NUSYM15, Krakow, 30. 6. 2015

Quantum statistical approach to few-nucleon correlations in nuclear systems

Gerd Röpke, Rostock

Quantum statistics

- Symmetry energy: well defined? (Phantasm – reality)

- System in equilibrium: temperature T, volume Ω, particle numbers N_c (conserved)

  Thermodynamic potential: free energy $F(T, \Omega, N_c)$

  Internal energy $U(T, \Omega, N_c)$

- Nuclear systems, N_c: neutrons n_n, protons n_p, electrons n_e, …

- Nuclear structure $T=0$,

  astrophysics, heavy ion reactions (HIC): finite T

- Interaction: strong - Coulomb

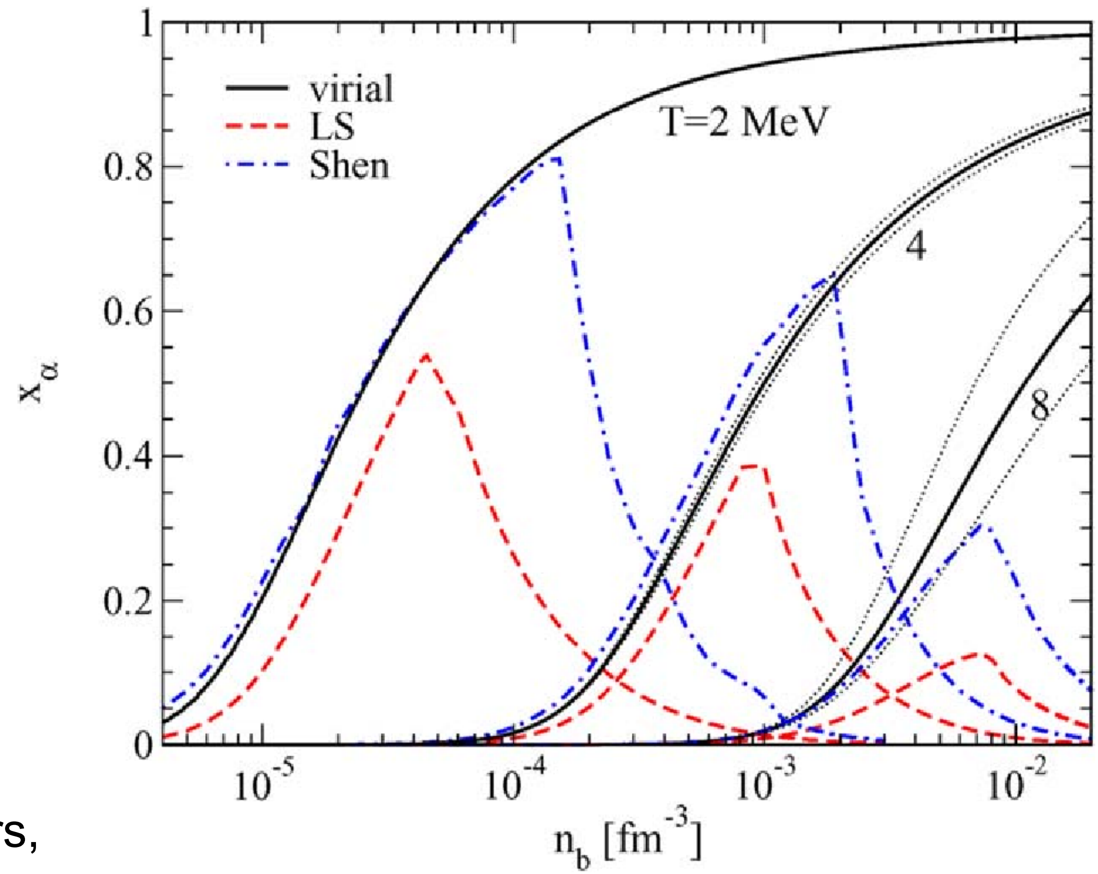
  Separation of the Coulomb part: Interaction energy and structure?

