Nusym15, Cracow, Poland
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SINAP

Spin dynamics in intermediate-energy heavy-ion collisions

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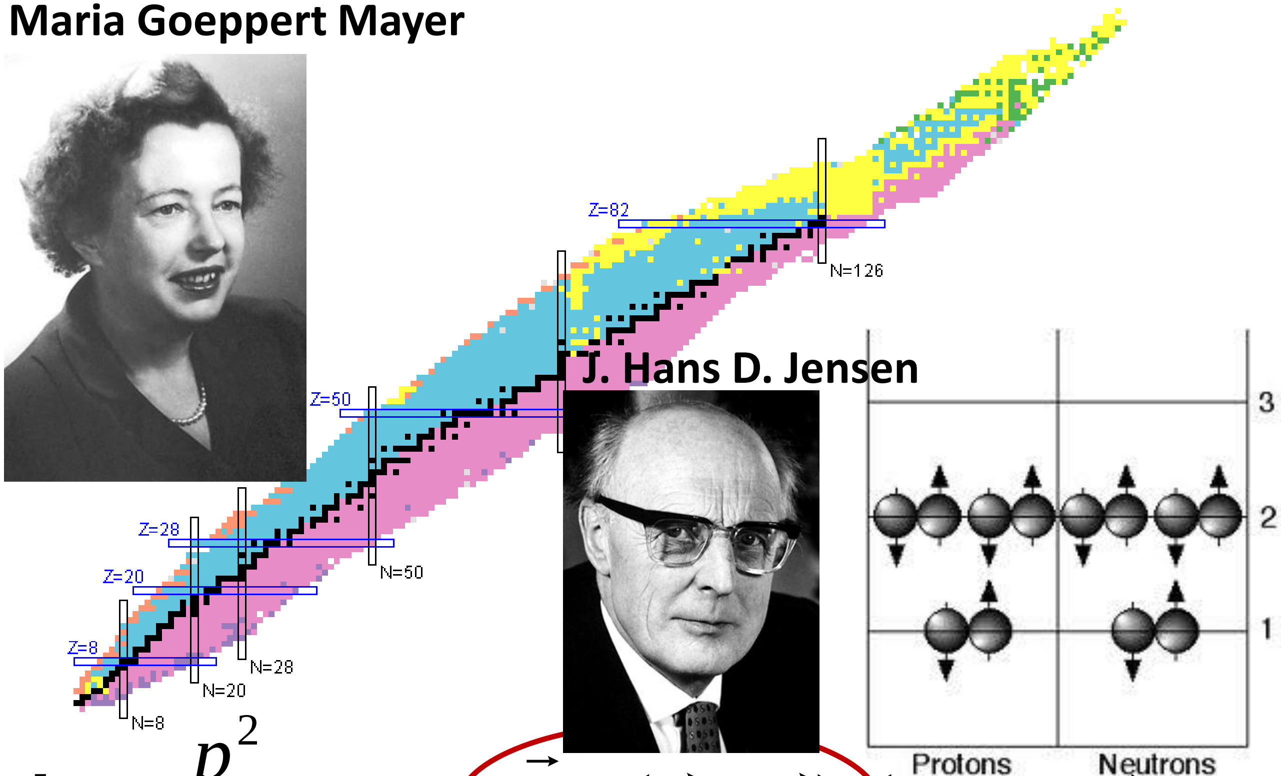
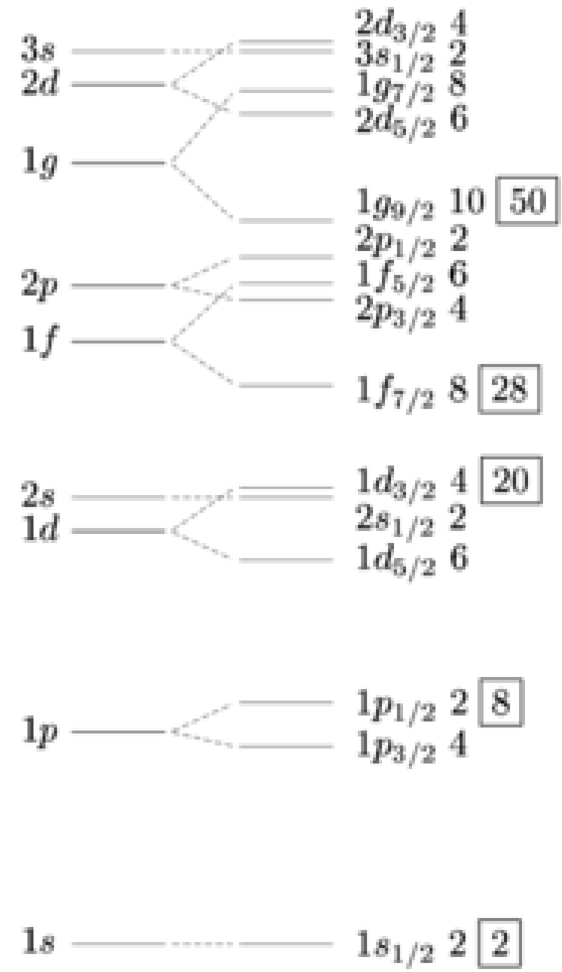
Spin-orbit potential and magic number

Maria Goeppert Mayer





$$h_q = \frac{p^2}{2m} + U_q + \vec{W}_q \cdot (\vec{p} \times \vec{\sigma}), (q = n, p)$$

Schrödinger equation: $h_q \varphi_q = e_q \varphi_q$

The spin-orbit (SO) potential $\vec{W}_q \cdot (\vec{p} \times \vec{\sigma})$ helps to explain the magic number and shell structure.

From in-medium spin-orbit nuclear interaction

- Skyrme-Hartree-Fock model

$$V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}'$$

Hartree-Fock method

$$\vec{W}_q = \frac{W_0}{2}\left(\nabla\rho + \nabla\rho_q\right)$$

- Relativistic mean field model

Dirac equation

Non-relativistic expansion

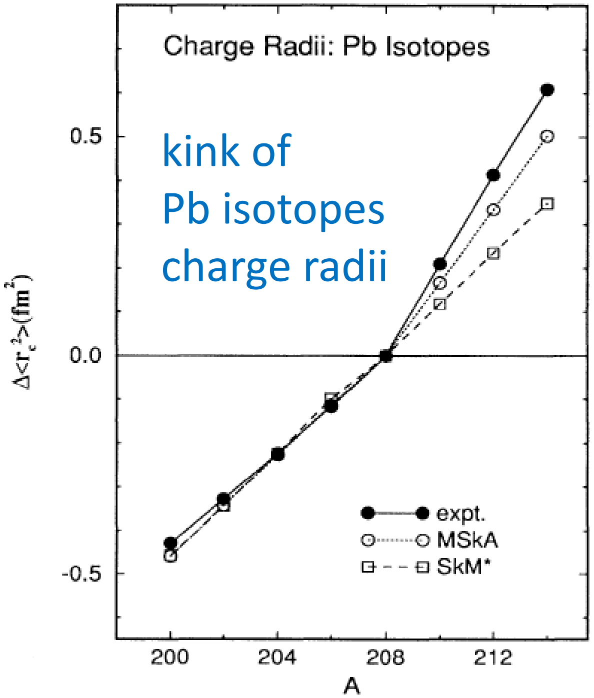
$$\vec{W}_q = \frac{C}{(2m - C\rho)^2}\nabla\rho, C = \frac{g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2}$$

P. G. Reinhard and H. Flocard, Nucl. Phys. A, 1995

the isospin dependence of the SO potential

$$\vec{W}_q = \frac{W_0}{2}(1 + \chi_w)\nabla\rho_q + \frac{W_0}{2}\nabla\rho_{q'} . (q \neq q')$$

M. M. Sharma *et al.*, Phys. Rev. Lett., 1995



the density dependence of the SO potential

$$v_{ij} = v_{ij}^0 + (i/\hbar^2)W_1(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot \mathbf{p}_{ij} \times (\rho_{q_i} + \rho_{q_j})^\gamma \delta(\mathbf{r}_{ij})\mathbf{p}_{ij}.$$

$$\vec{W}_q = \frac{W_0}{2}\nabla(\rho+\rho_q) + \frac{W_1}{2}[(\rho)^\gamma \nabla(\rho-\rho_q) + (2+\gamma)(2\rho_q)^\gamma \nabla\rho_q] + \frac{W_1}{4}\gamma\rho^{\gamma-1}(\rho-\rho_q)\nabla\rho$$

W_1 and γ fitted to reproduce the density dependence of the SO potential from the RMF model

⇓

Similar spin-orbit field in semi-infinite nuclear matter

J. M. Pearson and M. Farine, Phys. Rev. C 50, 185 (1994).

Generally $$\vec{W}_q = W_0 \left(\frac{\rho}{\rho_0}\right)^\gamma \left(a\nabla\rho_q + b\nabla\rho_{q'}\right) \qquad (q \neq q')$$

density dependence — isospin dependence

W_0= 80 ~ 150 MeVfm^5, γ, a, and b still under debate

T. Lesinski *et al.,* Phys. Rev. C 76, 014312 (2007).
M. Zalewski *et al.,* Phys. Rev. C 77, 024316 (2008).
M. Bender *et al.,* Phys. Rev. C 80, 064302 (2009).

The spin-orbit interaction may affect

1) Properties of drip-line nuclei

G. A. Lalazissis *et al.*, Phys. Rev. Lett., 1998

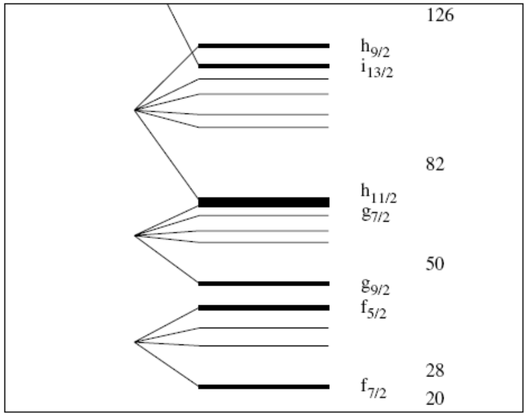
2) Astrophysical r-process

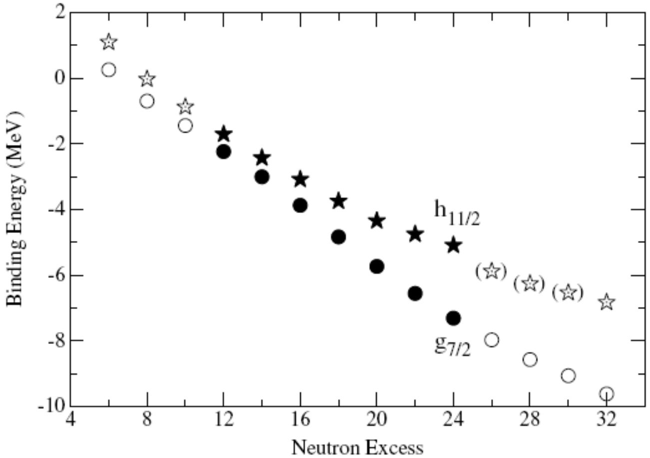
B. Chen *et al.*, Phys. Lett. B, 1995

3) Location of SHE

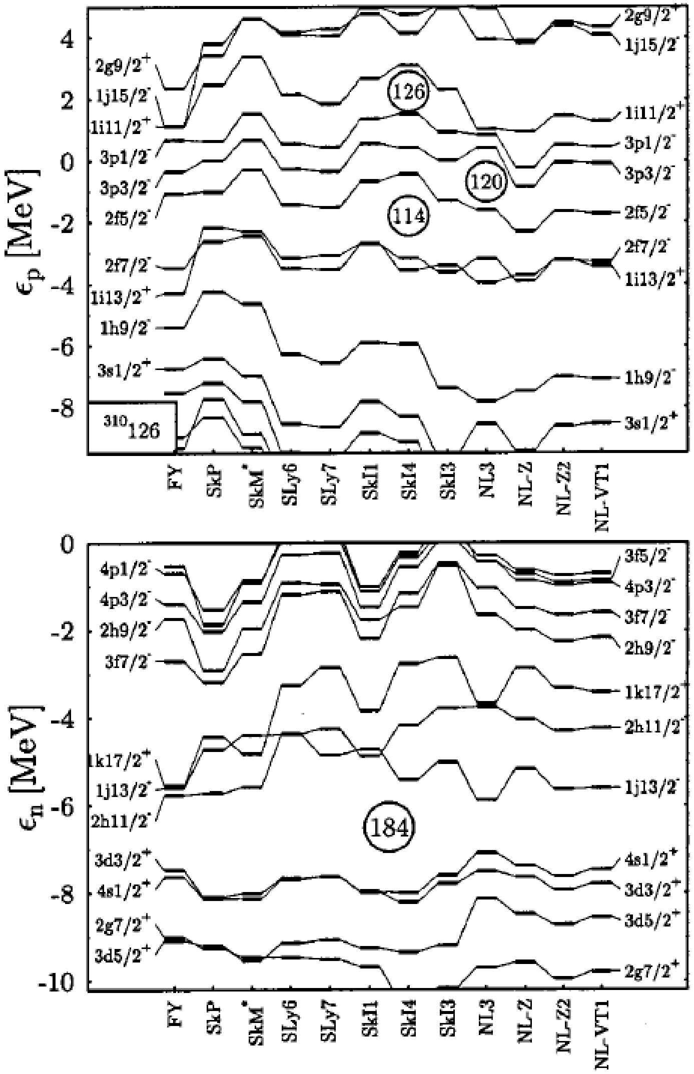
M. Morjean *et al.*, Phys. Rev. Lett., 2008

4) …





J.P. Schiffer *et al.*, Phys. Rev. Lett., 2004



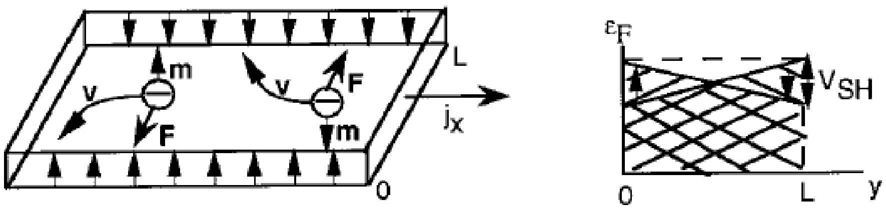
Shell structure of $^{310}_{184}126$.

