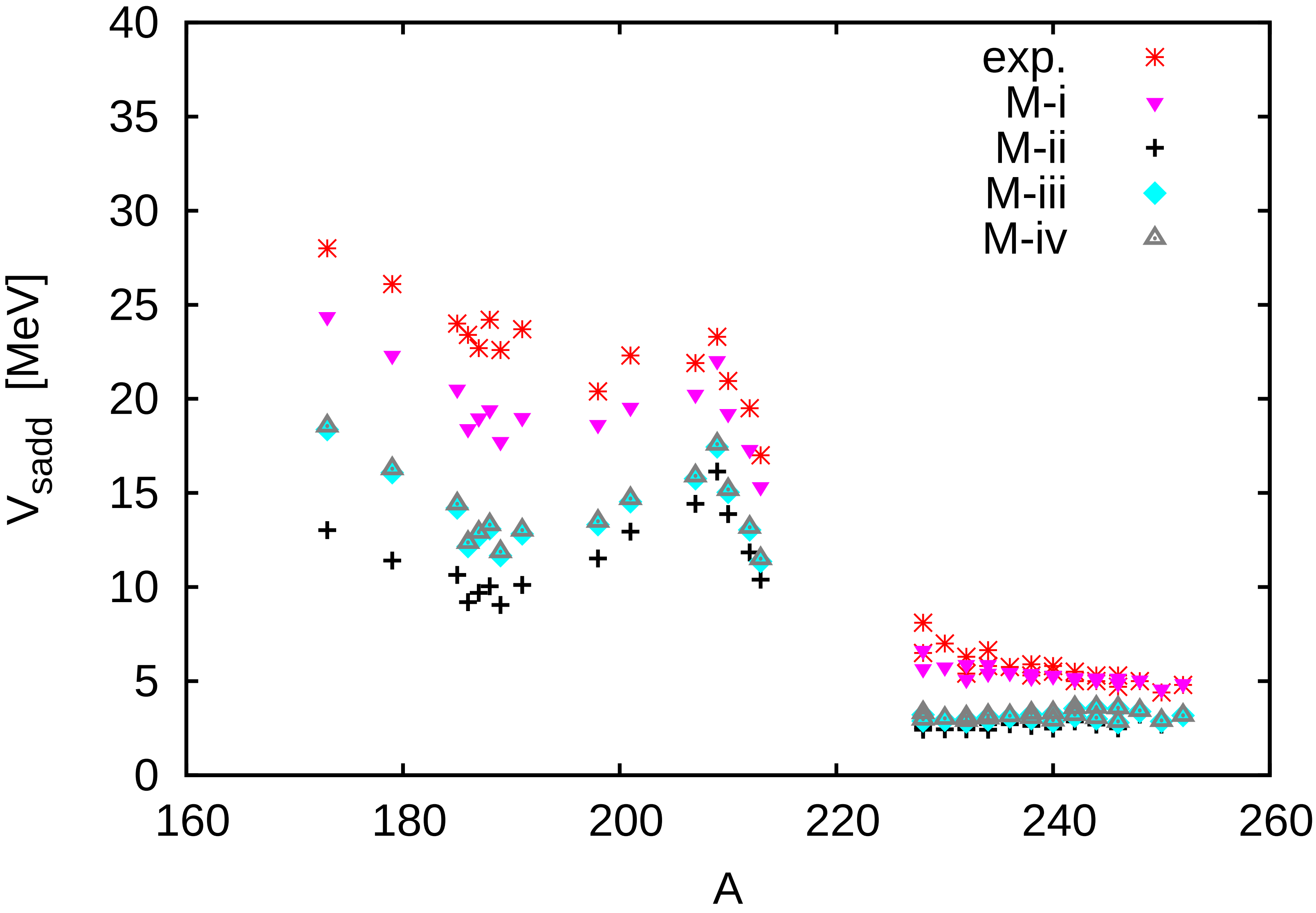
LSD barrier heights obtained using the topographical theorem of Swiatecki*



* W.D. Myers, W.J. Swiatecki, Nucl. Phys. **A601**, 141 (1996).