Known results for the EOS

- Low-density limit: ideal quantum gas
- Second virial coefficient: Beth—Uhlenbeck
- Nuclear statistical equilibrium (NSE)
- Cluster-virial expansion

Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



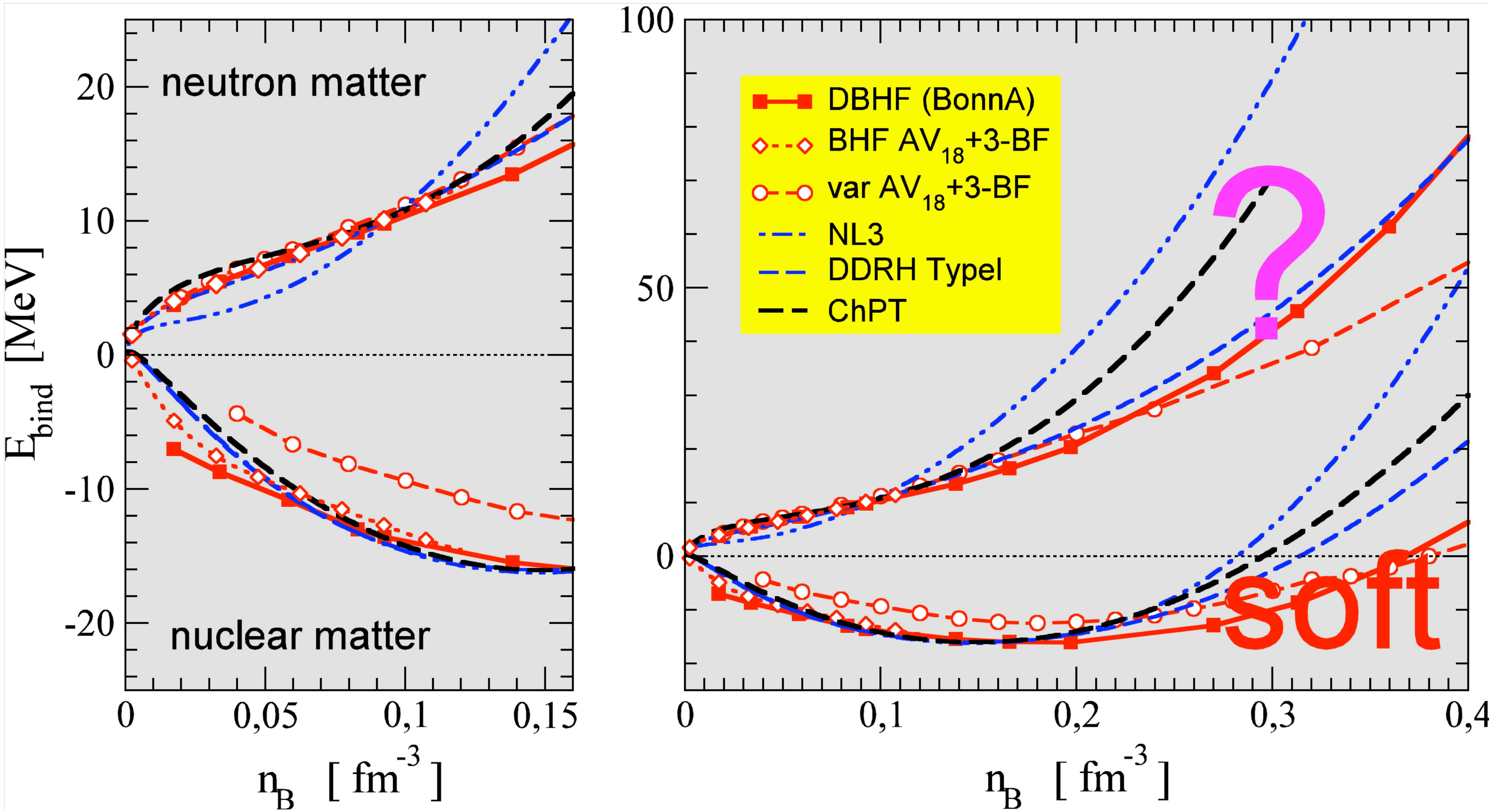
LS, Shen:
higher clusters,
excluded volume

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Known results for the EOS

- Low-density limit: ideal quantum gas
- Second virial coefficient: Beth—Uhlenbeck
- Nuclear statistical equilibrium (NSE)
- Cluster-virial expansion
- Saturation density
- Skyrme, relativistic mean-field: Quasiparticles
- Density functional theory

