Alpha Cluster Model of Atomic Nuclei

and the

Nuclear Equation of State

Zbigniew Sosin,[1] Jan Błocki,[2] Jinesh Kallunkathariyil,[1]

Jerzy Łukasik,[3] Piotr Pawłowski [3]

[1]Jagiellonian University, Institute of Physics, Kraków, Poland
[2]NCBJ, Theoretical Physics Division (BP2), Świerk, Poland
[3]Institute of Nuclear Physics PAN, Kraków, Poland

NuSYM15, Kraków, June 29 – July 2, 2015

Work supported by Polish National Science Centre (NCN),
Contract Nos. UMO-2013/10/M/ST2/00624, UMO-2013/09/B/ST2/04064

Plan:

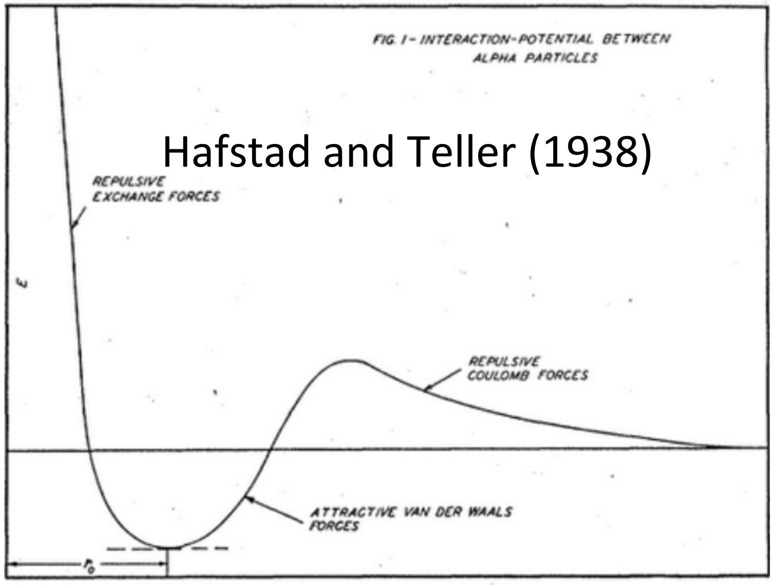
- Introduction
- What do we expect from the model?
- Description of the model
- Results of the model calculations

  - constraining the EoS parameters

  - description of the ground state binding energies and rms radii for the even-even nuclei with Z=N
- Conclusions

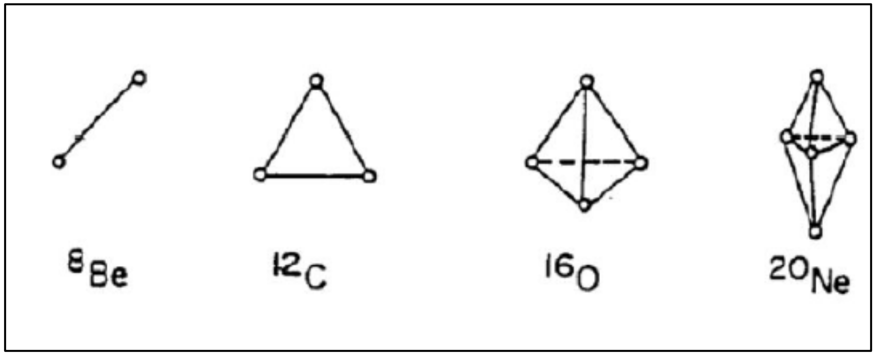
Introduction

Molecular viewpoint of alpha clustered nuclei - Bethe and Bacher (1936)



Hafstad and Teller (1938)

r



total binding energy (MeV)

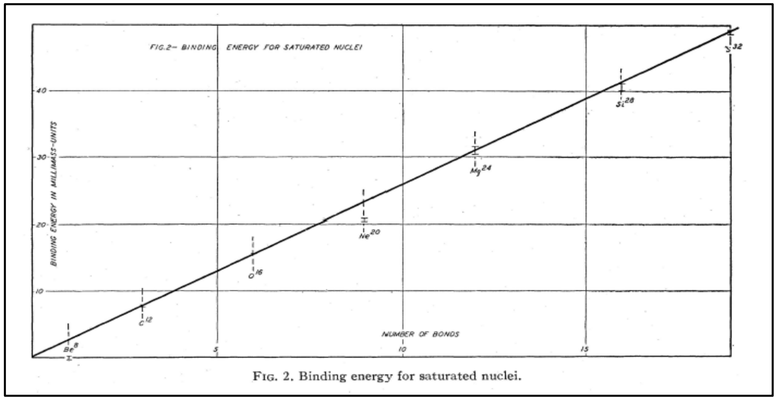
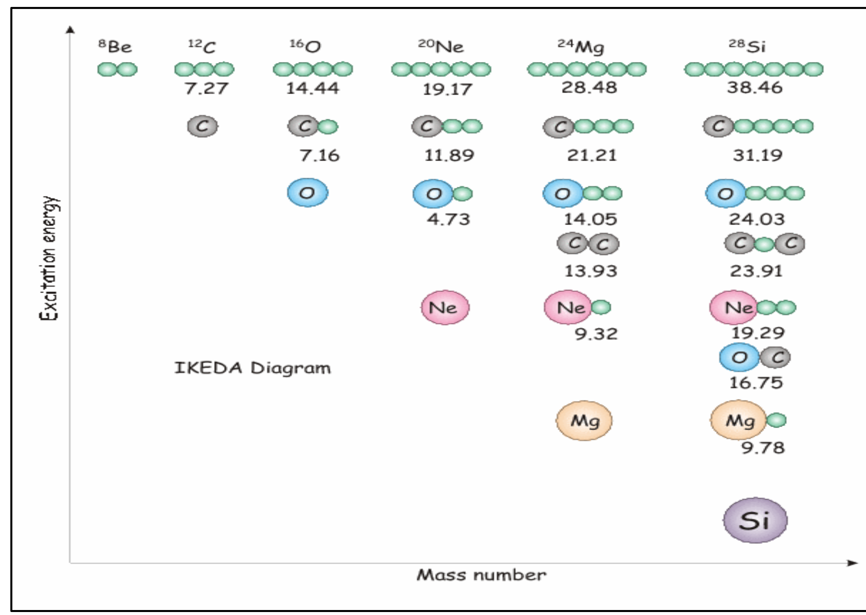


Fig. 2. Binding energy for saturated nuclei.