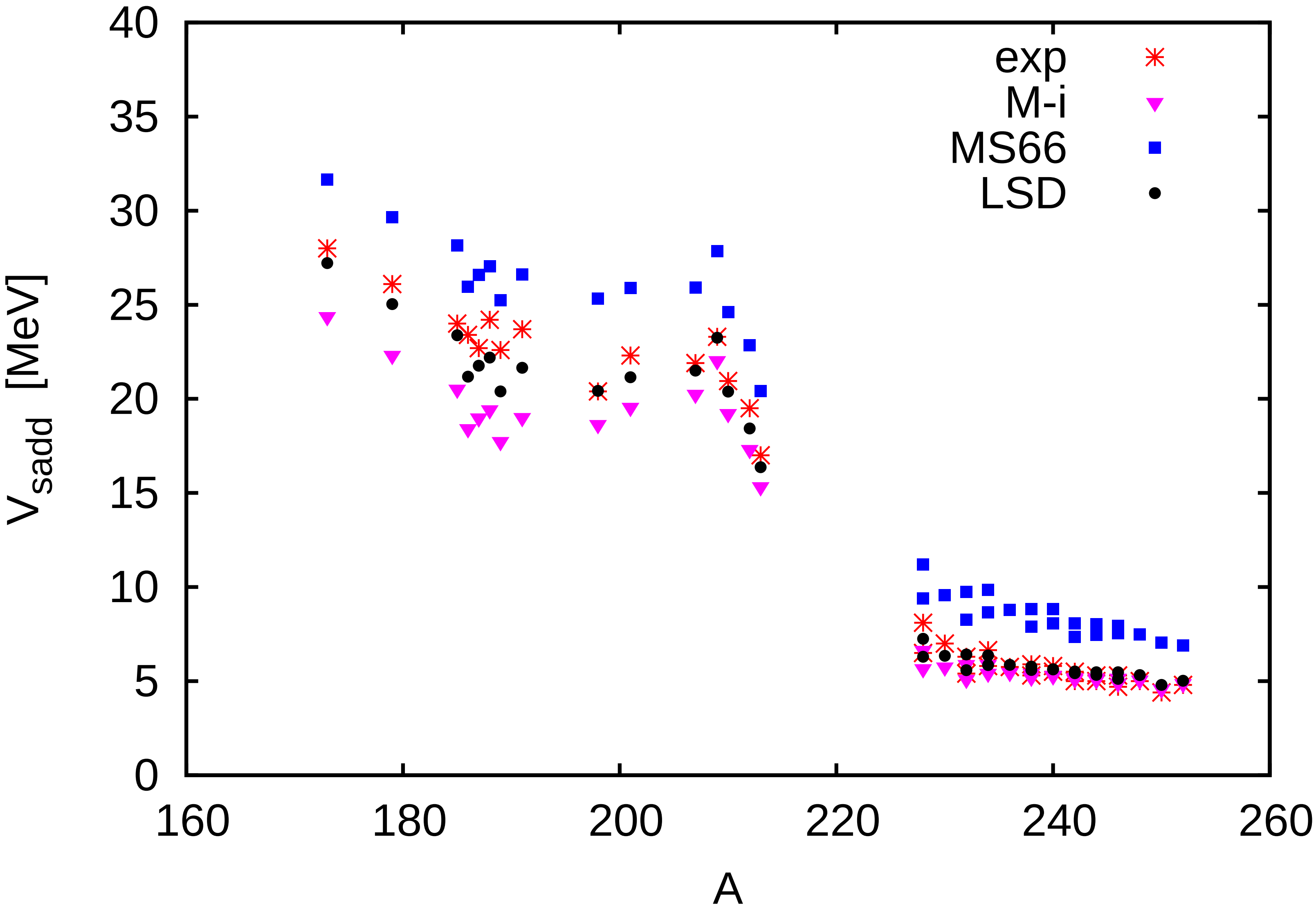
A. Dobrowolski, B. Nerlo-Pomorska, K. Pomorski, Acta Phys. Pol. **B40**, 705 (2009).