M. Bender *et al.*, Phys. Rev. C, 1999

A hot topic in the studies of nuclear structure!

Spin-orbit potential at low- and high-energy HIC

Spin Hall effect

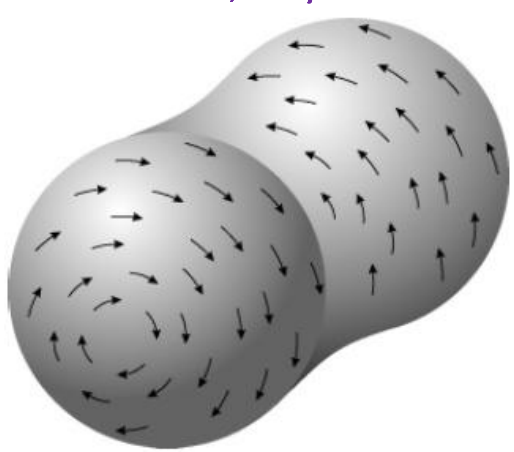


low energies (TDHF):

TABLE I. Thresholds for the inelastic scattering of $^{16}O + ^{16}O$ system.

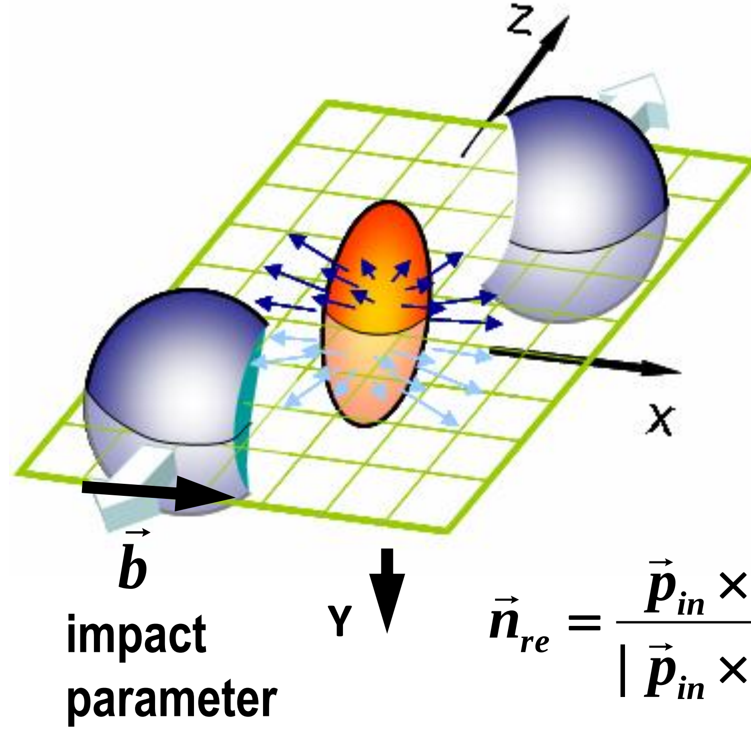
Force	Skyrme II (MeV)	Skyrme M* (MeV)
Spin orbit	68	70
No spin orbit	31	27

A. S. Umar *et al.*, Phys. Rev. Lett., 1986



J. A. Maruhn *et al.*, Phys. Rev. C, 2006

high energies:



$$\vec{n}_{re} = \frac{\vec{p}_{in} \times \vec{b}}{|\vec{p}_{in} \times \vec{b}|}$$

perpendicular to the reaction plane

Z. T. Liang and X. N. Wang,
Phys. Rev. Lett., 2005
Phys. Lett. B, 2005

Spin related experiments

Spin-polarized beam
at RIKEN, GSI, NSCL, GANIL
can be produced with
pick-up or removal reactions.

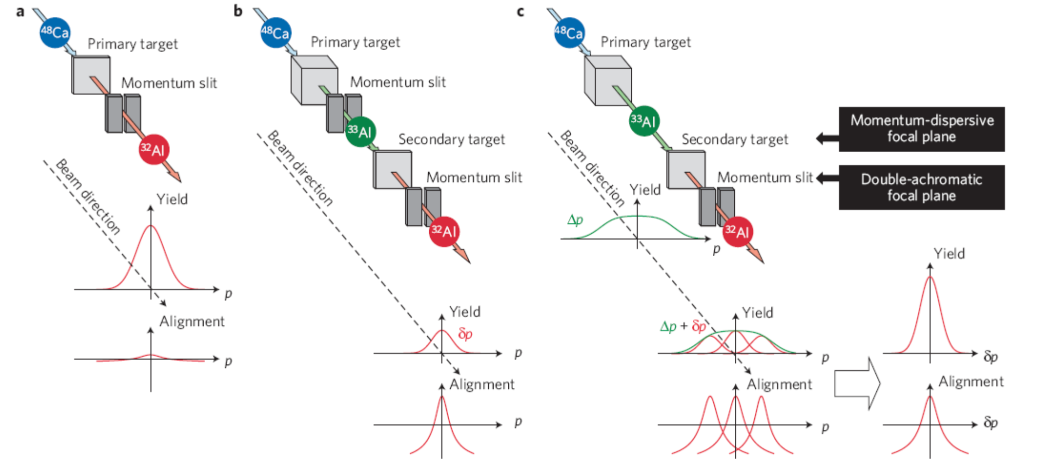
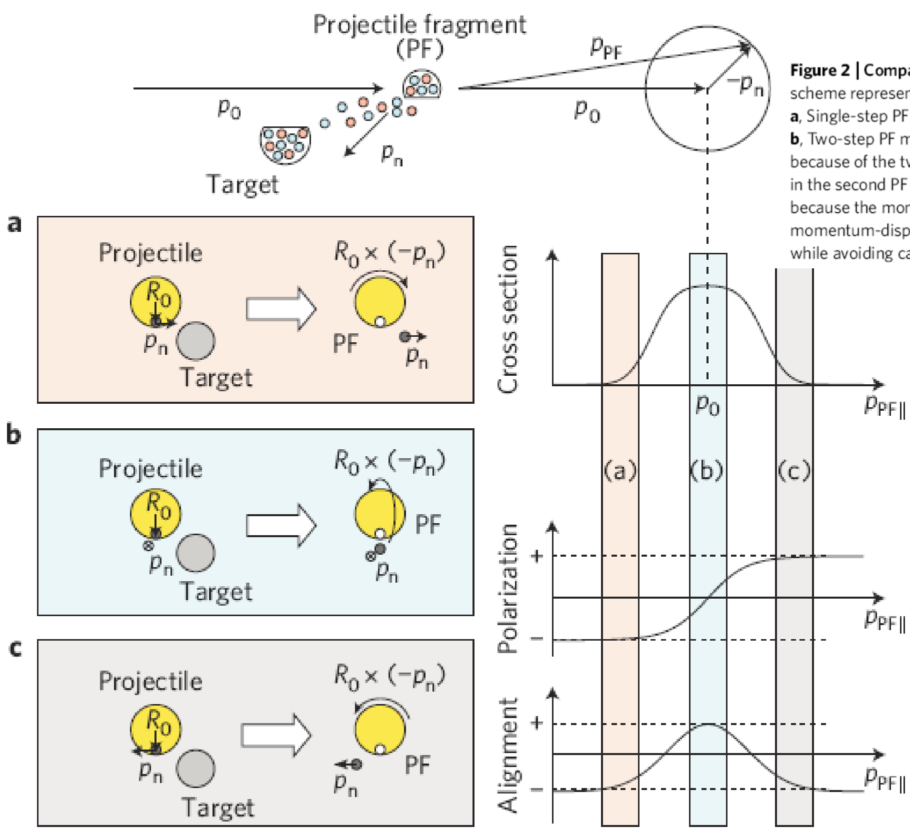


Figure 2 | Comparison of three schemes for producing a spin-aligned rare-isotopebeam of ^{32}Al from a primary beam of ^{48}Ca. The graphs below each scheme represent the typical momentum distribution and the corresponding alignment, with abscissas representing the momentum p of ^{32}Al. **a**, Single-step PF method. The ^{32}Al beam is directly produced from ^{48}Ca. As PF involves a large number of nucleons, the expected spin alignment is small. **b**, Two-step PF method. ^{32}Al is produced via an intermediate nucleus ^{33}Al. The expected spin alignment is high, whereas the production yield is low because of the two-fold selection with momentum slits. **c**, Two-step PF method with dispersion matching. Direct selection of the change in momentum δp in the second PF can be achieved by placing a secondary target in the momentum-dispersive focal plane and a slit in the double-achromatic focal plane, because the momentum spread Δp of the incident beam is compensated for by fulfilling the condition of momentum-dispersion matching. The effect of momentum-dispersion matching is represented by graphs connected by a broad arrow. This method yields an intense spin-aligned rare-isotope beam while avoiding cancellation between the opposite signs of spin alignment caused by the momentum spread Δp.



The spin alignment of projectile fragment can be measured through the angular distribution of its γ or β decay.

to be negligible in the present case of ^{32m}Al. Here, A denotes the degree of spin alignment

$$A = \sum_m \frac{3m^2 - I(I+1)}{I(2I-1)} a(m)$$

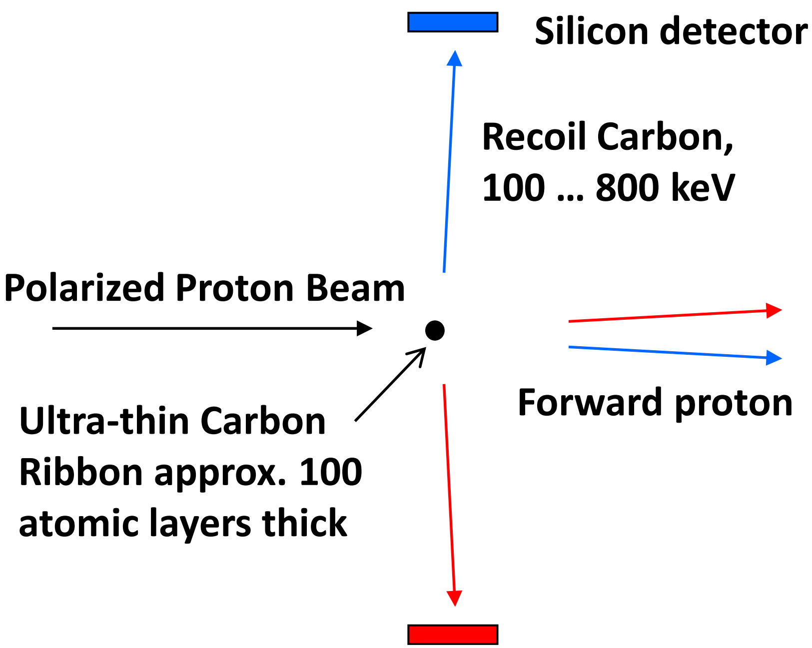
where $a(m)$ is the occupation probability for magnetic sublevel m, and I the nuclear spin. B_2 is the statistical tensor for complete

Analyzing power measurement at AGS and RHIC

$$A = \frac{\sqrt{LU}\sqrt{RD} - \sqrt{LD}\sqrt{RU}}{\sqrt{LU}\sqrt{RD} + \sqrt{LD}\sqrt{RU}}$$

The analyzing power can be as large as 100% at certain angles and energies

Providing a possible way of identifying nucleon spin



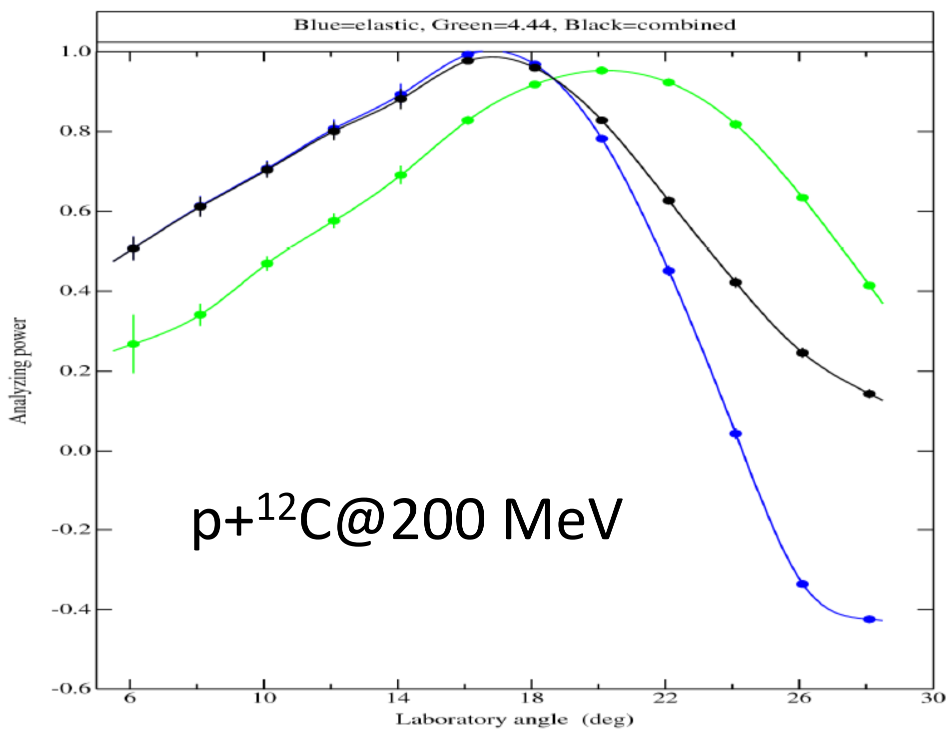


Figure 1: Measurements of the analyzing power for proton scattering from ^{12}C at 200 MeV. The blue (green) curves correspond to protons exiting from the ground (4.44-MeV) state. The black curve represents the sum of the two data sets / 7/.