Number of bonds

Hafstad and Teller (1938)



Ikeda diagram

Nuclear Equation of State (EoS) of cold nuclear matter (standard approach)

$$e(\rho,\delta) = e(\rho,0) + e_{sym}(\rho)\delta^2$$

dominant symmetric matter (N=Z) term (*isoscalar*):

$$e(\rho,0) \approx e_{00} + \frac{K_o}{18}\xi^2 + \cdots$$

symmetry term (*isovector*):

$$e_{sym}(\rho)\delta^2 \approx \left(a_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2 + \cdots\right)\delta^2$$

ρ_n, ρ_p →neutron, proton densities

$\rho = \rho_n + \rho_p$ →nucleon density

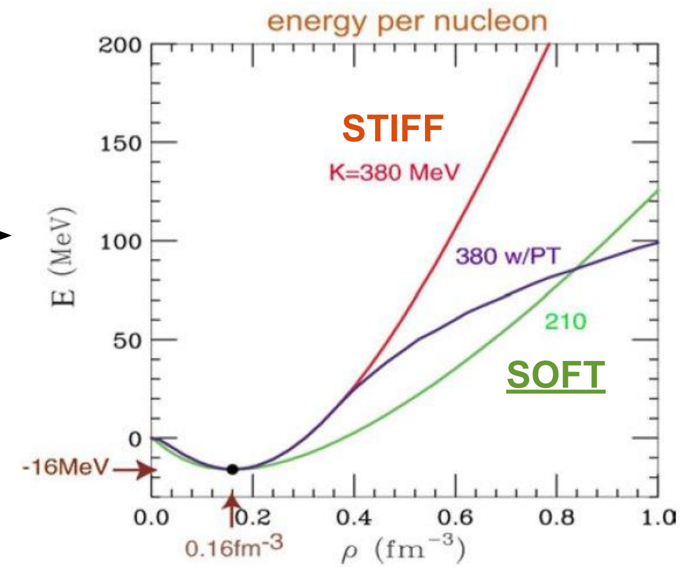
$\xi = \frac{\rho - \rho_o}{\rho_o}$ →deviation from saturation

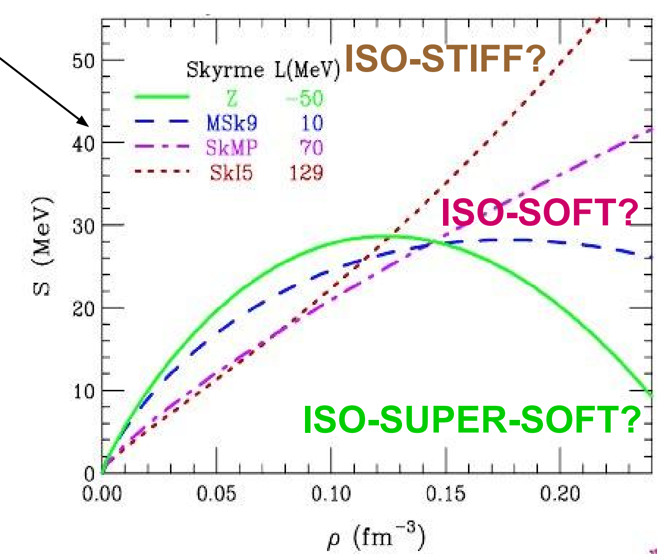
$\delta = \frac{\rho_n - \rho_p}{\rho}$ →isospin asymmetry

$K_o = 9\rho_o^2 \left.\frac{\partial^2 e}{\partial \rho^2}\right|_{\rho=\rho_o,\,\delta=0}$ →compressibility

$L_I = 3\rho_o \left.\frac{\partial e_{sym}}{\partial \rho}\right|_{\rho=\rho_o}$ →symmetry pressure

$K_I = 9\rho_o^2 \left.\frac{\partial^2 e_{sym}}{\partial \rho^2}\right|_{\rho=\rho_o}$ →symmetry compressibility





P. Danielewicz, arXiv:nucl-th/0512009

(*)P. Danielewicz, J. Lee, NPA 818(2009)36

Nuclear system in its ground state and at low excitations based on the equation of state (EoS) around the saturation density

System evolution

(Variational principle)

Wave function structure in the ground state of nuclei

(Minimization of Hamiltonian)

Binding energy and radii of nuclei

(Minimization of Hamiltonian)

Strong correlations in the position of nucleons:

Clusters

Constraining the EoS parameters according to the ground state properties of nuclei

^{3}H, ^{3}He, α

Description of the Model

- Nucleon wave packets and system wave function

- Interactions

- The form of the EoS

The system wave function and nucleon wave packets

$$\Phi = \prod_{k=1}^{A} {}^{k}\phi_{I_k S_k}$$

$${}^{k}\phi_{I_k S_k} = \frac{1}{\left(2\pi\sigma_k^2(r)\right)^{3/4}} \exp\left(\frac{-\left(\mathbf{r}_k - \langle \mathbf{r}_k \rangle\right)^2}{4\sigma_k^2(r)} + \frac{i}{\hbar}\mathbf{r_k} \langle \mathbf{p}_k \rangle \right)$$

$\langle \mathbf{r}_k \rangle$, $\langle \mathbf{p}_k \rangle$ and $\sigma_k^2(r)$ Are time dependent parameters

I_k Isospin S_k Spin

Nuclear matter as a four component fluid

-protons with the spin up - $\rho_{p\uparrow}$
-protons with the spin down - $\rho_{p\downarrow}$
-neutrons with the spin up - $\rho_{n\uparrow}$
-neutrons with the spin down - $\rho_{n\downarrow}$