Fission barrier heights in the Moretto et al. models*



* K. Pomorski, Phys. Scr. **T154** (2013) 014023.

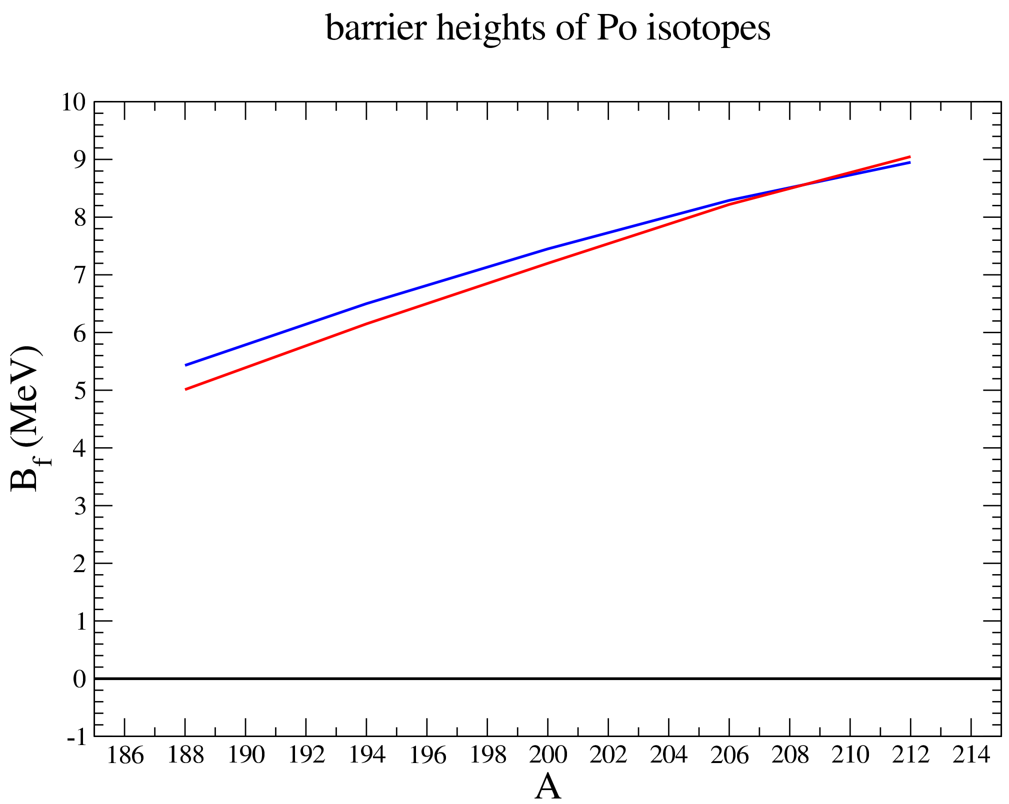
Fission barrier heights in diferent LD models*