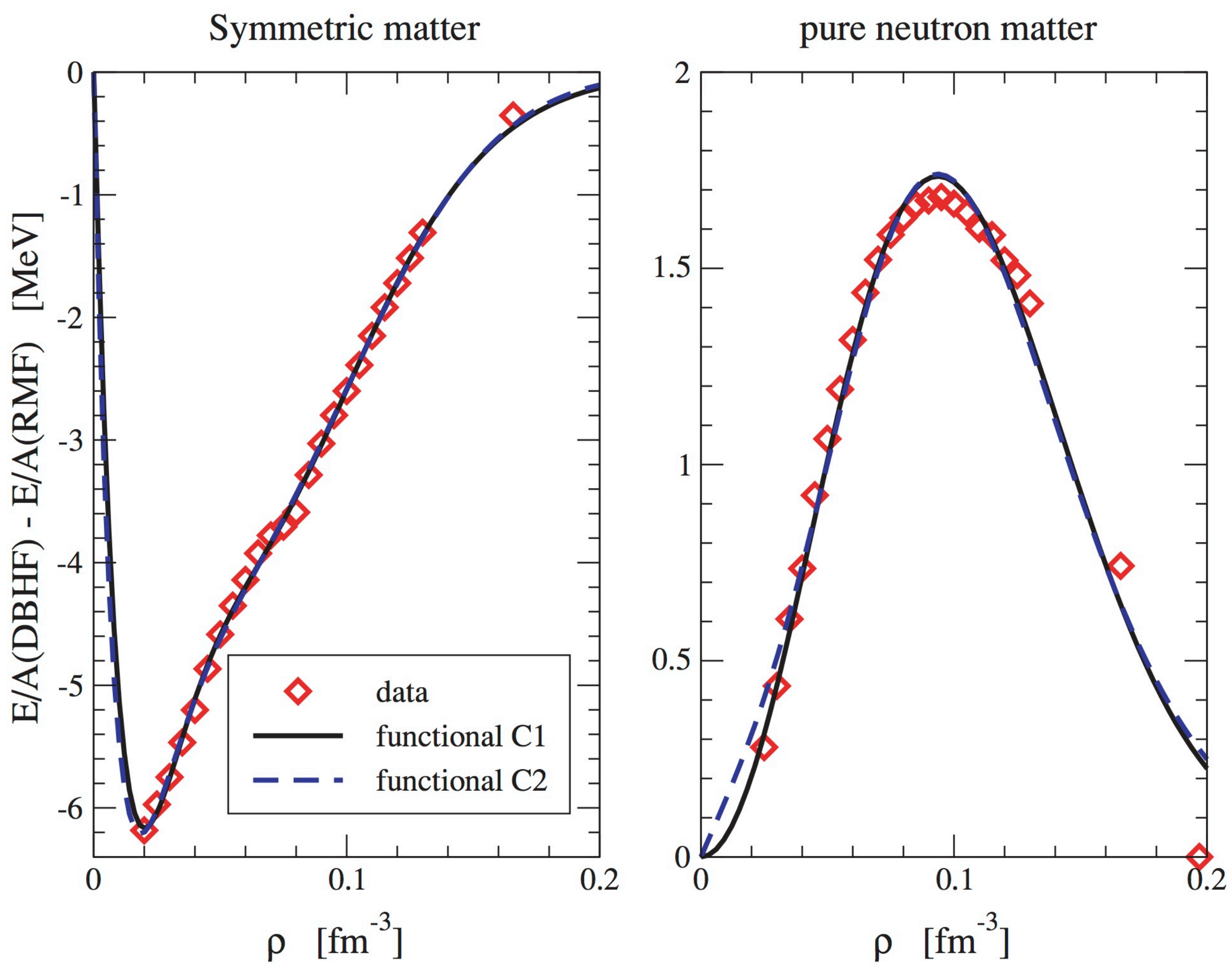
Quasiparticle picture: RMF and DBHF



But: cluster formation

C. Fuchs, H.H. Wolter, Eur. Phys. J. A 30, 5 (2006)

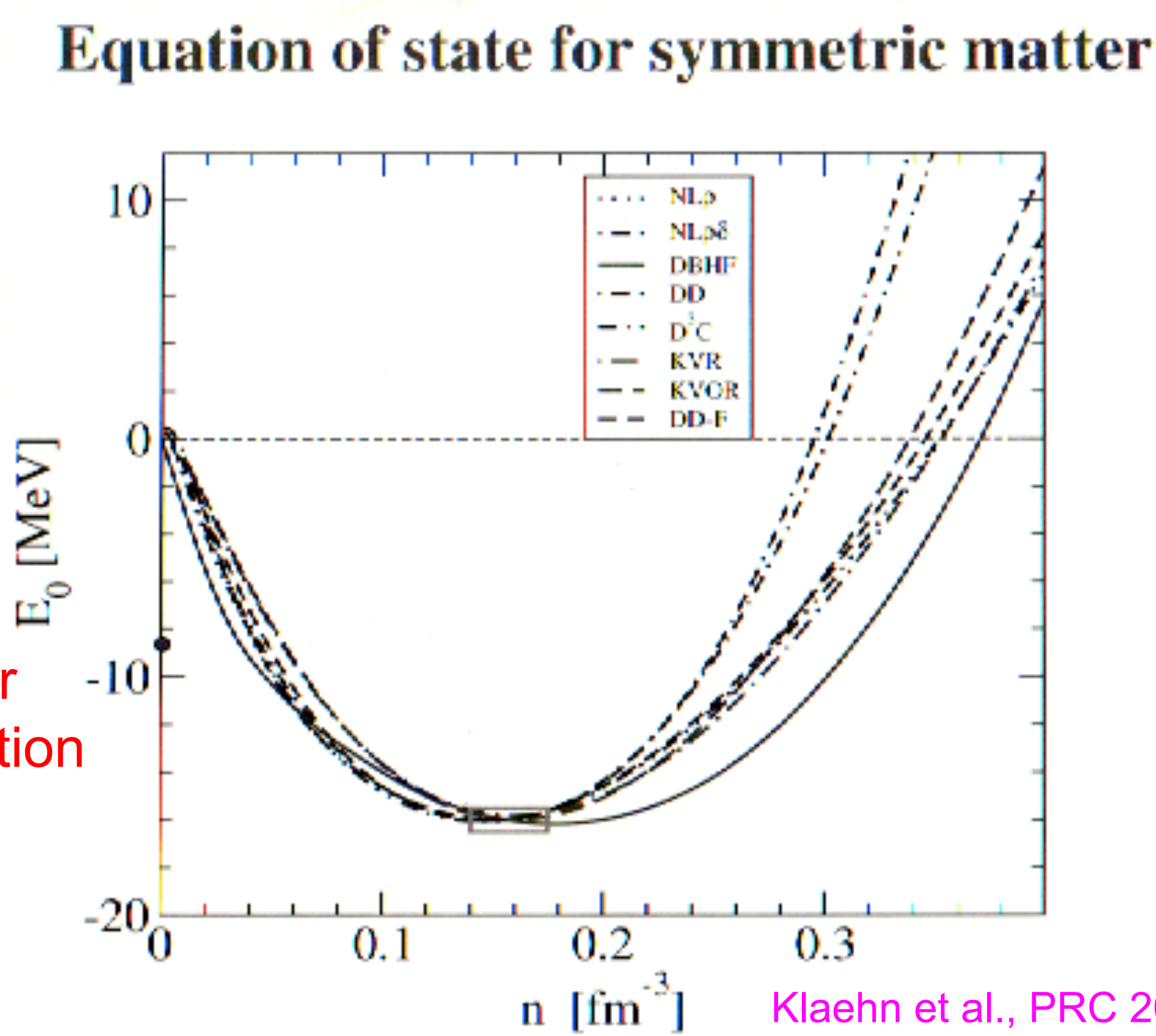
DBHF at low densities



Difference between DBHF calculation and low-density RMF fit (square symbols). The corrections (solid lines) are drawn for symmetric nuclear matter (left panel) and pure neutron matter (right panel)