Interaction description

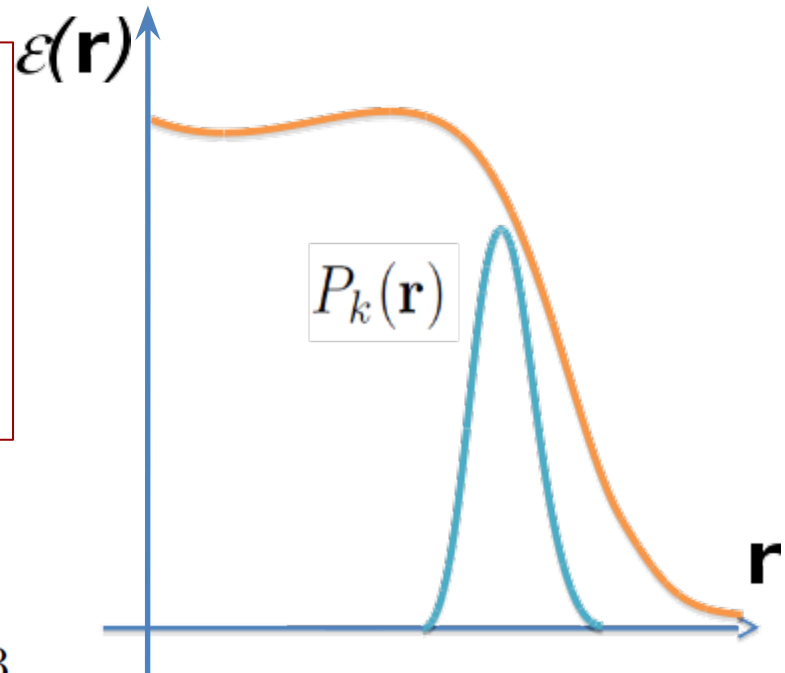
For every nucleon ***k*** with wave packet ${}^{k}\phi_{I_k S_k}$, Probability of finding a nucleon at ***r***

$$P_k(\mathbf{r}) = \left| {}^{k}\phi_{I_k S_k} \right|^2$$

This probability allows to calculate
$\rho_{p\uparrow}(\mathbf{r})$, $\rho_{p\downarrow}(\mathbf{r})$, $\rho_{n\uparrow}(\mathbf{r})$, $\rho_{n\downarrow}(\mathbf{r})$ at any point. From this one can calculate some scalar energy field

$$\boxed{\varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right)}$$

which will be defined later



The mean and variance of average energy per nucleon is given by

$$\langle \varepsilon \rangle_k = \int P_k(\mathbf{r}) \varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) d^3\mathbf{r}$$

$$\sigma_k^2(\varepsilon) = \int P_k(\mathbf{r}) \left(\varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) - \langle \varepsilon \rangle_k\right)^2 d^3\mathbf{r}$$

We assume that the average energy $\mathbf{e_k}$ associated with $\mathbf{k^{th}}$ nucleon is

$$e_k = \langle \varepsilon \rangle_k + \lambda \sigma_k (\varepsilon)$$

Where λ is a parameter related to the surface energy in finite systems

Z. Sosin, International Journal of Modern Physics E. Vol. 19, No. 4 (2010)

For nuclear matter, variance disappear thus energy per nucleon

$$e_k = \int P_k(\mathbf{r}) \varepsilon \left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) d^3\mathbf{r}$$

Total energy $\sum_{k=1}^{k=A} \langle \varepsilon \rangle_k$ but $\rho(\mathbf{r}) = \sum_{k=1}^{k=A} P_k(\mathbf{r}),$

Total energy $= \int \varepsilon \left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) \rho(\mathbf{r})\, d^3\mathbf{r}$

$\boxed{\varepsilon \left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right)}$ **is EoS depends on local density**

Proposed form of the EoS around ρ_o

Z. Sosin, J. Kallunkathariyil, Acta Phys. Polon. B45 (2014)

$$e = e_{00} + \frac{K_0}{18}\xi^2 +$$

balanced system - zero isospin and spin

$$\delta^2 \left(e_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2 \right) +$$

isospin symmetry energy

$$(\eta_n^2 + \eta_p^2) \left(e_{ii0} + \frac{L_{ii}}{3}\xi + \frac{K_{ii}}{18}\xi^2 \right) +$$

proton or neutron spin symmetry energy

$$2\eta_n\eta_p \left(e_{ij0} + \frac{L_{ij}}{3}\xi + \frac{K_{ij}}{18}\xi^2 \right)$$

mutual proton - neutron spin symmetry energy

$$\xi = \frac{\rho - \rho_0}{\rho_0} \qquad \eta_n = \frac{\rho_{n\uparrow} - \rho_{n\downarrow}}{\rho}$$

$$\delta = \frac{\rho_n - \rho_p}{\rho} \qquad \eta_p = \frac{\rho_{p\uparrow} - \rho_{p\downarrow}}{\rho}$$

Proposed form of the EoS around ρ_o

STANDARD

$$e = e_{00} + \frac{K_0}{18}\xi^2 +$$

balanced system - zero isospin and spin

$$\delta^2 \left(e_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2 \right) +$$

isospin symmetry energy

NEW

$$(\eta_n^2 + \eta_p^2) \left(e_{ii0} + \frac{L_{ii}}{3}\xi + \frac{K_{ii}}{18}\xi^2 \right) +$$

proton or neutron spin symmetry energy

$$2\eta_n \eta_p \left(e_{ij0} + \frac{L_{ij}}{3}\xi + \frac{K_{ij}}{18}\xi^2 \right)$$

mutual proton - neutron spin symmetry energy

$$\xi = \frac{\rho - \rho_0}{\rho_0} \qquad \eta_n = \frac{\rho_{n\uparrow} - \rho_{n\downarrow}}{\rho}$$

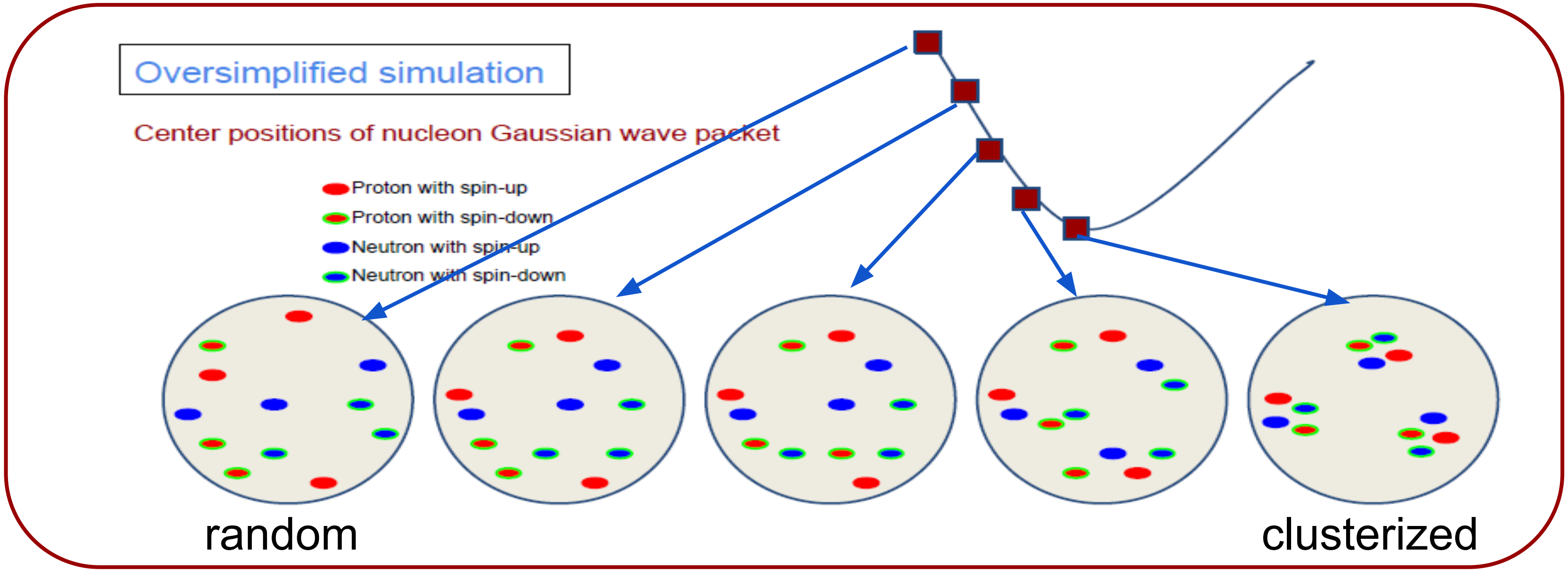
$$\delta = \frac{\rho_n - \rho_p}{\rho} \qquad \eta_p = \frac{\rho_{p\uparrow} - \rho_{p\downarrow}}{\rho}$$