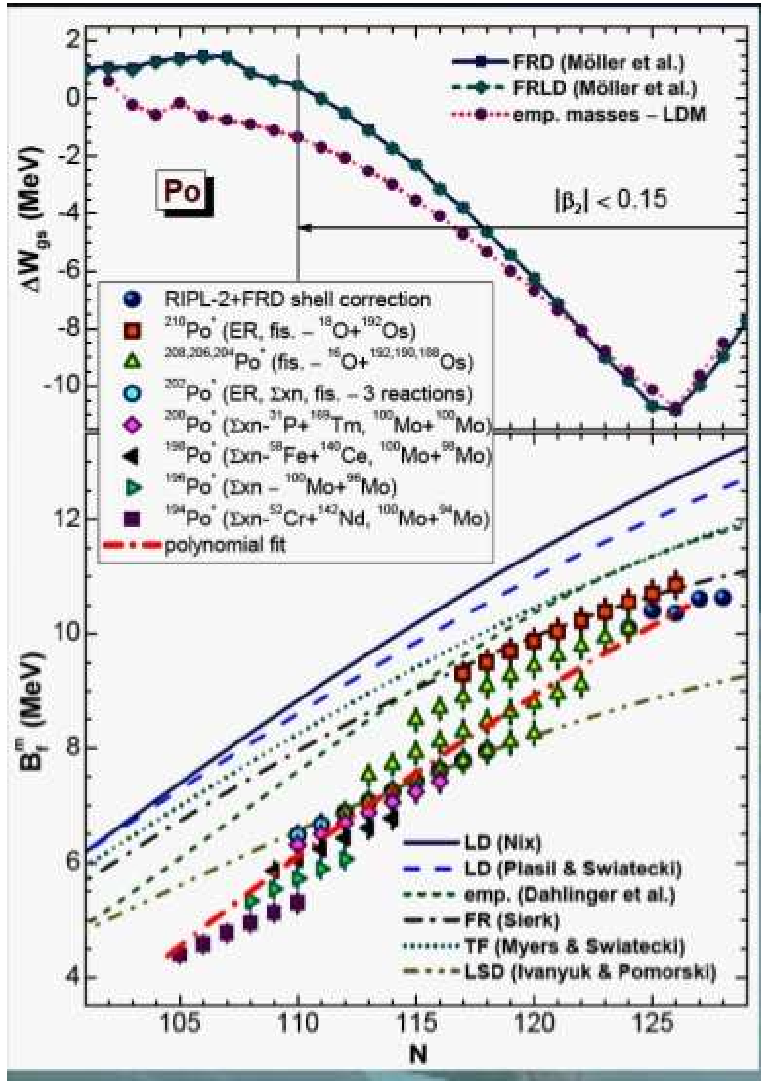


* K. Pomorski, Phys. Scr. **T154** (2013) 014023.

Fission barrier heights of Po isotopes in LSD and M_i *

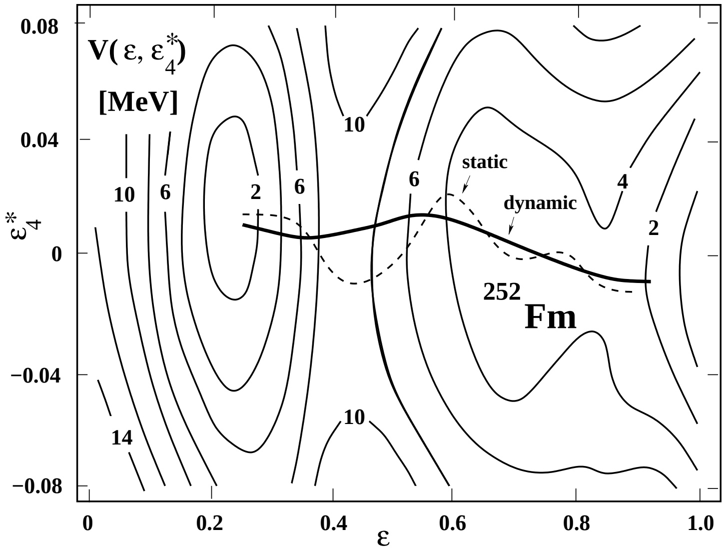
Sagaidak and Andreyev has made the analysis of the barrier heights shown in the neighbouring figure taken from [Phys. Rev. **C79** (2009)