Introduce spin-orbit interaction to IBUU

Additional spin-dependent mean-field potential

$$U_q^s = -\frac{W_0^\star(\rho)}{2}[\nabla\cdot(a\vec{J}_q + b\vec{J}_{q'})] - \frac{W_0^\star(\rho)}{2}\vec{p}\cdot[\nabla \times (a\vec{s}_q + b\vec{s}_{q'})] - \frac{W_0^\star(\rho)}{2}\vec{\sigma}\cdot[\nabla\times(a\vec{j}_q + b\vec{j}_{q'})],$$

$$U_q^{so} = \frac{W_0^\star(\rho)}{2}(a\nabla\rho_q + b\nabla\rho_{q'})\cdot(\vec{p}\times\vec{\sigma}).(q \neq q')$$

$q = n, p$

$W_0^\star(\rho) = W_0(\rho/\rho_0)^\gamma$

number density $\rho = \sum_i \phi_i^\star \phi_i,$ time-even

spin density $\vec{s} = \sum_i \sum_{\sigma,\sigma'} \phi_i^\star \langle\sigma|\vec{\sigma}|\sigma'\rangle\phi_i,$ time-odd

momentum density $\vec{j} = \frac{1}{2i}\sum_i(\phi_i^\star\nabla\phi_i - \phi_i\nabla\phi_i^\star),$ time-odd

spin-current density $\vec{J} = \frac{1}{2i}\sum_i\sum_{\sigma,\sigma'}(\phi_i^\star\nabla\phi_i - \phi_i\nabla\phi_i^\star)\times\langle\sigma|\vec{\sigma}|\sigma'\rangle$ time-even

$\rho, \vec{J}, \vec{S},$ and $\vec{j}$ from test particle method (C. Y. Wong, PRC 25, 1460 (1982))

Single-particle Hamiltonian: $h_q = \frac{p^2}{2m} + U_q + U_q^s + U_q^{so}$

$$h \sim -\vec{s} \cdot \vec{B}$$

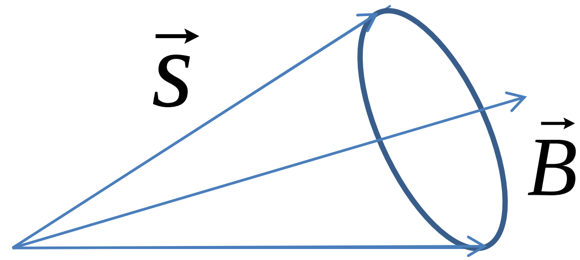
$$\frac{d\vec{r}}{dt} = \nabla_p h_q$$

$$\frac{d\vec{p}}{dt} = -\nabla h_q$$

$$\frac{d\vec{\sigma}}{dt} = \frac{1}{i}\left[\vec{\sigma}, h_q\right]$$

Similar to

$$\frac{d\vec{s}}{dt} \sim \vec{s} \times \vec{B}$$



$$\left[\sigma_i, \sigma_j\right] = 2i\varepsilon_{ijk}\sigma_k$$

Equations of motion:

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} + \frac{W_0^\star(\rho)}{2}\vec{\sigma} \times (a\nabla\rho_q + b\nabla\rho_{q'}) - \frac{W_0^\star(\rho)}{2}\nabla \times (a\vec{s}_q + b\vec{s}_{q'}),$$

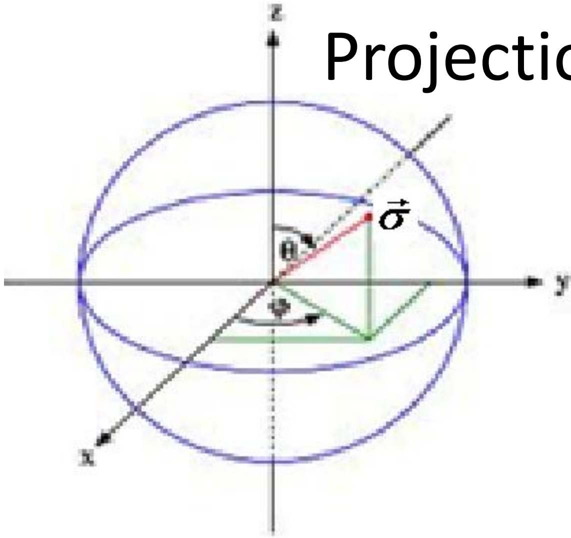
$$\frac{d\vec{p}}{dt} = -\nabla U_q - \nabla U_q^s - \nabla U_q^{so},$$

$$q = n, p$$

$$\frac{d\vec{\sigma}}{dt} = W_0^\star(\rho)[(a\nabla\rho_q + b\nabla\rho_{q'}) \times \vec{p}] \times \vec{\sigma} - W_0^\star(\rho)[\nabla \times (a\vec{j}_q + b\vec{j}_{q'})] \times \vec{\sigma}.$$

$\vec{\sigma}$ a unit vector for each nucleon

expectation value of the spin $\vec{s} = \frac{\hbar}{2}\vec{\sigma}$



Projection on y direction (total angular momentum)

$$\sigma_y = \sin\theta \sin\varphi$$

$(1+\sigma_y)/2$ probability $s_y = \frac{\hbar}{2}$ **Spin-up**

$(1-\sigma_y)/2$ probability $s_y = -\frac{\hbar}{2}$ **Spin-down**

Spin- and isospin-dependent phase space distribution function

$$f_{\sigma\tau}(ix, iy, iz, ipx, ipy, ipz)$$

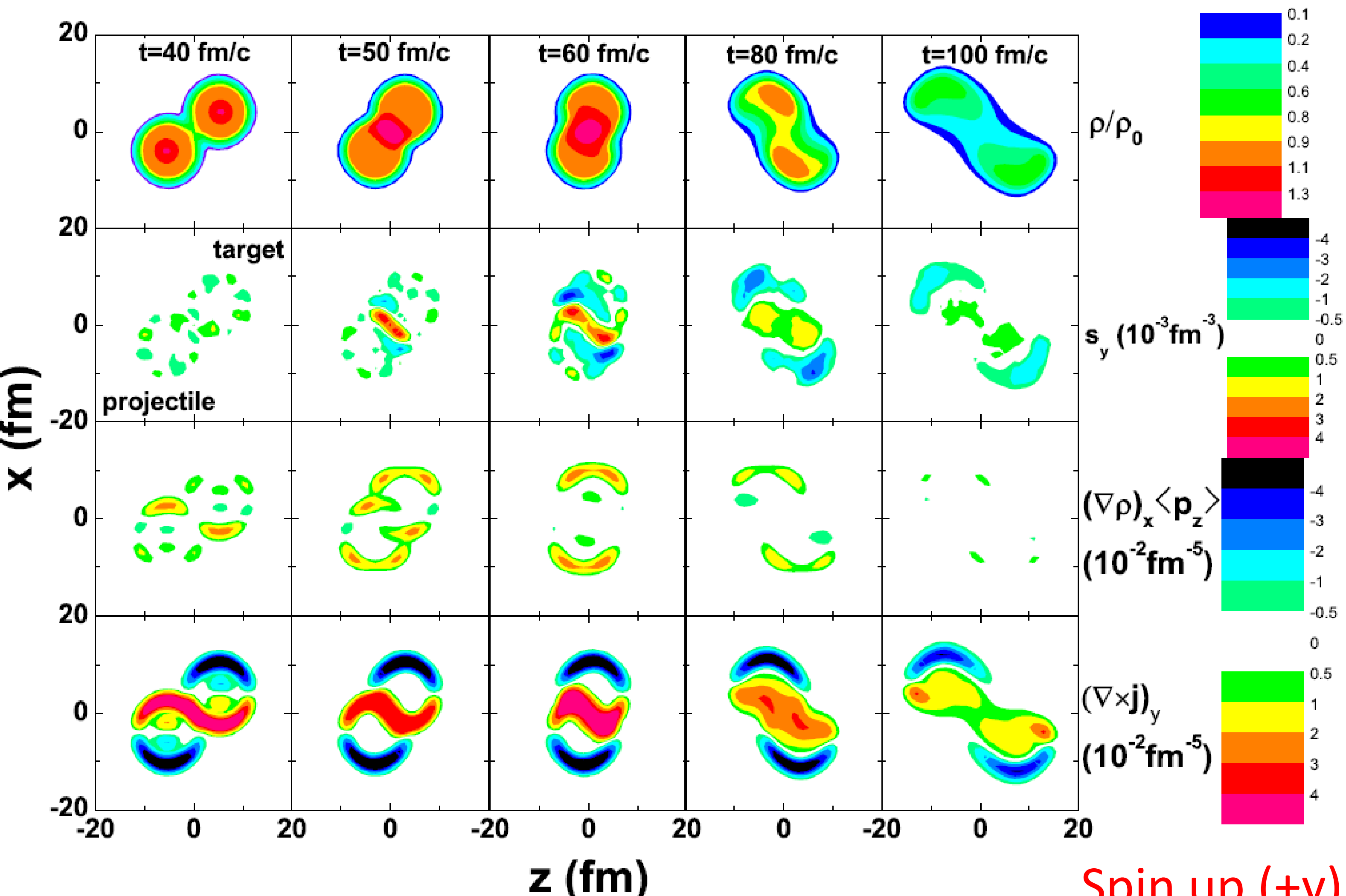
spin- and isospin-dependent Pauli blocking

$$n_{occup} = \frac{h^3}{d * dx * dy * dz * dpx * dpy * dpz} f_{\sigma\tau}(ix, iy, iz, ipx, ipy, ipz), d = 1$$

Nucleon spin may flip after nucleon-nucleon scattering (randomized?)

Local spin polarization

Au+Au@100MeV/A b = 8 fm W_0 = 150 MeVfm^5 γ = 0 a = 2 b = 1



Time-even:

$\vec{\sigma} // \vec{p} \times \nabla\rho$

Time-odd:

$\vec{\sigma} // \nabla \times \vec{j}$

$(\nabla\rho)_x\langle p_z\rangle$

$\wedge$

$(\nabla\times \mathbf{j})_y$

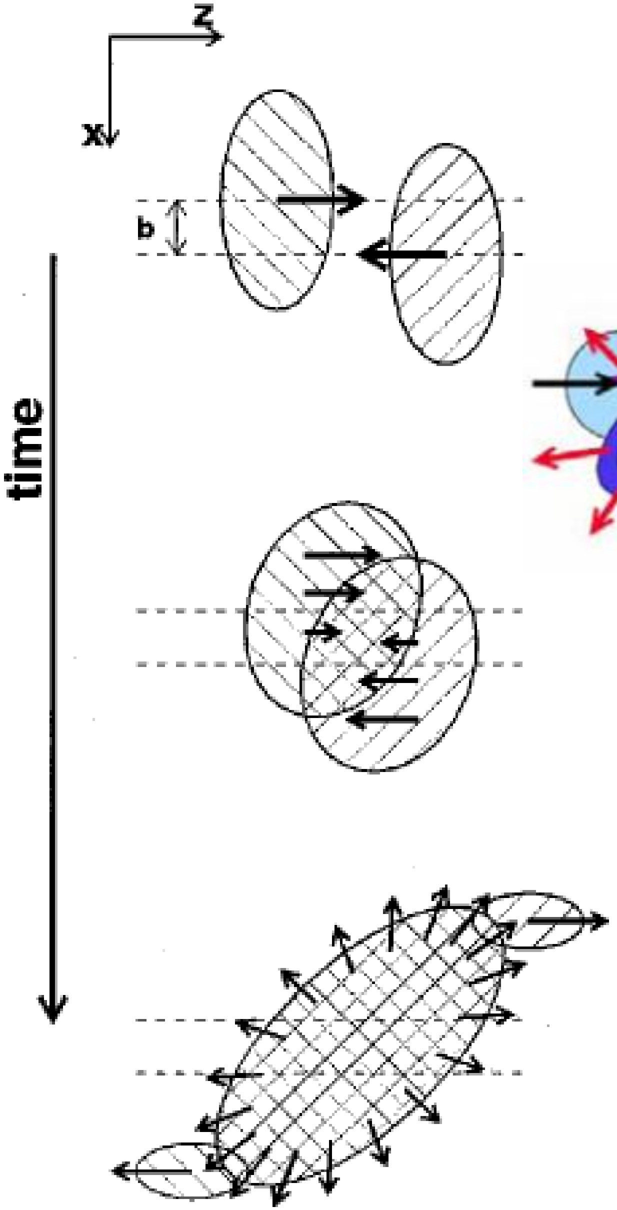
Spin up (+y): attractive

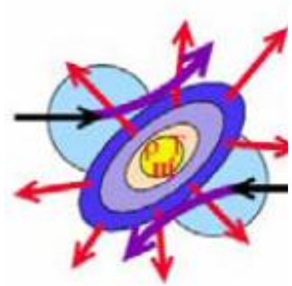
Spin down (-y): repulsive

Time-odd terms overwhelm time-even terms

Transverse flow $\langle p_x \rangle \sim y$

sensitive to nuclear interaction





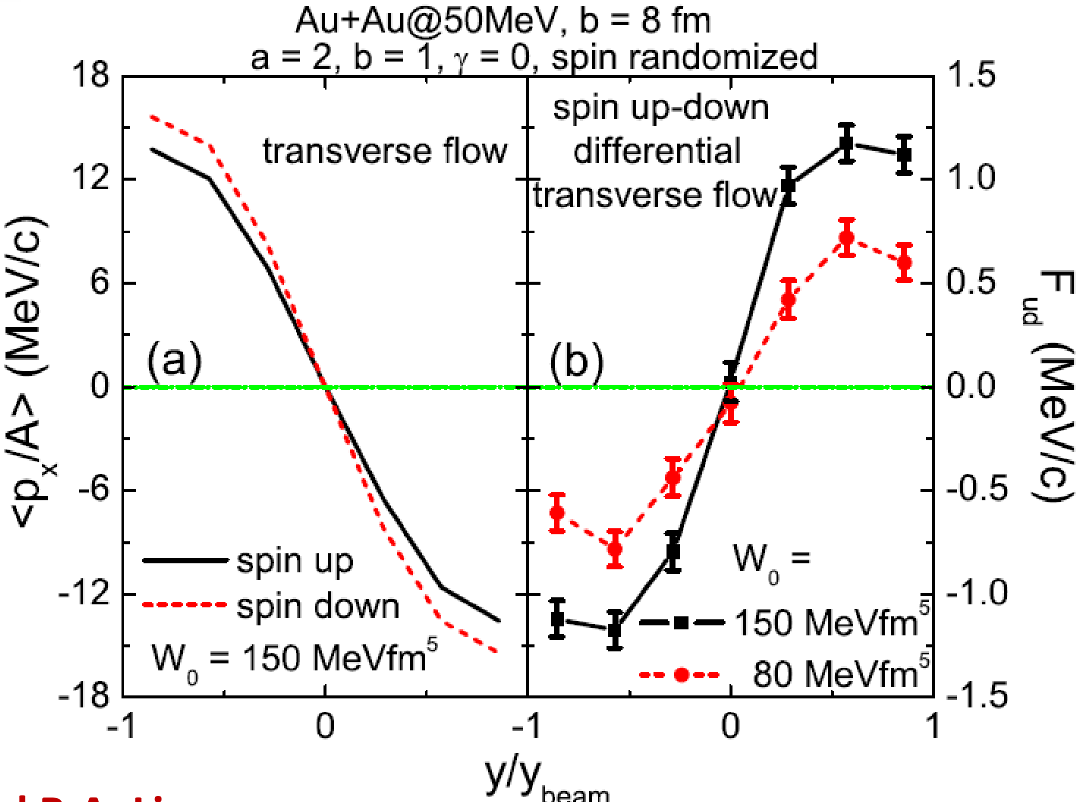
Spin up-down differential transverse flow

$$U = U_0 + \sigma U_{spin}$$

$$\sigma = 1(\uparrow) or -1(\downarrow)$$

$$F_{ud}(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} \sigma_i (p_x)_i$$

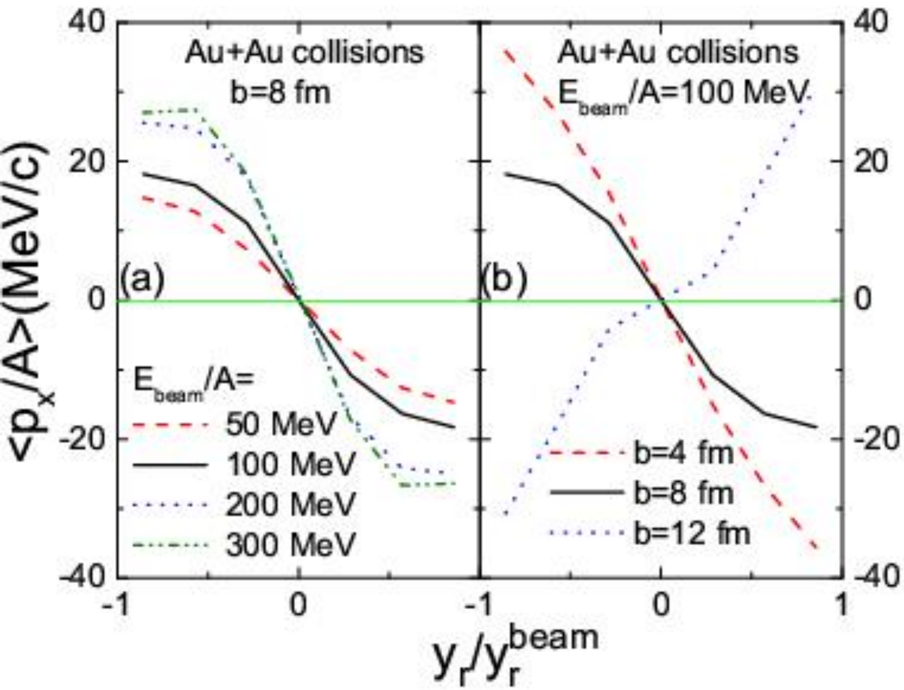
reflects different transverse flows of spin-up and spin-down nucleons

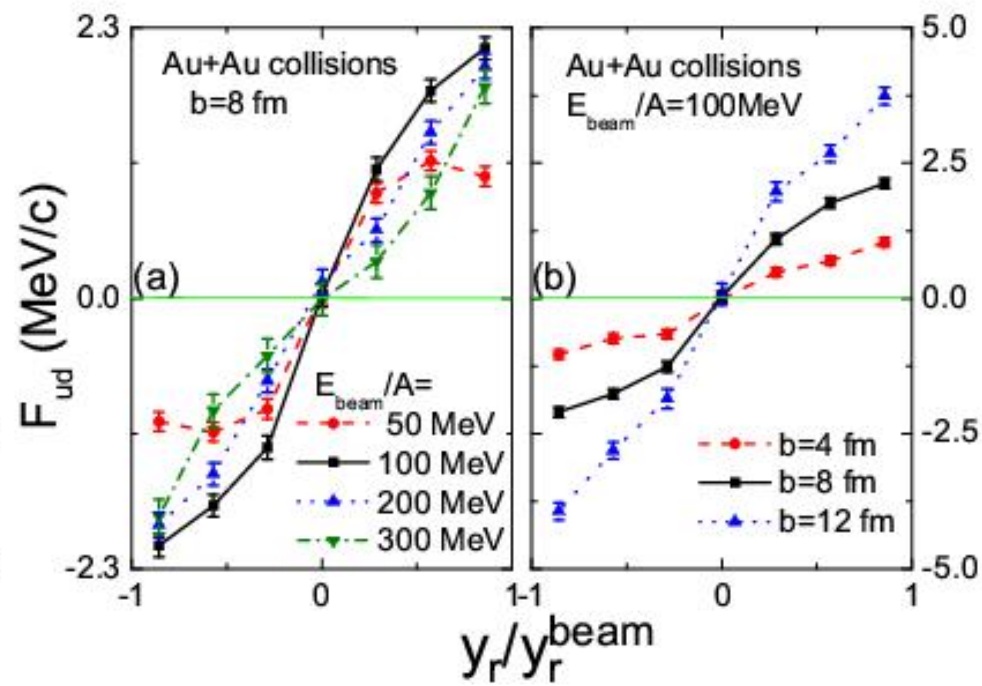


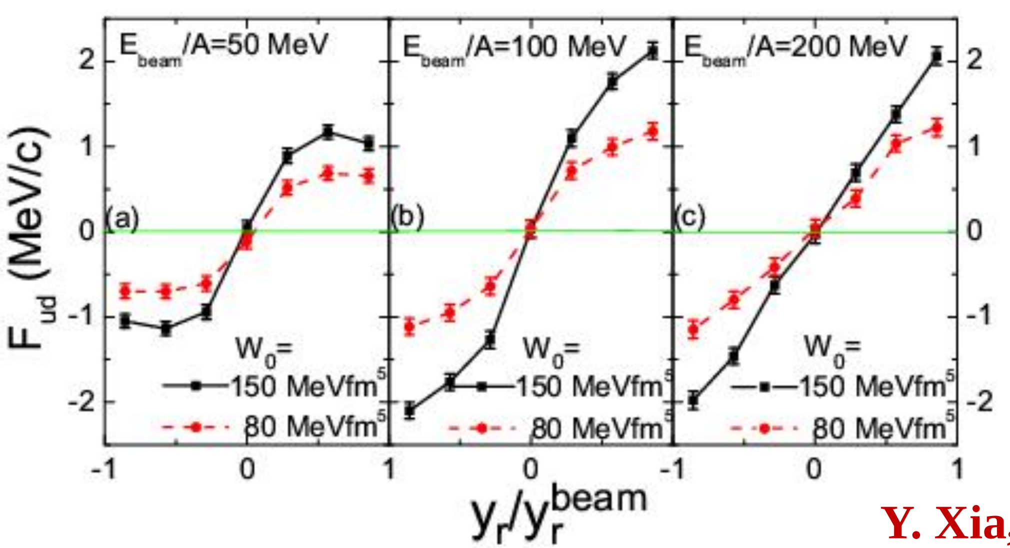
JX and B.A. Li PLB (2013)

F_{ud} is sensitive to W_0, the strength of the spin-orbit interaction.

Energy and impact parameter dependence







The transverse flow

repulsive NN scatterings

attractive mean-field potential

Spin up-down differential transverse flow

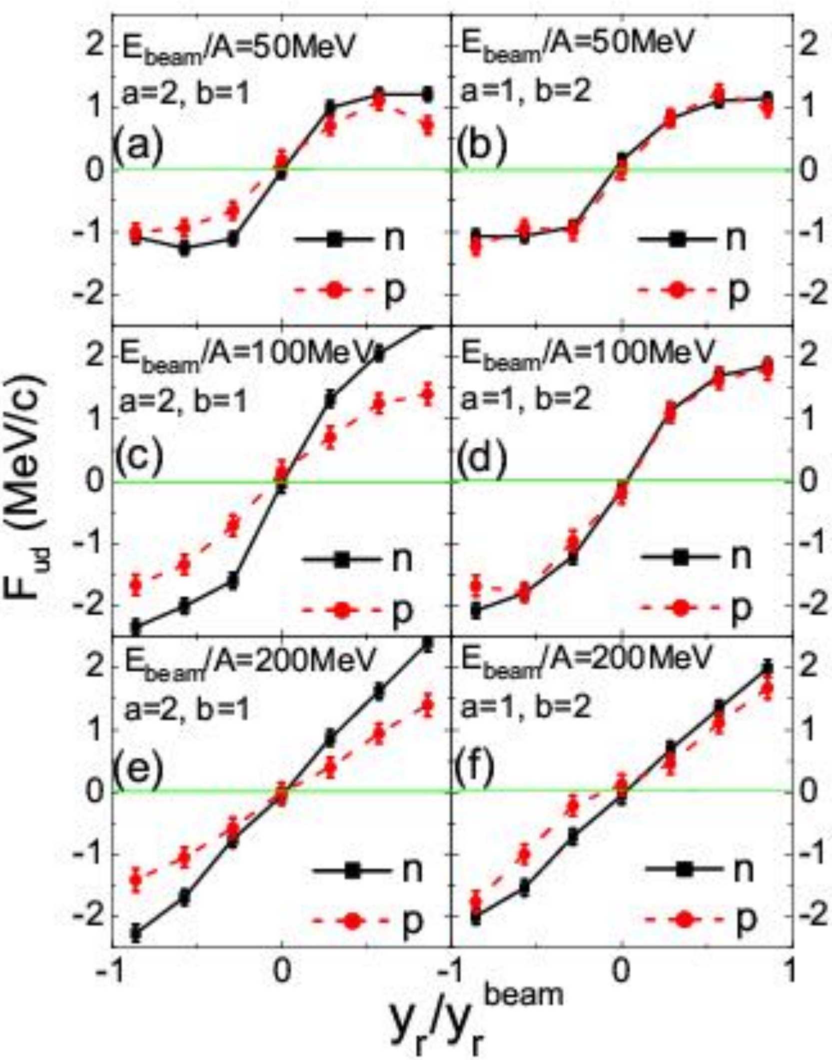
density gradient (surface)

angular momentum (current)

violent NN scatterings

Y. Xia, JX*, B.A. Li, and W.Q. Shen, PRC (2014)

isospin dependence of SO coupling



$$\frac{W_0}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}\left(a\nabla\rho_q + b\nabla\rho_{q'}\right)+\ldots$$