Hamiltonian

$$\langle \Phi | H | \Phi \rangle = \sum_{k=1}^{k=A} \frac{\langle \mathbf{p}_k \rangle^2}{2m_N} + \sum_{k=1}^{k=A} \langle \varepsilon \rangle_k + \lambda \sum_{k=1}^{k=A} \sigma_k(\varepsilon) + \langle \Phi | V_C | \Phi \rangle$$

For the ground state, one can omit the kinetic energy

$$\langle \Phi | H | \Phi \rangle = \int \varepsilon \left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow} \right) \rho \left(\mathbf{r} \right) d^3\mathbf{r} + \lambda \sum_{k=1}^{k=A} \sigma_k(\varepsilon) + \langle \Phi | V_C | \Phi \rangle$$



Constraining the EoS parameters

To describe the properties of an alpha particle it is sufficient to use the EoS in the form:

$$e = e_{00} + \frac{K_0}{18}\xi^2$$

with **4** parameters:

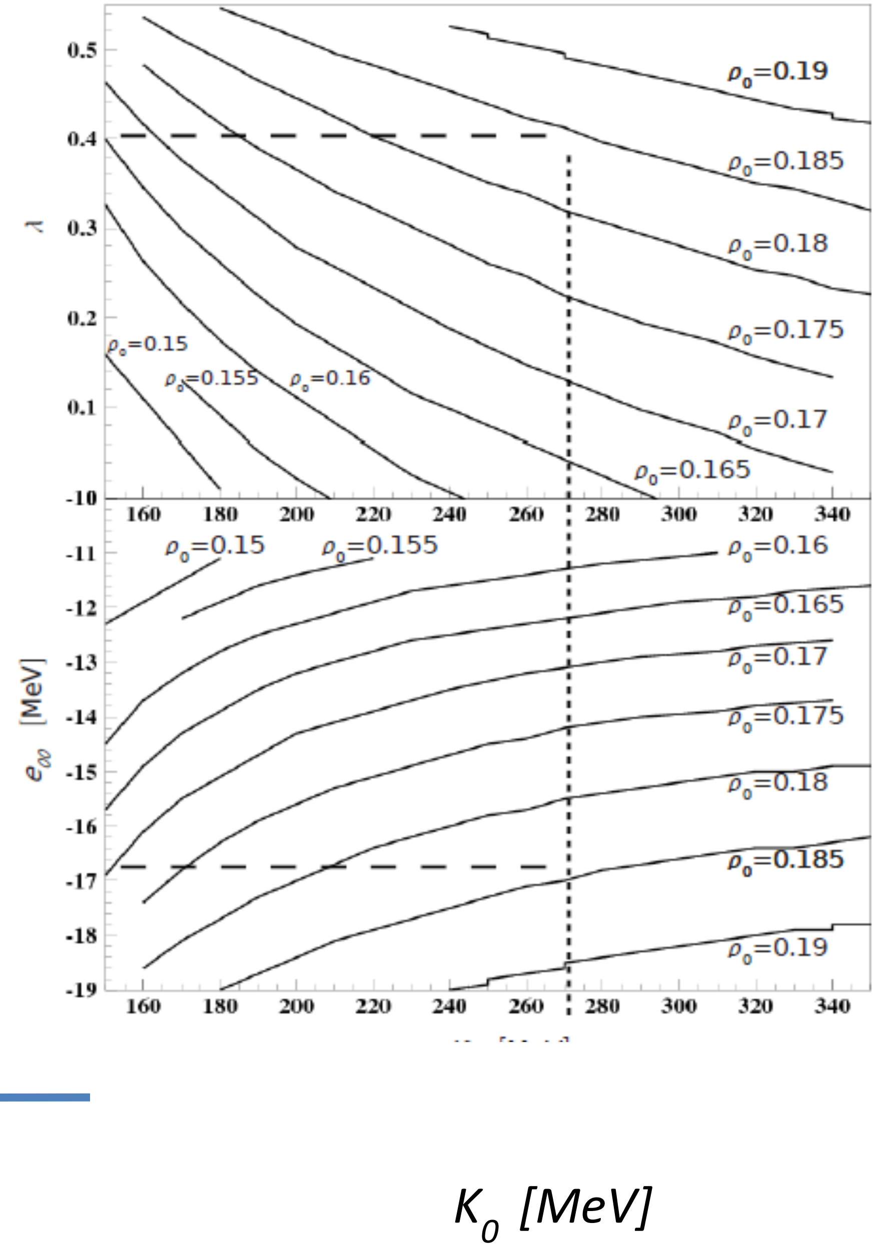
$\boldsymbol{e_{00}},\ \boldsymbol{K_0},\ \boldsymbol{\rho_0},\ \boldsymbol{\lambda}$

Ground state energy and radius of an **α** constrains 2 parameters

2 parameters remain free

There is a two dimensional subspace for which the model reproduces the energy and the radius of the ground state.