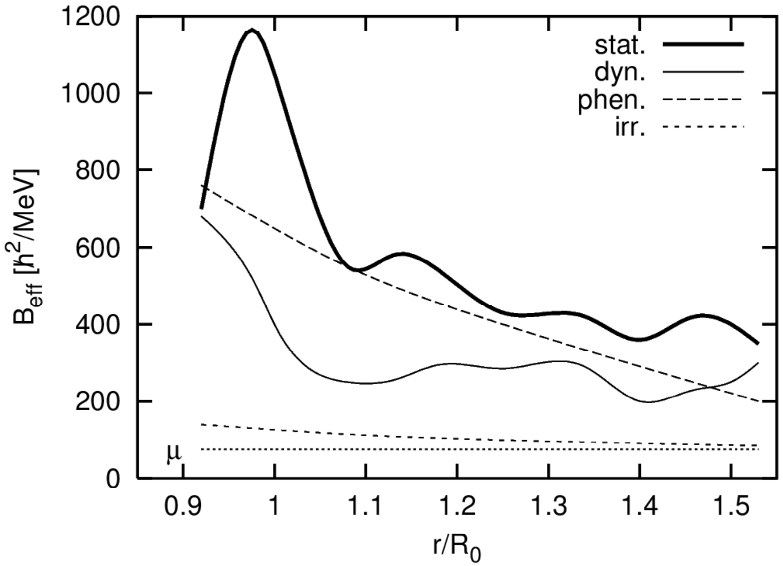




* J. Bartel, B. Nerlo-Pomorska, K. Pomorski, C. Schmitt, Phys. Scr. (2015) in print.

Spontaneous fission life-times systematics



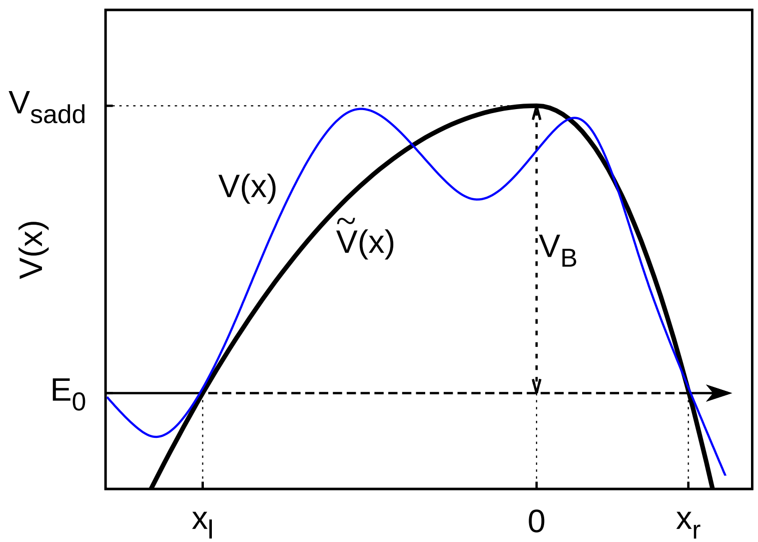


The transformation:

$$x(s) = \int_{s_{\rm sadd}}^{s} \sqrt{\frac{B_{ss}(s')}{m}}\, ds' \ ,$$

ansures that $B_{xx} = m$=const.
The potential $V[s(x)]$ in the new coordinate x can be approximated by:

$$\tilde{V}(x) = \begin{cases} V_{\rm sadd} - \frac{1}{2}\, C_l\, x^2 & \text{for } x < 0 \ , \\ V_{\rm sadd} - \frac{1}{2}\, C_r\, x^2 & \text{for } x > 0 \ , \end{cases}$$



WKB approximation

The spontaneous fission half–life is given by:

$$T^{\text{sf}}_{1/2} = \frac{\ln 2}{nP} \; , \quad \text{where} \quad P = \frac{1}{1 + \exp\{2S(L)\}.}$$

The WKB action-integral along the fission path $L(x)$ is given by:

$$S(L) = \int_{s_l}^{s_r} \sqrt{\frac{2}{\hbar^2} B_{ss}[V(s) - E_0]ds} \approx \int_{-x_l}^{x_r} \sqrt{\frac{2m}{\hbar^2}[\widetilde{V}(x) - E_0]} \, dx$$