J. Margeron, E. van Dalen, C. Fuchs, Phys. Rev. C **76,** 034309 (2007)

Quasiparticle approximation for nuclear matter

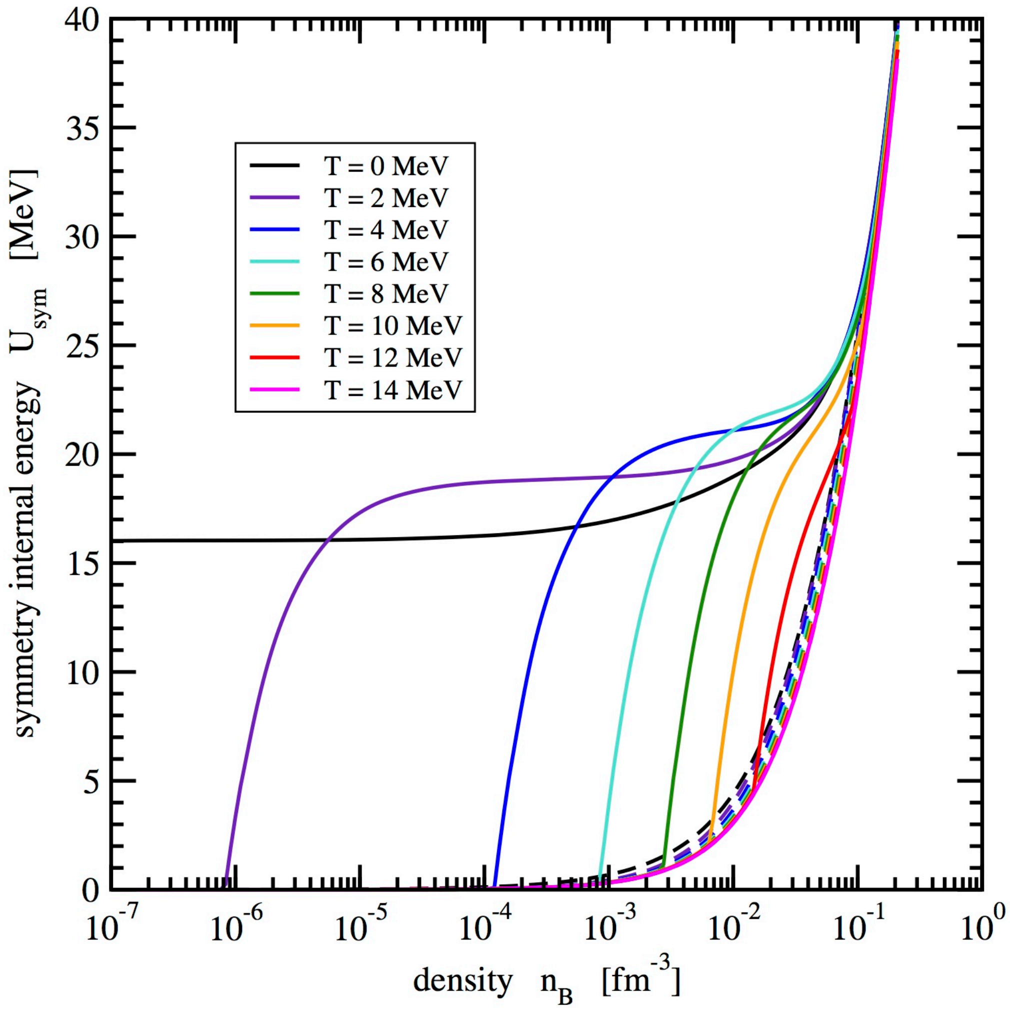


But: cluster formation

Incorrect low-density limit

Klaehn et al., PRC 2006

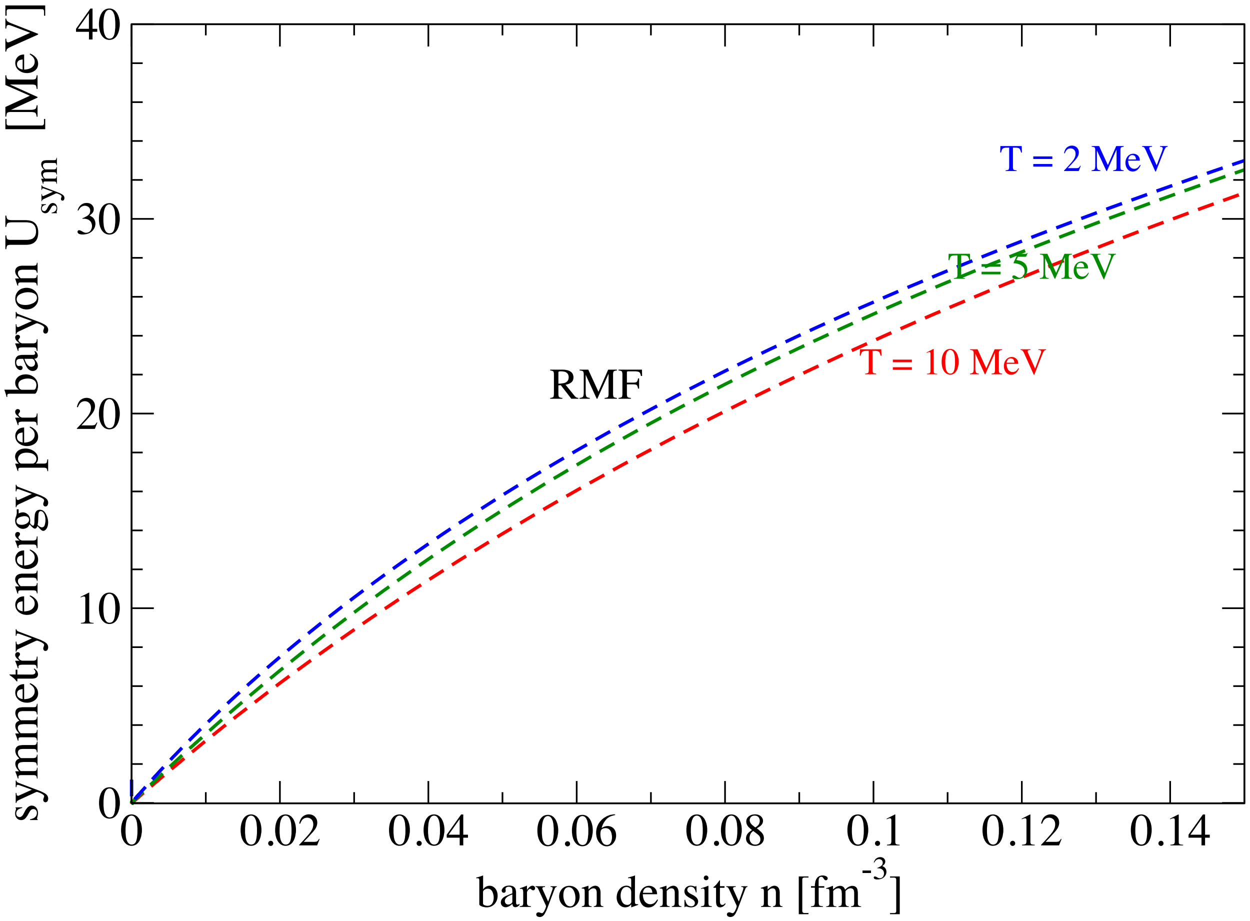