Assuming the value of ρ_0 and K_0 one can uniquely determine the parameters λ and e_{00}



Constraining the EoS parameters

To describe the properties of ^{3}H , ^{3}He particles it is sufficient to use EoS in form:

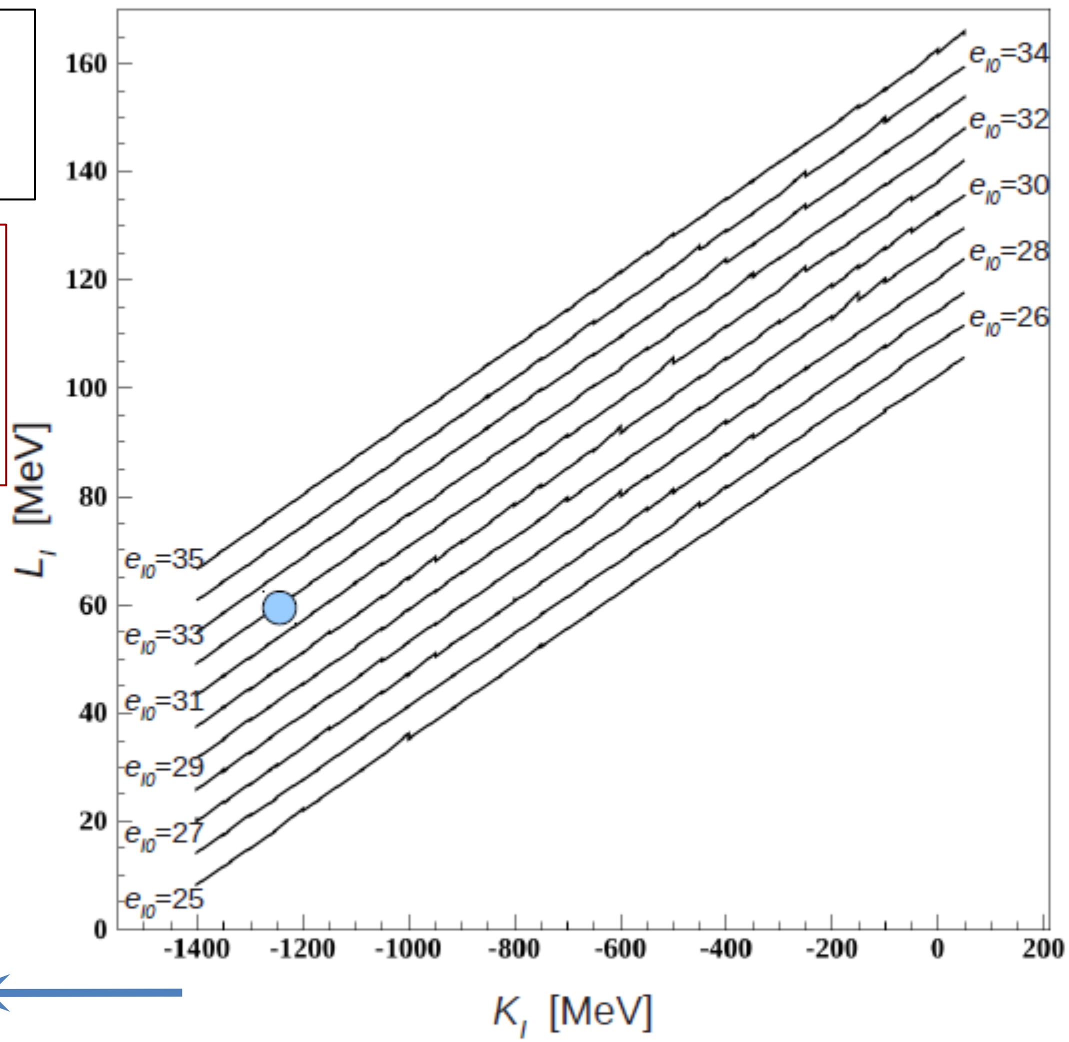
$$e = e_{00} + \frac{K_0}{18}\xi^2 + \delta^2\left(e_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2\right) + \left(\eta_n^2 + \eta_p^2\right)\left(e_{ii0} + \frac{L_{ii}}{3}\xi + \frac{K_{ii}}{18}\xi^2\right)$$

with predetermined parameters
$\boldsymbol{\rho_0}$, $\mathbf{K_0}$, $\mathbf{e_{00}}$, $\boldsymbol{\lambda}$ (4 parameters)

Ground state energies and radii of **^{3}H , ^{3}He** give **4** equations, for **6** new parameters
$\mathbf{e_{I0}}$, $\mathbf{L_I}$, $\mathbf{K_I}$, $\mathbf{e_{ii0}}$, $\mathbf{L_{ii}}$, $\mathbf{K_{ii}}$

Therefore we have also same correlations between
$\mathbf{e_{I0}}$, $\mathbf{L_I}$ and $\mathbf{K_I}$

Assuming the value of $\mathbf{e_{I0}}$ and $\mathbf{L_I}$ one can uniquely determine the parameter $\mathbf{K_I}$



Constraining the EoS parameters

For considered nuclei one can omit the $2\eta_n\eta_p\left(e_{ij0}+\frac{L_{ij}}{3}\xi+\frac{K_{ij}}{18}\xi^2\right)$ term in EoS

because $(\eta_n = 0$ or $\eta_p = 0)$

Thus, the constraints derived from the ground state properties of light charged particles: 3H , 3He and reduce the number of free parameters for defining the Hamiltonian to 4.

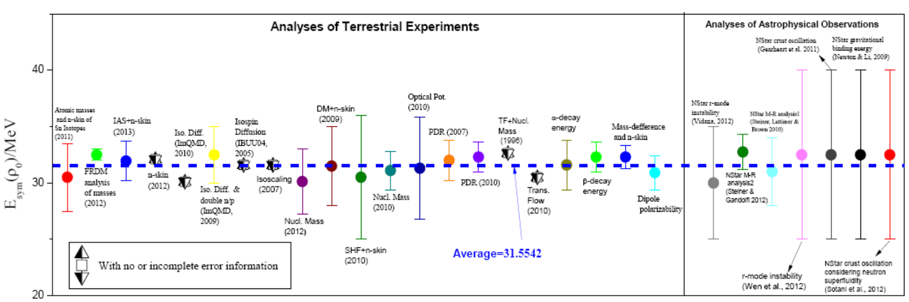
The values of e_{I0} and L_I have been assumed according to the recent experimental constraints of [Bao-An Li and Xiao Han, Phys. Lett. B 727 (2013) 276]:

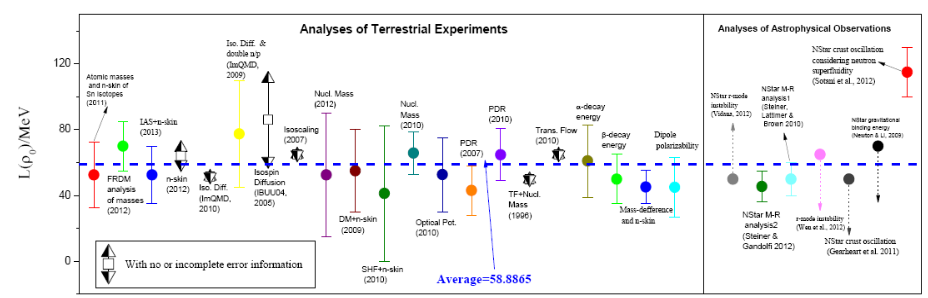
e_{I0} = 32 ± 1 MeV

L_I = 59 ± 17 MeV

which yielded the value of the symmetry compressibility

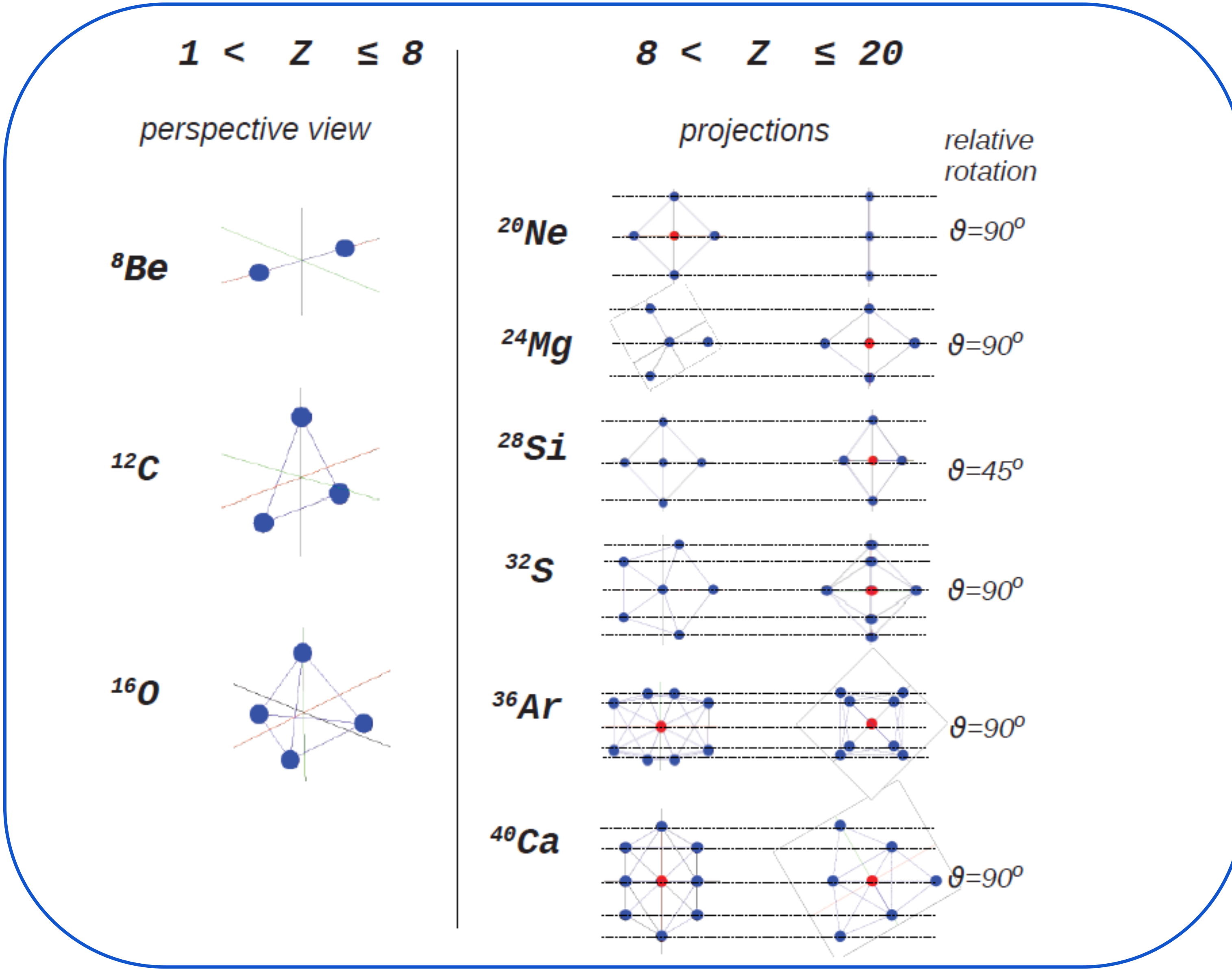
K_I = -1250 MeV (see blue circle on the previous slide)





Results of the model calculations – even- even nuclei with Z=N

In preprint: arXiv:1506.06731



Results of the model calculations – even- even nuclei with Z=N

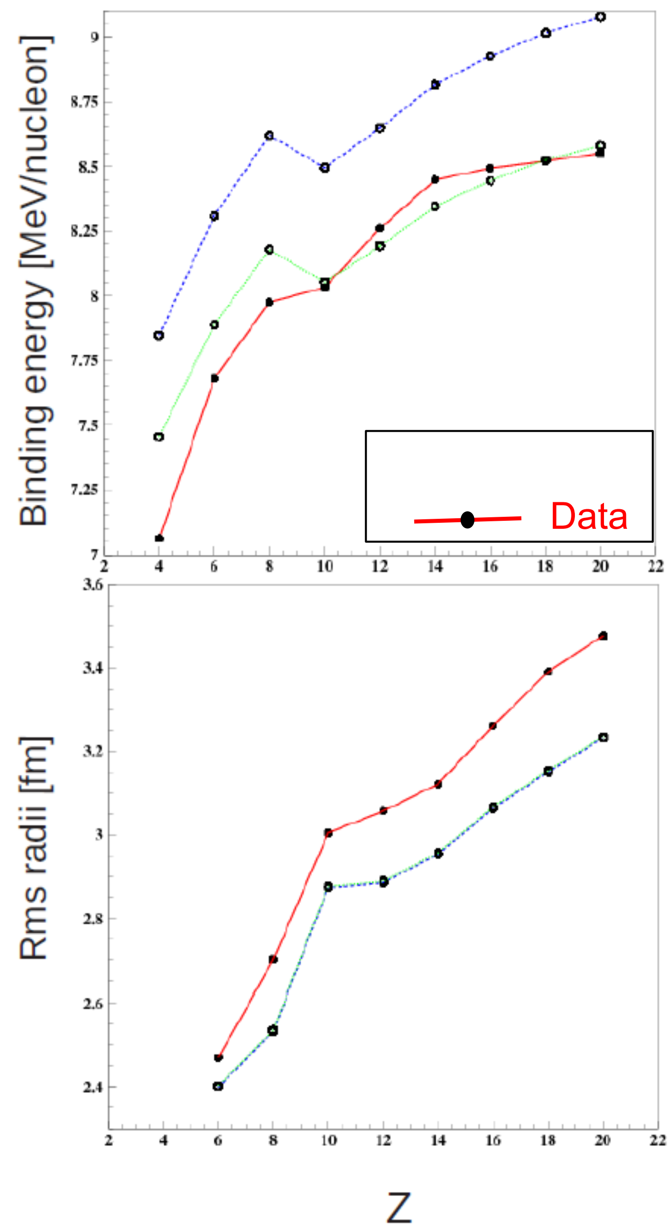
: model calculations for 4 free parameters (rest determined by ^{3}H , ^{3}He and **α** ground state properties)