Globally a neutron-rich system

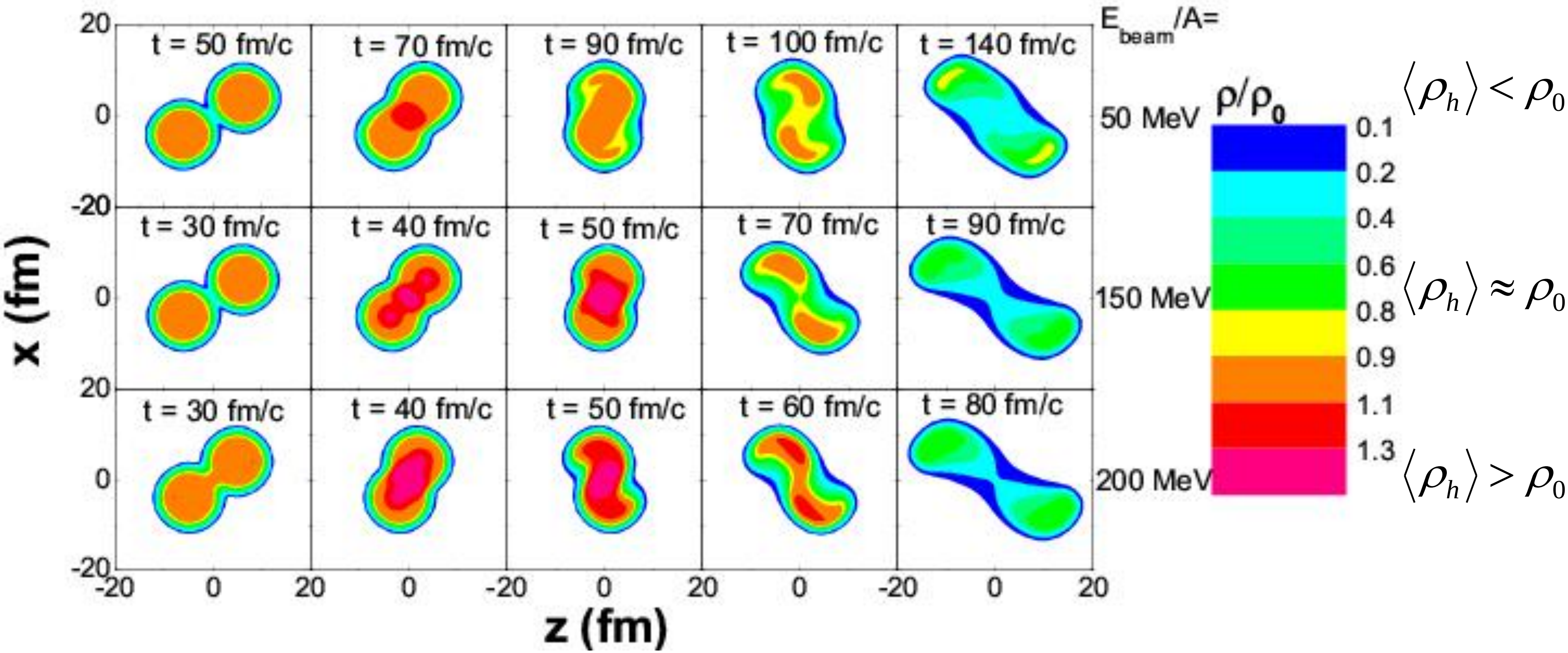
$$|\nabla\rho_n| > |\nabla\rho_p|$$

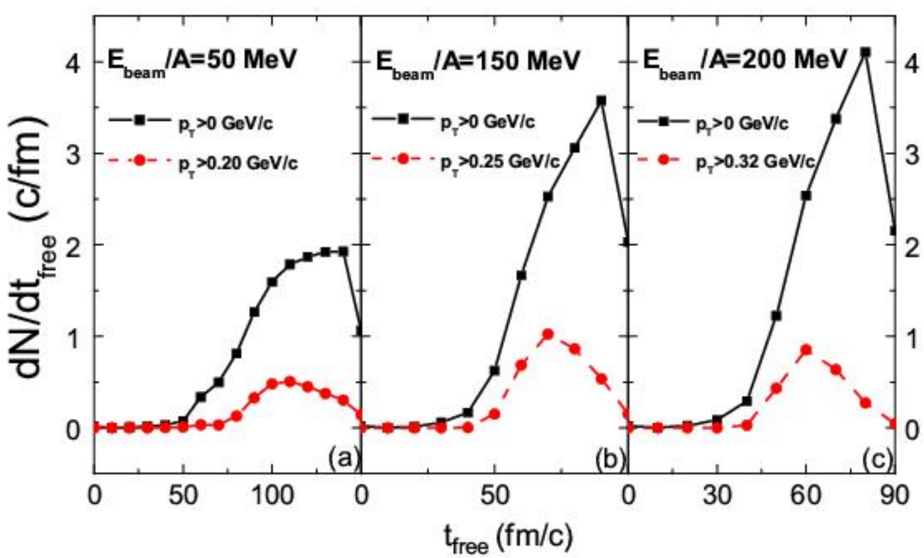
$$|\nabla\times\vec{j}_n| > |\nabla\times\vec{j}_p|$$

By comparing the spin up-down differential transverse flow for neutrons and protons using different isospin dependence of SO coupling.

$$F' = \left[\frac{dF_{ud}}{d(y_r/y_r^{beam})}\right]_{y_r=0} \qquad \delta' = \frac{F_n' - F_p'}{F_n' + F_p'}$$

	$E_{beam} = 50$ (AMeV)		$E_{beam} = 100$ (AMeV)		$E_{beam} = 200$ (AMeV)	
	$a/b = 2$	$a/b = 1/2$	$a/b = 2$	$a/b = 1/2$	$a/b = 2$	$a/b = 1/2$
F_n'	4.17 ± 0.09	3.41 ± 0.53	5.62 ± 0.35	4.43 ± 0.24	2.60 ± 0.50	2.37 ± 0.28
F_p'	2.59 ± 0.36	3.58 ± 0.34	2.55 ± 0.33	3.74 ± 0.75	1.68 ± 0.23	1.10 ± 0.39
δ'	0.23 ± 0.06	-0.02 ± 0.09	0.38 ± 0.06	0.08 ± 0.10	0.21 ± 0.08	0.36 ± 0.09





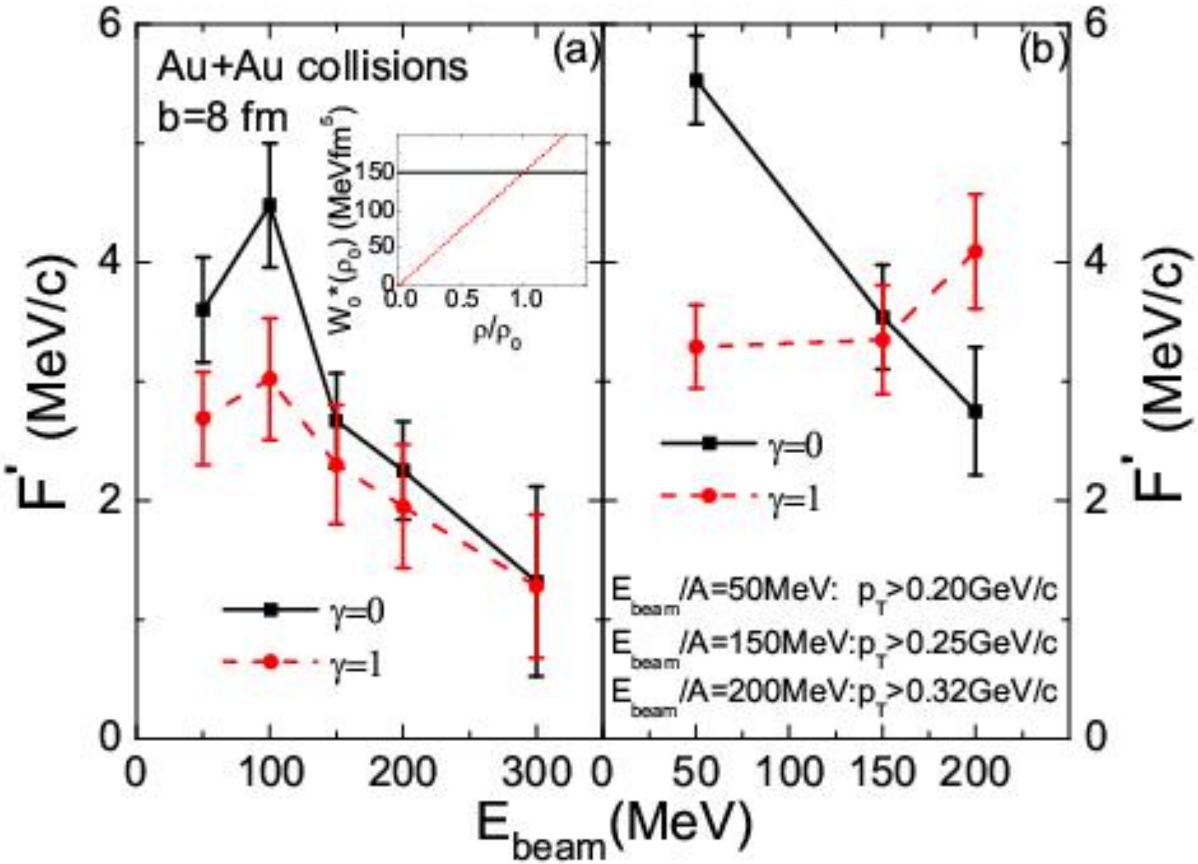
Free nucleons: $\rho < \rho_0 / 8$

Different densities are reached at different beam energies

Nucleons of high transverse momentum (p_T) are emitted at early stages.

density dependence of SO coulping

$$\frac{W_0}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}\left(a\nabla\rho_q + b\nabla\rho_{q'}\right)+\ldots$$



$$F' = \left[\frac{dF_{ud}}{d(y_r/y_r^{beam})}\right]_{y_r=0}$$

Low-p_T nucleons: emitted at later stages carry information of **lower densities**

high-p_T nucleons: emitted at early stages carry information of **higher densities**

The strength of the SO coupling at a certain density can be extracted from HIC at the corresponding collision energy.

In this way the strength and density dependence of SO coupling can be **disentangled**.

System size dependence

Directed flow:

$$v_1 = \langle \cos(\phi) \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