Symmetry energy and phase transition



Symmetry internal energy
U_{sym}
in nuclear matter
without cluster formation,
without (dashed lines)
and with (full lines)
liquid-gas phase transition,
as a function of the
baryon density n_B

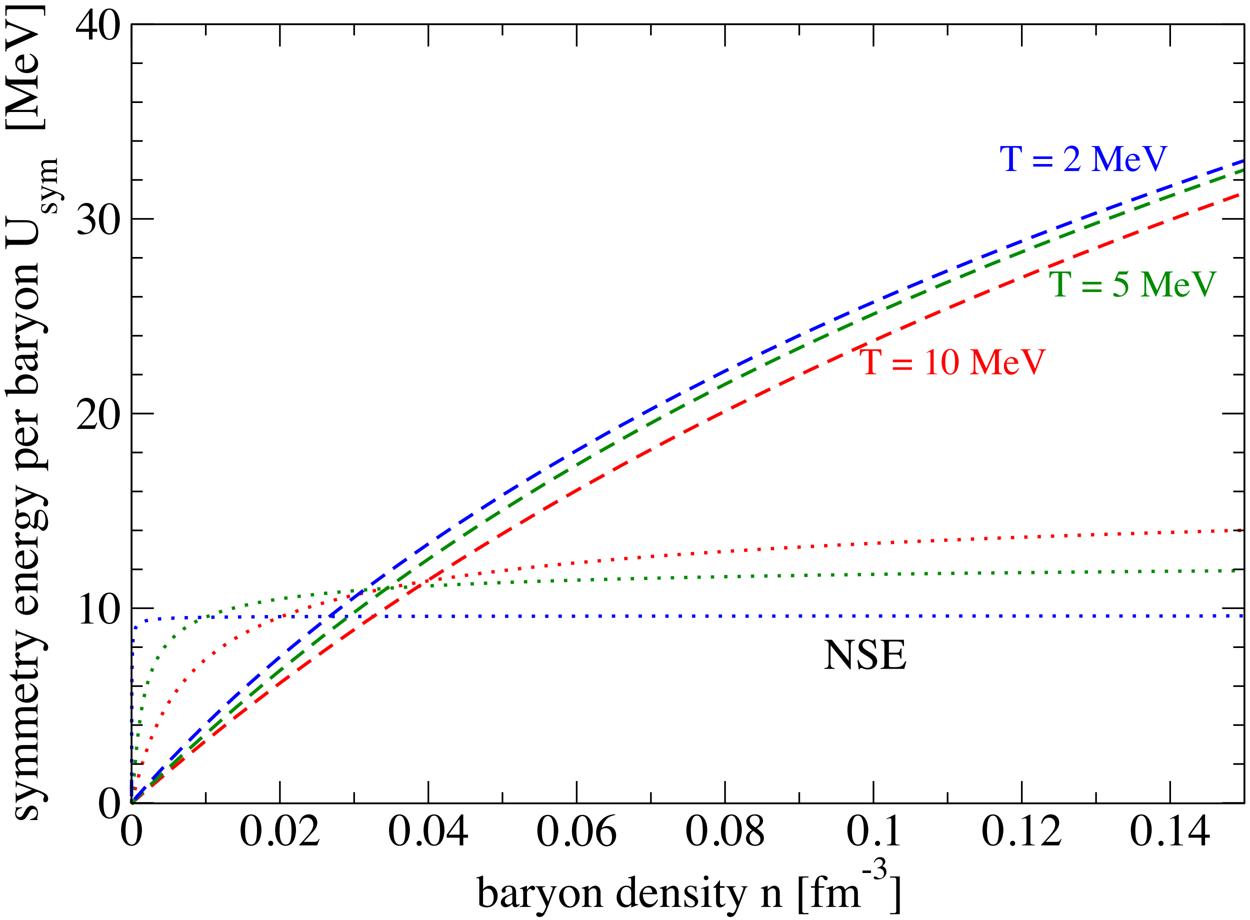
S. Typel et al., Eur. Phys. J. A (2014) 50: 17