: calculations for smaller value of $\mathbf{e_{00}}$

The results are not satisfactory !!!

The results are not satisfactory but follow the experimental trends

Fine corrections for Hamiltonian are needed

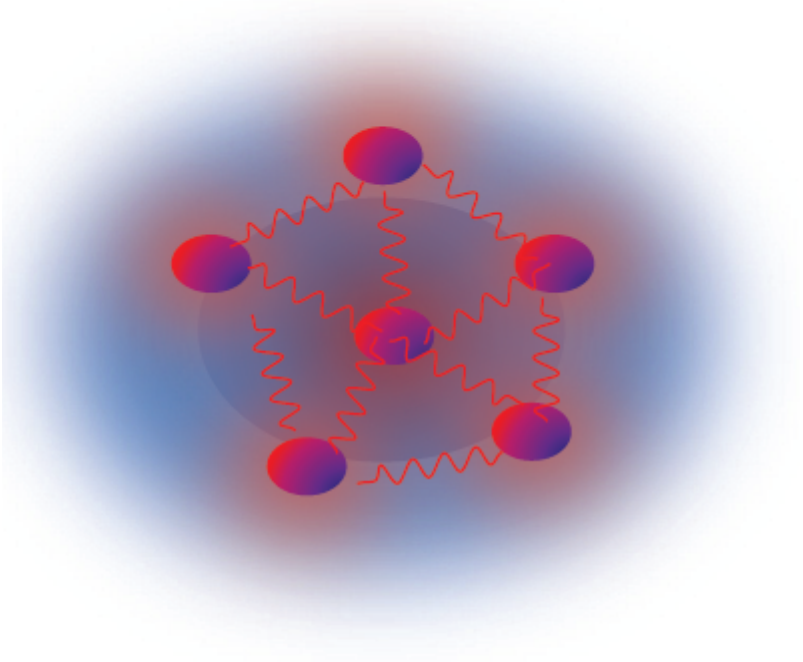


Results of the model calculations – even- even nuclei with Z=N,

Corrections for Hamiltonian

cluster interaction defined by harmonic oscillator potential

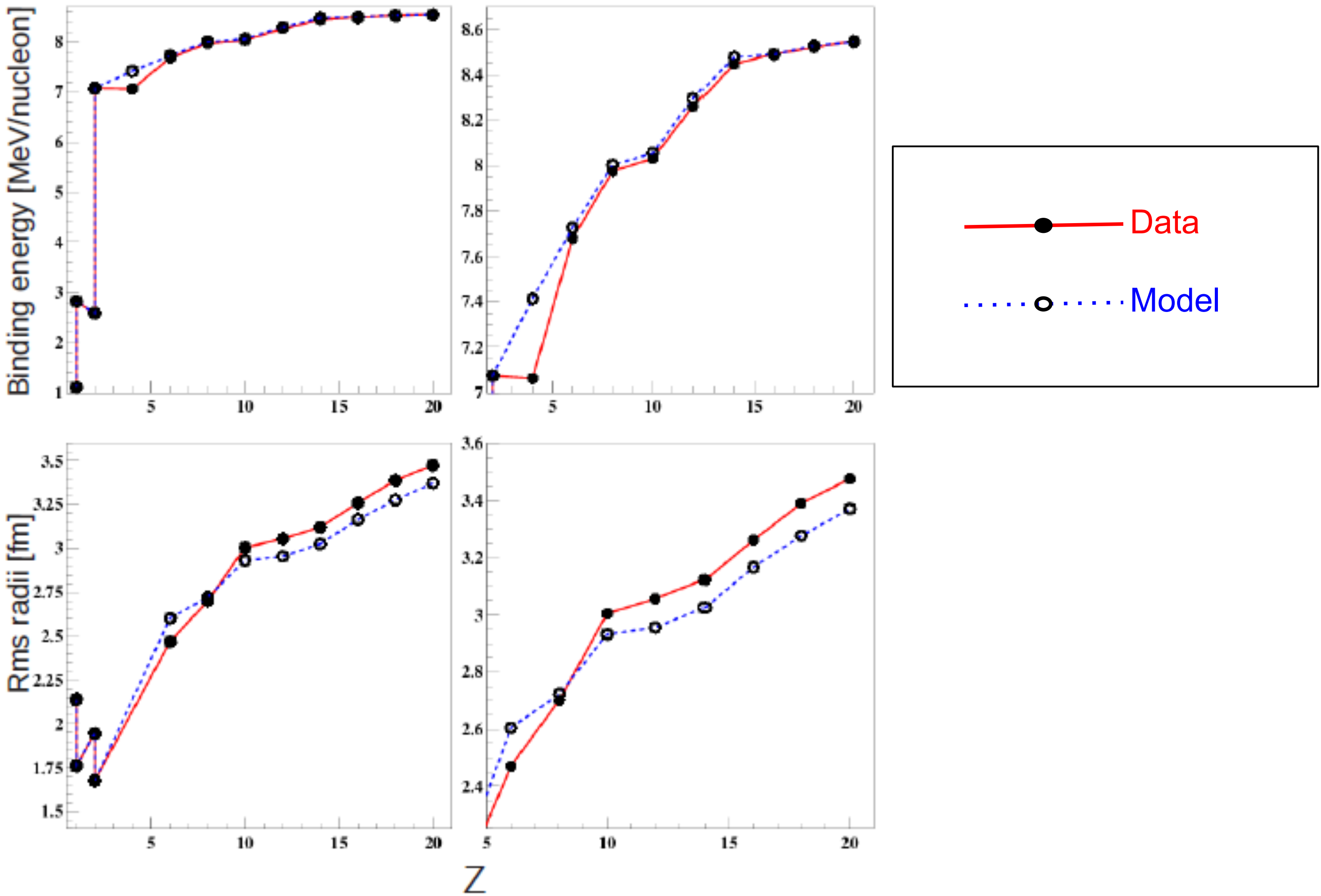
$$\langle \Phi \,|\Delta H_1|\, \Phi \rangle = \sum_{i \neq j} P_\alpha\,(i)\, P_\alpha\,(j)\, V_{\alpha\alpha}\,(d_{ij})$$