heavier system

higher density, pressure

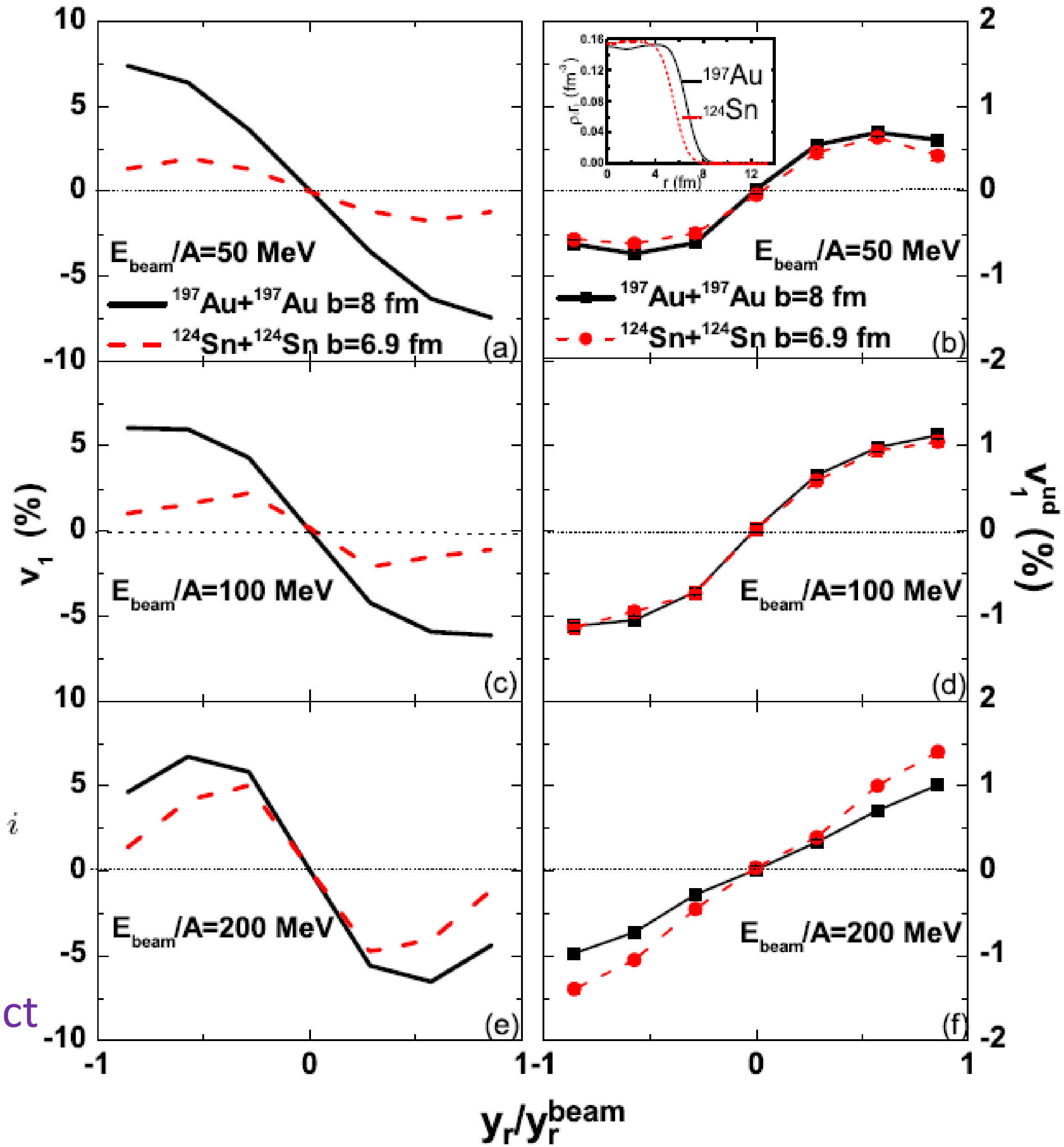
Larger v_1

Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T}\right)_i$$

Surface effect

NN scatterings wash out spin effect



Effects of spin-orbit interaction on v_2

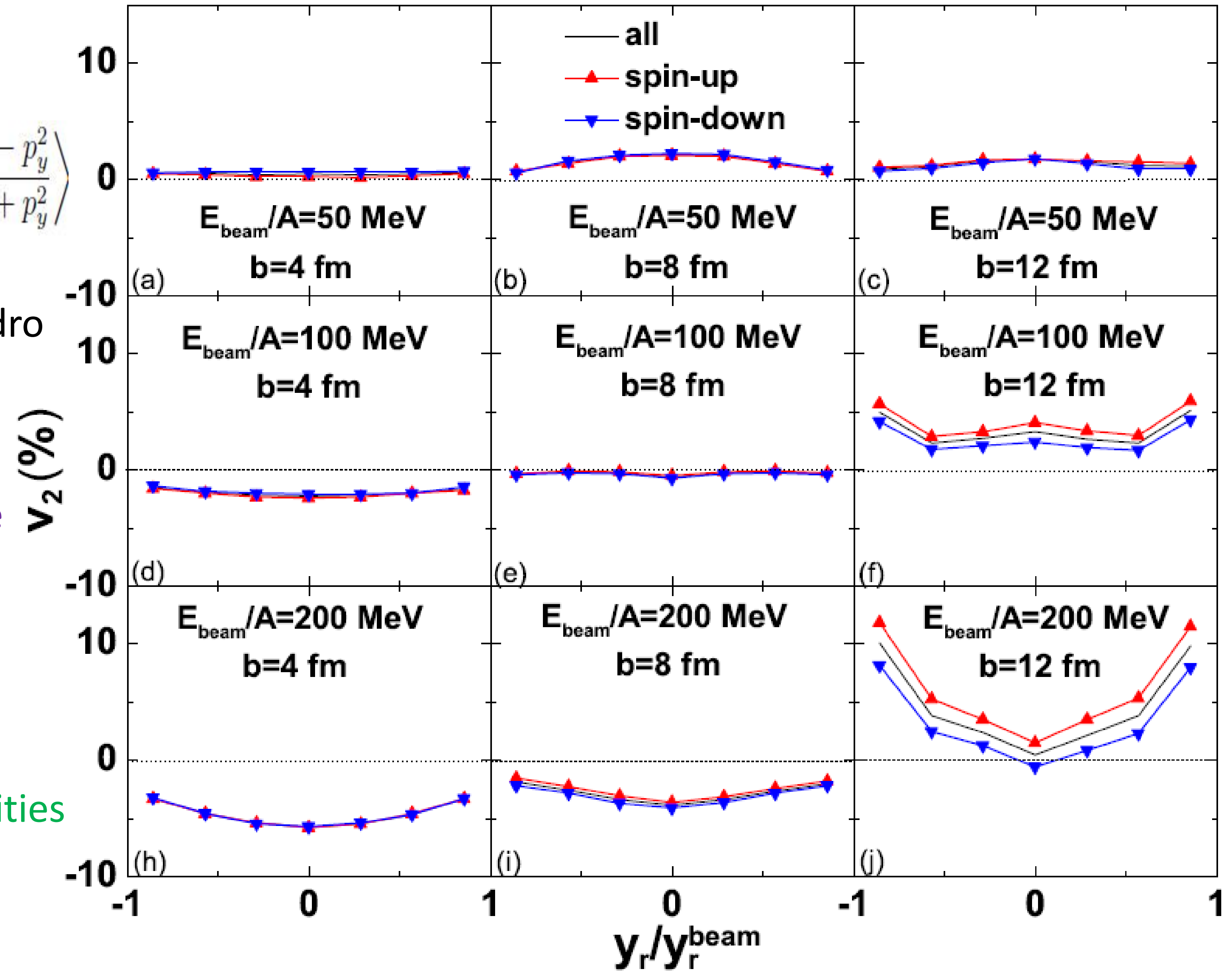
Elliptic flow:

$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

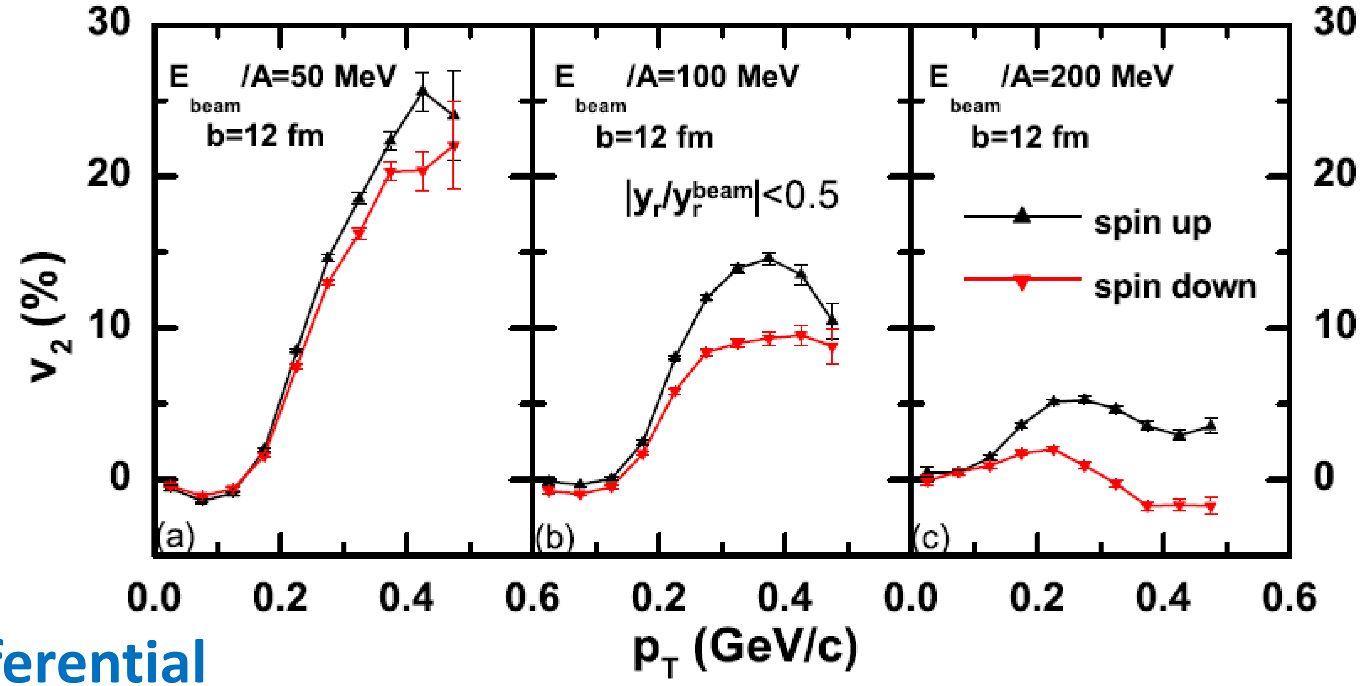
+: in-plane hydro
-: squeeze out

Dynamics more complicated than v_1

Spin splitting at large centralities



Spin splitting at higher p_T

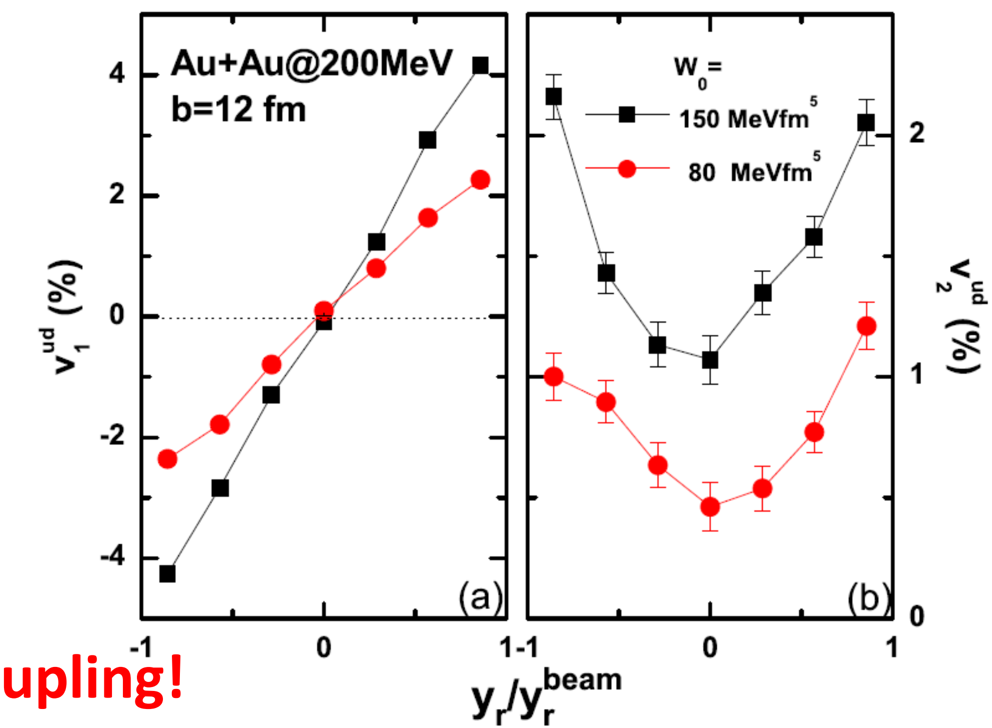


Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T} \right)_i$$

Spin up-down differential elliptic flow:

$$v_2^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x^2 - p_y^2}{p_T^2} \right)_i$$



Both are sensitive probes of SO coupling!

The spin-relevant light cluster production in intermediate energy HIC

the SIBUU model

the modified coalescence model
(spin and isospin coalescence)

→ the spin-relevant light cluster

The probability for producing a cluster is determined by **the overlap of its Wigner phase-space density** with the nucleon phase-space distributions at freeze out.

emission criteria: local densities are less than $\rho_0/8$ → freeze out

The multiplicity of a M-nucleon cluster is

$$N_M = G \int \sum_{i_1>i_2>\ldots>i_M} d\mathbf{r}_{i_1} d\mathbf{k}_{i_1} \cdots d\mathbf{r}_{i_{M-1}} d\mathbf{k}_{i_{M-1}} \times \langle \rho_i^W(\mathbf{r}_{i_1}, \mathbf{k}_{i_1} \cdots \mathbf{r}_{i_{M-1}}, \mathbf{k}_{i_{M-1}}) \rangle.$$

R. Mattiello et al.,
Phys. Rev. Lett 1995
Phys. Rev. C 1997.

ρ^W is the Wigner phase-space density of the M-nucleon cluster, G is the spin-isospin statistical factor (spin-averaged),

$$G = (2S_A + 1) Z!N!/2^A A!$$

- 3/8 for deuteron
- 1/12 for triton
- 1/12 for helium3

In SIBUU model, each nucleon get a spin degree of freedom.