Light clusters and symmetry energy



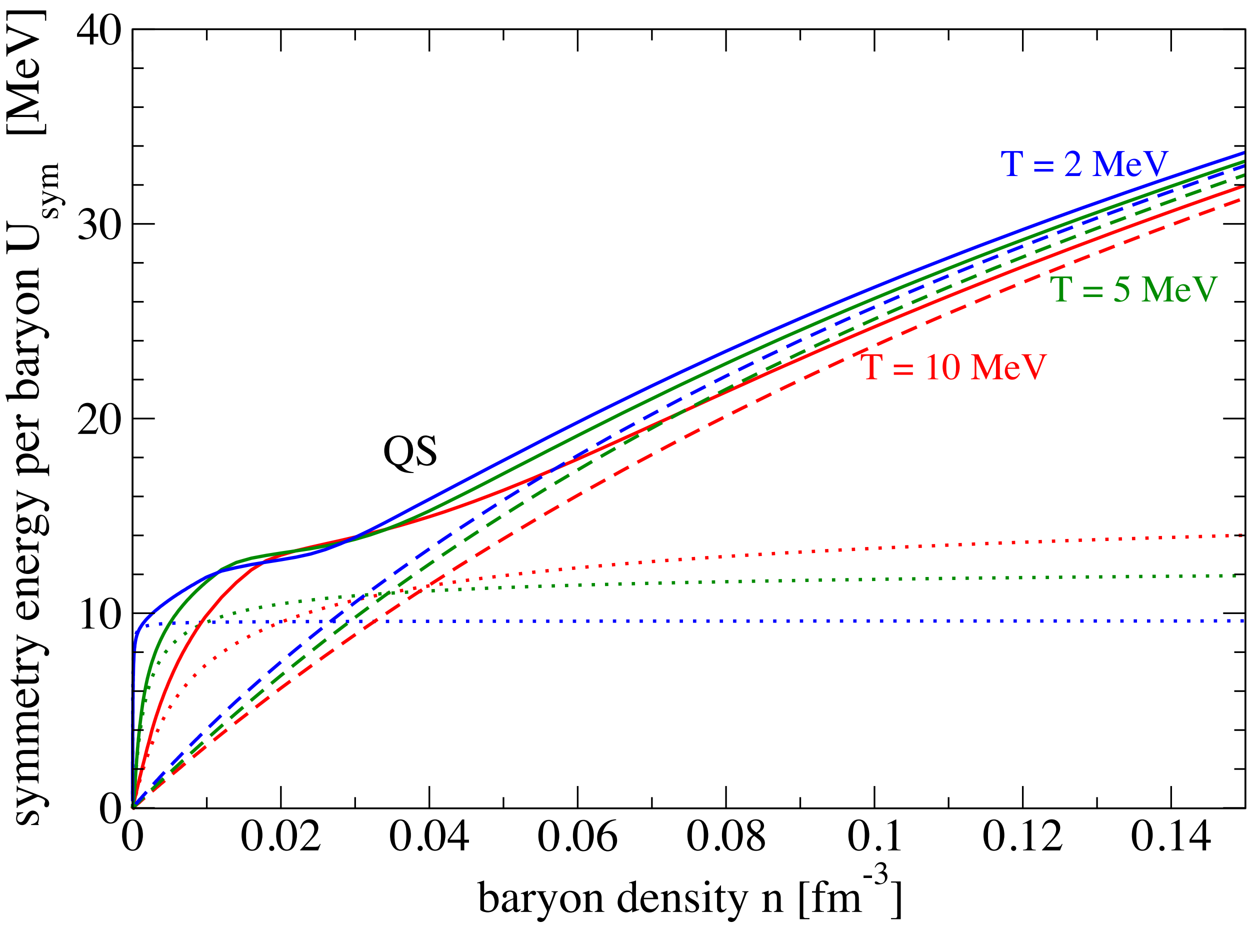
K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Light clusters and symmetry energy