After a small algebra the action integral becomes

$$S = \frac{\pi}{2\hbar} V_B \left(\sqrt{\frac{m}{C_l}} + \sqrt{\frac{m}{C_r}} \right) = \frac{\pi}{\hbar} V_B \frac{\omega_l + \omega_r}{2\,\omega_l\,\omega_r} = \frac{\pi}{\hbar} V_B \, \tilde{\omega} \; ,$$

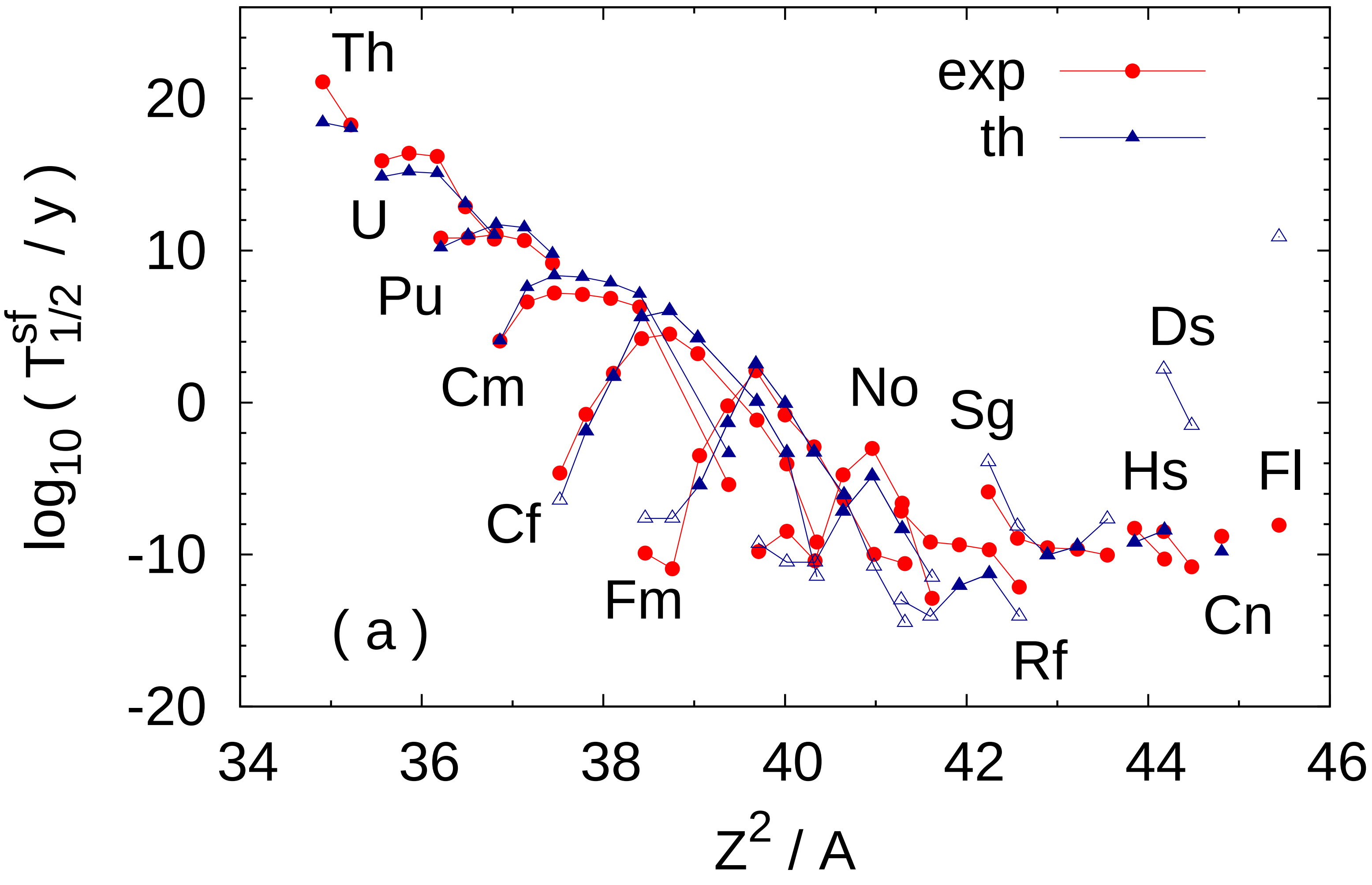
where $\omega_l = \sqrt{C_l/m}$ and $\omega_r = \sqrt{C_r/m}$ the inveted H.O. frequencies.