- G : coalescence with **a given isospin**
- G': coalescence with **a given spin and isospin**

${}^2_1H(S=1)$	G'	${}^3_1H(S=1/2)$	G'	${}^3_2He(S=1/2)$	G'
$p\uparrow \& n\uparrow$	$1/2\ (S_Z=1)$	$p\uparrow \& n\uparrow \& n\downarrow$	$1/3\ (S_Z=1/2)$	$n\uparrow \& p\uparrow \& p\downarrow$	$1/3\ (S_Z=1/2)$
$p\uparrow \& n\downarrow$	$1/4\ (S_Z=0)$				
$p\downarrow \& n\uparrow$	$1/4\ (S_Z=0)$	$p\downarrow \& n\uparrow \& n\downarrow$	$1/3\ (S_Z=-1/2)$	$n\downarrow \& p\uparrow \& p\downarrow$	$1/3\ (S_Z=-1/2)$
$p\downarrow \& n\downarrow$	$1/2\ (S_Z=-1)$				

Wigner phase-space density

deuteron

$$\rho_d^W(\mathbf{r},\mathbf{k}) = \int \phi\left(\mathbf{r}+\frac{\mathbf{R}}{2}\right)\phi^*\left(\mathbf{r}-\frac{\mathbf{R}}{2}\right)\exp(-i\mathbf{k}\cdot\mathbf{R})\,d\mathbf{R},$$

$$\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2 \qquad \mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

Internal wave function $\phi(r)$ ➡ root-mean-square radius of 1.96 fm

Triton or Helium3

$$\rho^W_{t(^3\mathrm{He})}(\rho,\lambda,\mathbf{k}_\rho,\mathbf{k}_\lambda) = \int \psi\left(\rho+\frac{\mathbf{R}_1}{2},\lambda+\frac{\mathbf{R}_2}{2}\right)\psi^*\left(\rho-\frac{\mathbf{R}_1}{2},\lambda-\frac{\mathbf{R}_2}{2}\right) \times \exp(-i\mathbf{k}_\rho\cdot\mathbf{R}_1)\exp(-i\mathbf{k}_\lambda\cdot\mathbf{R}_2)3^{3/2}\,d\mathbf{R}_1\,d\mathbf{R}_2$$

$$\begin{pmatrix}\mathbf{R}\\ \boldsymbol{\rho}\\ \boldsymbol{\lambda}\end{pmatrix} = J\begin{pmatrix}\mathbf{r}_1\\ \mathbf{r}_2\\ \mathbf{r}_3\end{pmatrix} \quad J = \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix} \quad \begin{pmatrix}\mathbf{K}\\ \mathbf{k}_\rho\\ \mathbf{k}_\lambda\end{pmatrix} = J^{-,+}\begin{pmatrix}\mathbf{k}_1\\ \mathbf{k}_2\\ \mathbf{k}_3\end{pmatrix} \quad J^{-,+} = \begin{pmatrix}1 & 1 & 1\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix}$$

Internal wave function $\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ ➡ RMS radius 1.61 and 1.74 fm for triton and ^{3}He,

^2_1H wave function

S=1 T=0

$$\left|{}^2_1H\right\rangle \sim \left|spin\right\rangle\left|isospin\right\rangle$$

	S	**T**	
$S=1$	$\uparrow\uparrow$	pp	$T=1$
	$\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn+np)$	
	$\downarrow\downarrow$	nn	
$S=0$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn-np)$	$T=0$

Assign all many-nucleon states which are allowed from the Pauli principle the same weight .

8 wave function(considering the spin-isospin and exchange of antisymmetric), 3 of 8 are feasible.

G= 3/8 (no information about spin)

S_z = +1: $\psi_1 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow - n\uparrow p\uparrow)$

S_z = 0: $\psi_2 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow - n\uparrow p\downarrow - n\downarrow p\uparrow)$

S_z = -1: $\psi_3 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow - n\downarrow p\downarrow)$

$\psi_4 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow - n\uparrow p\downarrow + n\downarrow p\uparrow)$

$\psi_5 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow + n\uparrow p\uparrow)$

$\psi_6 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow + n\uparrow p\downarrow + n\downarrow p\uparrow)$

$\psi_7 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow + n\downarrow p\downarrow)$

$\psi_8 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow + n\uparrow p\downarrow - n\downarrow p\uparrow)$

$p\uparrow \& n\uparrow \longrightarrow G' = 1/2(S_z = +1)$

$p\uparrow \& n\downarrow \longrightarrow G' = 1/4(S_z = 0)$

$p\downarrow \& n\uparrow \longrightarrow G' = 1/4(S_z = 0)$

$p\downarrow \& n\downarrow \longrightarrow G' = 1/2(S_z = -1)$

${}^3_1H$ & ${}^3_2He$ wave function

S=1/2 T=1/2

$$\left|{}^3_1H\,/\,{}^3_2He\right\rangle \sim \left|spin\right\rangle \left|isospin\right\rangle \qquad S_\rho T_\lambda - S_\lambda T_\rho$$

	S	T	
$S=3/2$	$\uparrow\uparrow\uparrow$	ppp	$T=3/2$
	$\tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow+\downarrow\uparrow\uparrow+\uparrow\downarrow\uparrow)$	$\tfrac{1}{\sqrt{3}}(ppn+npp+pnp)$	
	$\tfrac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow+\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow)$	$\tfrac{1}{\sqrt{3}}(nnp+pnn+npn)$	
	$\downarrow\downarrow\downarrow$	nnn	
$S=1/2$ ρ	$\tfrac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\tfrac{1}{\sqrt{6}}(2ppn-pnp-npp)$	$T=1/2$
	$\tfrac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow-2\downarrow\downarrow\uparrow)$	$\tfrac{1}{\sqrt{6}}(pnn+npn-2nnp)$	
$S=1/2$ λ	$\tfrac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\tfrac{1}{\sqrt{2}}(pnp-npp)$	$T=1/2$
	$\tfrac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$	$\tfrac{1}{\sqrt{2}}(pnn-npn)$	

$\left|{}^3_2He\right\rangle (S_z\uparrow)$

$$\psi_1 \sim \tfrac{1}{\sqrt{6}}(p\uparrow n\uparrow p\downarrow - p\downarrow n\uparrow p\uparrow - n\uparrow p\uparrow p\downarrow + n\uparrow p\downarrow p\uparrow - p\uparrow p\downarrow n\uparrow + p\downarrow p\uparrow n\uparrow)$$

$$\psi_2 \sim \tfrac{1}{2}(p\uparrow n\uparrow p\downarrow + n\uparrow p\uparrow p\downarrow - p\uparrow p\downarrow n\uparrow - p\downarrow p\uparrow n\uparrow)$$

$$\psi_3 \sim \tfrac{1}{\sqrt{12}}(-p\uparrow n\uparrow p\downarrow - 2p\downarrow n\uparrow p\uparrow + n\uparrow p\uparrow p\downarrow + 2n\uparrow p\downarrow p\uparrow + p\uparrow p\downarrow n\uparrow - p\downarrow p\uparrow n\uparrow)$$

$\{S({}^3_1H)=1/2\ \&\ S({}^3_2He)=1/2\}$

24 wave function(considering the spin-isospin and exchange of antisymmetric), 2 of 24 are feasible.

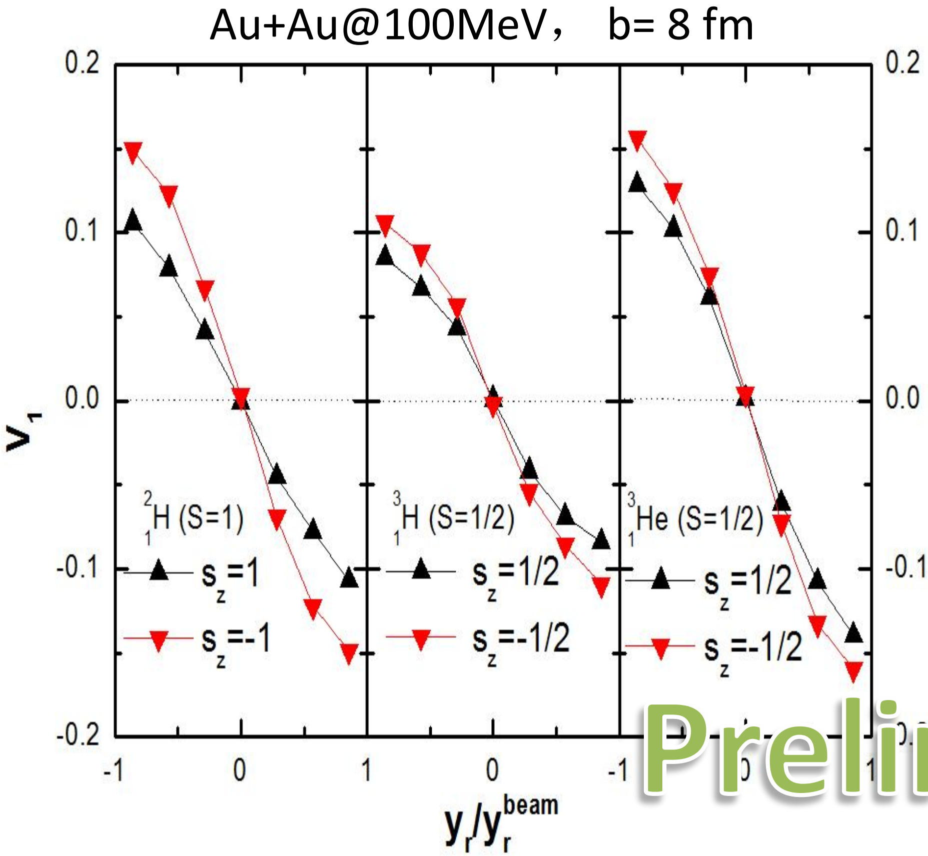
G= 1/12 (no information of spin)

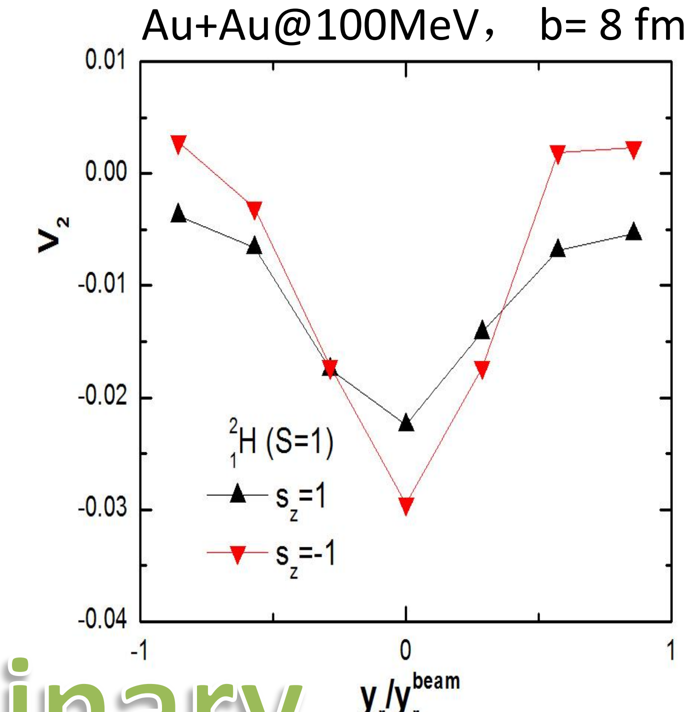
${}^3_2He$ $\qquad G'$

$n\uparrow \& p\uparrow \& p\downarrow \longrightarrow 1/3(S_z=+1/2)$

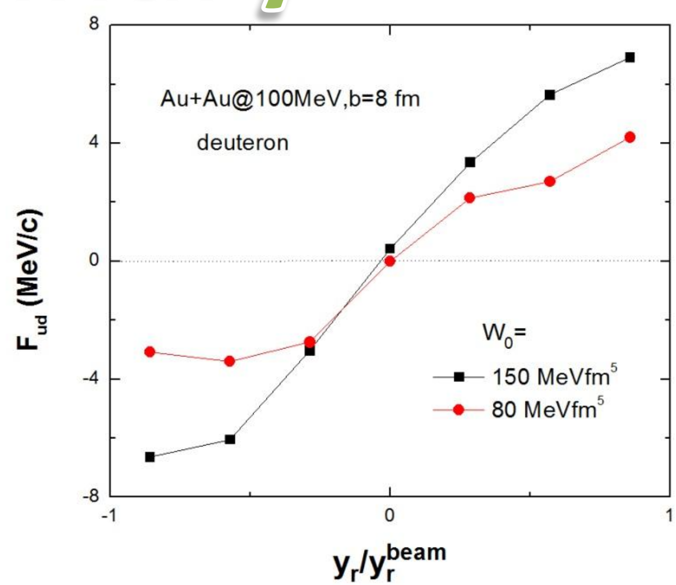
$n\downarrow \& p\uparrow \& p\downarrow \longrightarrow 1/3(S_z=-1/2)$