In preprint: arXiv:1506.06731

Results of the model calculations – even- even nuclei with Z=N,
Corrections for Hamiltonian

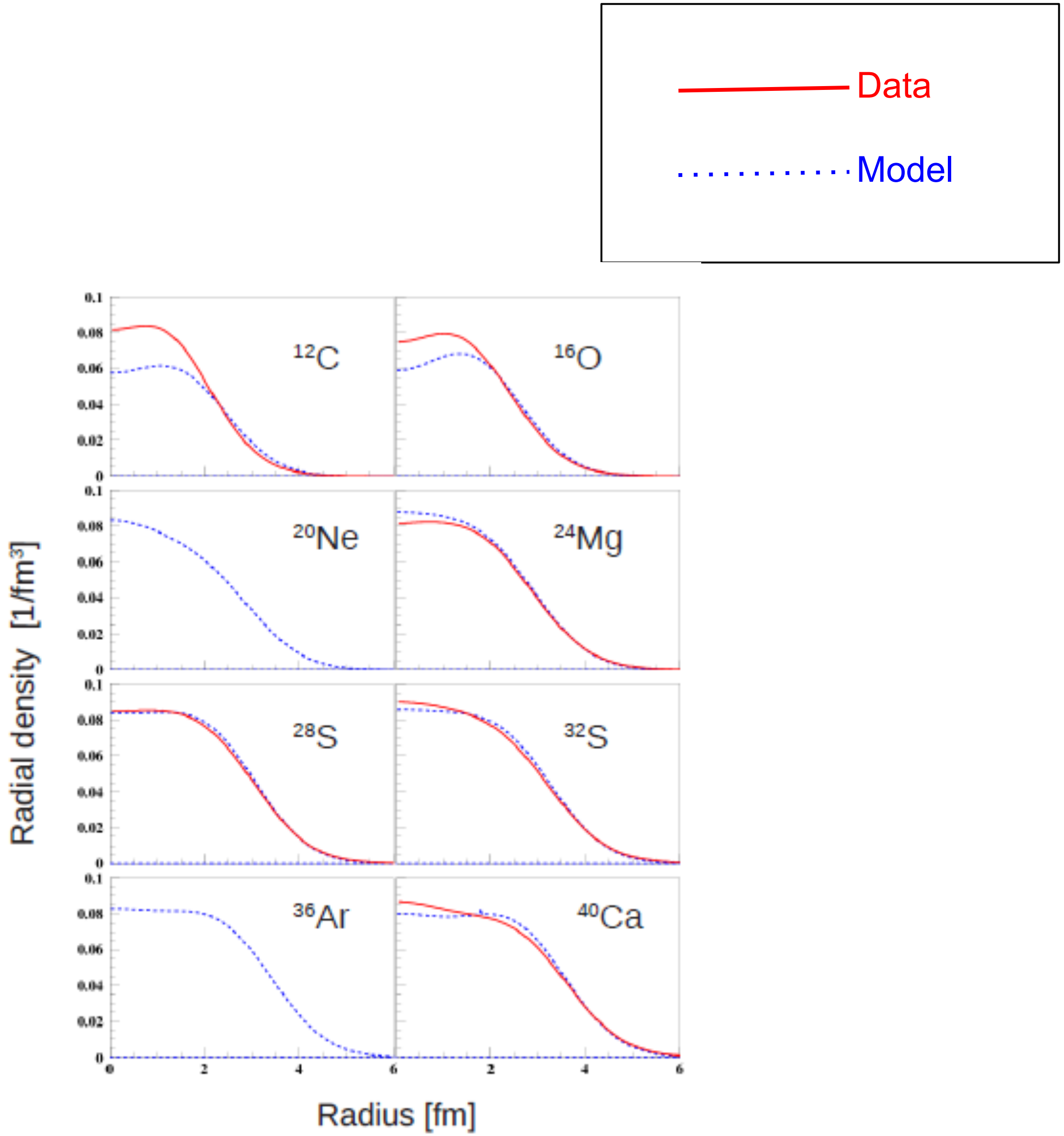
8 parameters - 44 ground state properties accurately reproduced, as a result of constraining of EoS parameters



Results of the model calculations – even- even nuclei with Z=N,

Corrections for Hamiltonian

Density profiles:



Summary and conclusions

- EoS of asymmetric nuclear matter around saturation density has been extended by introducing the spin dependent interactions.
- When the Hamiltonian is minimized while searching for the ground state configuration, the assumed interaction forces nucleons to form α-like structures in the α-like nuclei.
- At the low density nuclear surface, alpha-cluster sizes are comparable to the alpha particle size.
- Additional correction terms accounting for the cluster-cluster interactions were introduced in the Hamiltonian.
- The parameters of the Hamiltonian were constrained using the ground state properties of ^{3}H, ^{3}He and α particles and the available experimental constraints. Strong correlations between parameters have been found.
- The value of the symmetry compressibility parameter K_I has been found, from the analysis of the correlations between the other parameters, to be about -1250 MeV.
- The ground state binding energies, rms radii and density profiles of the α-like nuclei up to ^{40}Ca have been precisely reproduced using the model Hamiltonian with 8 free parameters.
- The ground state configurations of the α-like nuclei exhibit a high level of symmetry with an α-like structure in the “core” for 8 < Z ≤ 20 nuclei and a “core” of a mass of a double α-structure for 20 < Z ≤ 28 nuclei.
- The experimental binding energies for α-like nuclei with Z>8 were reproduced with an order of magnitude better accuracy than when using the LDM.