For $S > 1$ the logarithm of the s.f. half-lives takes the form:

$$\log(T^{\text{sf}}_{1/2}) = \frac{2\pi}{\hbar} V_B \, \tilde{\omega} - \log(n) - \log[\ln 2] \; ,$$

where n is the frequency of assaults against the fission barrier.

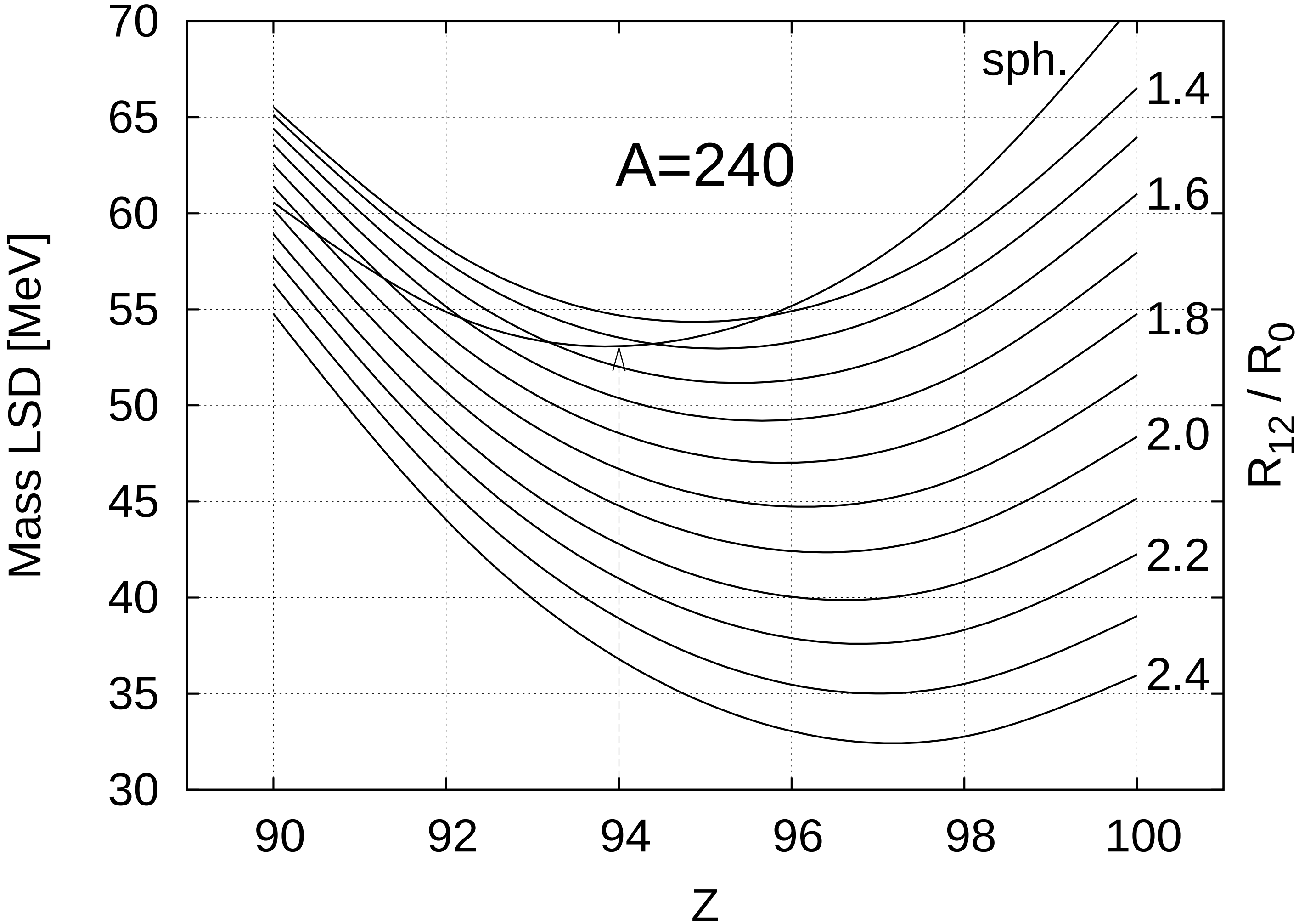
Spontaneous fission halve-lives for e-e isotopes*