Similar for ${}^3_1He$





Preliminary



Spin splitting of light clusters collective flows observed

Easily experimentally measured/identified

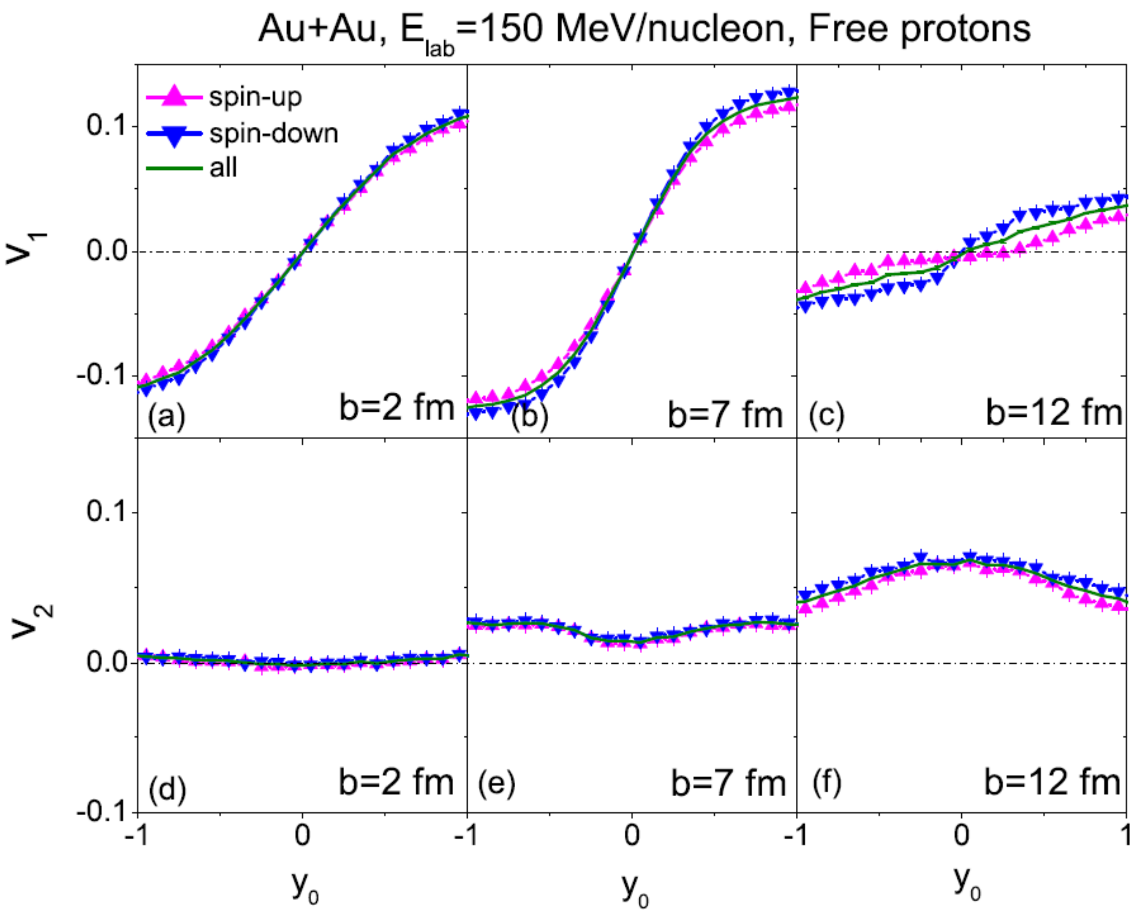
Useful probe of SO coupling

Spin dynamics from QMD model

$$u_{so}^{even} = -\frac{1}{2}W_0(\rho\nabla\cdot\vec{J} + \rho_n\nabla\cdot\vec{J}_n + \rho_p\nabla\cdot\vec{J}_p)$$

$$u_{so}^{odd} = -\frac{1}{2}W_0[\vec{s}\cdot(\nabla\times\vec{j}) + \vec{s}_n\cdot(\nabla\times\vec{j}_n) + \vec{s}_p\cdot(\nabla\times\vec{j}_p)]$$

C.C. Guo, Y.J. Wang, Q.F. Li, and F.S. Zhang, Phys. Rev. C 90, 034606 (2014)



$$\rho(\vec{r}) = \sum_i \rho_i(\vec{r}) = \sum_i \frac{1}{(2\pi L)^{3/2}} e^{[-(\vec{r}-\vec{r}_i)^2/(2L)]}$$

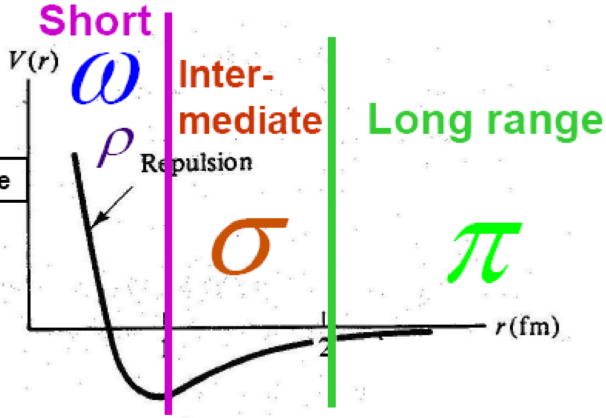
$$\vec{s}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{\sigma}_i,$$

$$\vec{j}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i,$$

$$\vec{J}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i \times \vec{\sigma}_i,$$

Effects of SO coupling on spin dynamics are robust and model independent.

Tensor force

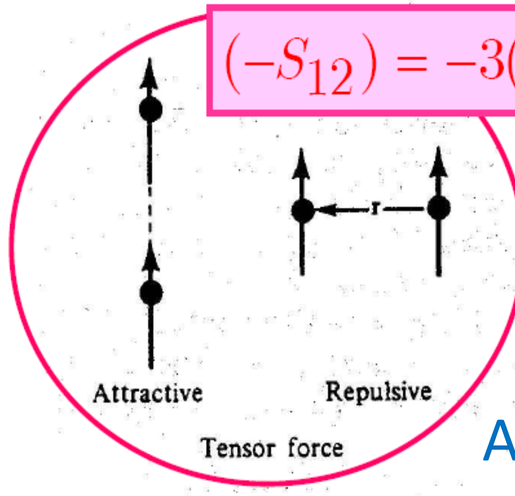


Central force

Tensor force π ρ

Spin-orbit force ω σ

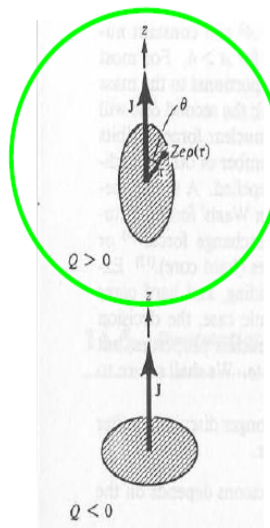
$$(-S_{12}) = -3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



Another interaction related to nucleon spin

$\pi(138)$	$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$	Long-ranged tensor force
$\sigma(600)$	$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1-\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$	intermediate-ranged, attractive central force plus LS force
$\omega(782)$	$V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1-3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$	short-ranged, repulsive central force plus strong LS force
$\rho(770)$	$V_\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$	short-ranged tensor force, opposite to pion

Tensor Force: First evidence from the deuteron



prolate

Deuteron

oblate

Add tensor force to transport?

Skyrme-type tensor force:

$$v_T = \frac{t_e}{2}\{\left[3\left(\vec{\sigma}_1 \cdot \vec{k}'\right)\left(\vec{\sigma}_2 \cdot \vec{k}'\right) - \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)k'^2\right]\delta(\vec{r})$$

$$+ \delta(\vec{r})\left[3\left(\vec{\sigma}_1 \cdot \vec{k}\right)\left(\vec{\sigma}_2 \cdot \vec{k}\right) - \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)k^2\right]\}$$

$$+ t_0\left[3\left(\vec{\sigma}_1 \cdot \vec{k}'\right)\delta(\vec{r})\left(\vec{\sigma}_2 \cdot \vec{k}\right) - \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\vec{k}' \cdot \delta(\vec{r})\vec{k}\right]$$

Hartree-Fock framework:

$$E_T = \frac{1}{2}\sum_{i,j}\langle ij \,|\, v_T(1 - P_r P_\sigma P_\tau) \,|\, ij\rangle = \int H_T(\vec{r})d^3r$$

$$\frac{\delta H_T}{\delta \varphi_i^*}\varphi_i \sim h_T \varphi_i$$

$$s_\mu = \sum_i \varphi_i^* \sigma_\mu \varphi_i$$

Spin density

$$T_\mu = \sum_i \nabla \varphi_i^* \cdot \nabla \varphi_i \sigma_\mu$$

Spin kinetic density

$$J_{\mu\nu} = \frac{1}{2i} \sum_i \sigma_\nu \left(\varphi_i^* \nabla_\mu \varphi_i - \nabla_\mu \varphi_i^* \varphi_i \right)$$

Spin current density

$$F_\mu = \frac{1}{2} \sum_i \sigma_\nu \left(\nabla_\nu \varphi_i^* \nabla_\nu \varphi_i + \nabla_\mu \varphi_i^* \nabla_\nu \varphi_i \right)$$

Pseudovector tensor kinetic density

Only consider vector component of $J_{\mu\nu}$

$$J_{\mu\mu}^2 = 0, J_{\mu\nu} J_{\mu\nu} = \frac{1}{2} J^2, J_{\mu\nu} J_{\nu\mu} = -\frac{1}{2} J^2$$

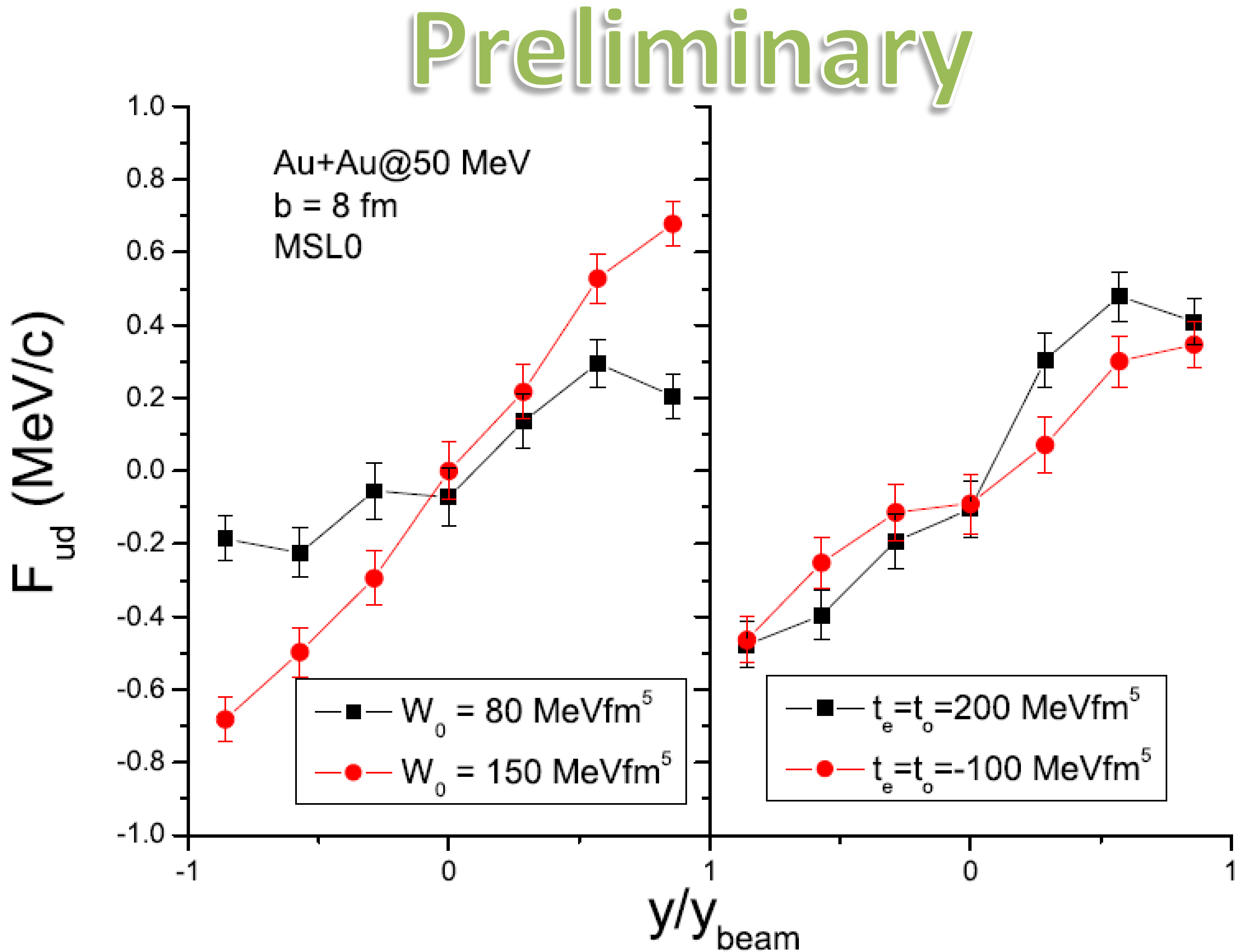
Potential energy density

$$H_T = \frac{3}{16}(3t_e - t_o)(\nabla \cdot \vec{s})^2 - \frac{3}{16}(3t_e + t_o)\sum_q (\nabla \cdot \vec{s}_q)^2$$
$$- \frac{1}{4}(t_e + t_o)\left(\vec{s} \cdot \vec{T} - \frac{1}{2} J^2 \right) + \frac{1}{4}(t_e - t_o)\sum_q \left(\vec{s}_q \cdot \vec{T}_q - \frac{1}{2} J_q^2 \right)$$
$$+ \frac{3}{4}(t_e + t_o)\left(\vec{s} \cdot \vec{F} + \frac{1}{4} J^2 \right) - \frac{3}{4}(t_e - t_o)\sum_q \left(\vec{s}_q \cdot \vec{F}_q + \frac{1}{4} J_q^2 \right)$$
$$+ \frac{1}{16}(3t_e - t_o)\vec{s} \cdot \nabla^2 \vec{s} - \frac{1}{16}(3t_e + t_o)\sum_q \vec{s}_q \nabla^2 \vec{s}_q \rightarrow h_T \rightarrow$$

Equation of motion

$$q = n, p$$

Spin-orbit interaction+tensor interaction



Conclusion and outlook

1) Introduce spin and spin-orbit interaction to a transport model for intermediate-energy heavy-ion collisions

2) Local spin polarization observed

3) Spin up-down differential flow (v1, v2,…) is a sensitive probe for in-medium spin-orbit interaction

5) System size effect

4) Different spin states of light clusters

6) Tensor force in heavy-ion collisions

Experimental measurement:

Using nuclei with known analyzing power as the detector …

Tensor force:

More effective probes with spin polarized beam

Heavy-ion collisions has the advantages of constructing the energy, density, and momentum current of the system, and might be a more promising way to study the properties of spin-related nuclear force!

Recent invited review:

Jun Xu, Bao-An Li, Wen-Qing Shen, and Yin Xia, arXiv: 1506.06860

Thank you!

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