We have assumed here the linear dependence of $\tilde{\omega}$ in Z.

K. Pomorski, M. Warda and A. Zdeb, Phys. Scr. in print; **arXiv:1501.03912v1**.
Confer also: W.J. Swiatecki, Phys. Rev. **100**, 937 (1955).

Masses of A=240 nuclei at different deformations



Very deformed stable nuclei becomes β-unstable! Read more in:

K.Pomorski, B. Nerlo=Pomorska and P. Quentin, Phys. Rev. C **91**, 054605 (2015).

Conclusions:

- The liquid drop like phenomenological models with realistic microscopic corrections are able to reproduce the binding energies of know isotopes with accuracy of the order of 0.7 MeV.
- The fitted in volume symmetry energy varies from 27 to 30 MeV dependent on the LD model.
- The good accuracy in reproduction of the ground state binding energies does not guarantee that the fission barrier heights are also reproduces.
- The different isospin dependence of the Wigner energy in considered models will influence significantly predictions of masses for the heavy neutron reach nuclei.
- Experimental study of the super and hyper-deformed shape isomers could bring important informations on the surface energy symmetry term.
- Systematics of the fission barrier heights and the $T_{1/2}^{\rm sf}$ confirms in addition the right isospin dependence of the LSD terms.

Thank you for your attention!

Photo Mariusz Cieszewski, https://www.flickr.com/photos/polandmfa/

XXII NUCLEAR PHYSICS WORKSHOP "MARIE & PIERRE CURIE"

22 – 27 September 2015, Kazimierz Dolny, Poland

ESSENTIAL PROBLEMS IN NUCLEAR PHYSICS

TOPICS

Nuclear decays, fission, fusion
Large-scale collective nuclear motion
Nuclei far from stability
Nuclear models
Symmetries in nuclei
Nuclear structure
Novel nuclear excitations
Gamma spectroscopy

ADVISORY BOARD

Sven Åberg, LTH Lund, Sweden
Anatoli Afanasjev, Mississippi State University, USA
Jacek Dobaczewski, Warsaw University, Poland
Gianluca Colò, INFN Milan, Italy
Jan Kvasil, Charles University, Prague, Czech Republic
Adam Maj, IFJ-PAN Kraków, Poland
Jie Meng, Peking University, China
Valentin Nesterenko, JINR Dubna, Russia
Wojciech Satuła, Warsaw University, Poland
Janusz Skalski, NCBJ Świerk, Poland
Adam Sobiczewski, NCBJ Świerk, Poland
Xavier Viñas, University of Barcelona, Spain

ORGANIZING COMMITTEE

Michał Warda
Andrzej Baran
Johann Bartel
Artur Dobrowolski
Jerzy Dudek
Andrzej Góźdź
Marek Góźdź
Jerzy Kraśkiewicz
Rafał Księżopolski
Aleksandra Pędrak
Krzysztof Pomorski
Agnieszka Szulerecka
Anna Wielgus
Krystyna Zając
Anna Zdeb

UMCS
MARIA CURIE-SKLODOWSKA UNIVERSITY IN LUBLIN
KATEDRA FIZYKI TEORETYCZNEJ UMCS LUBLIN

http://kft.umcs.lublin.pl/wfj/
e-mail: wfj@kft.umcs.lublin.pl