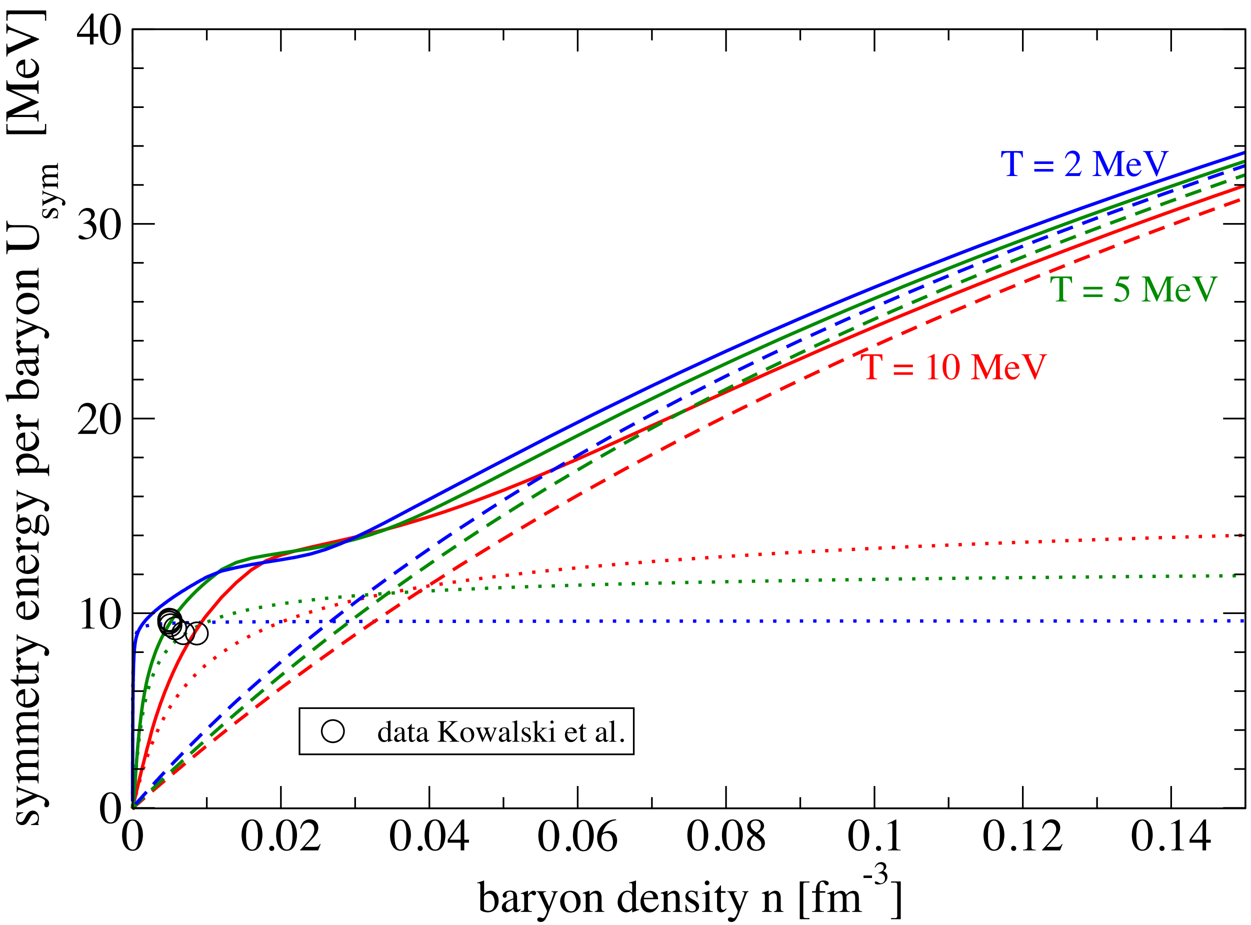
K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Light clusters and symmetry energy



K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Light clusters and symmetry energy



K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1, \omega) = \frac{2\text{Im}\, \Sigma(1, \omega + i0)}{[\omega - E(1) - \text{Re}\, \Sigma(1, \omega)]^2 + [\text{Im}\, \Sigma(1, \omega + i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations, correct virial expansions

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

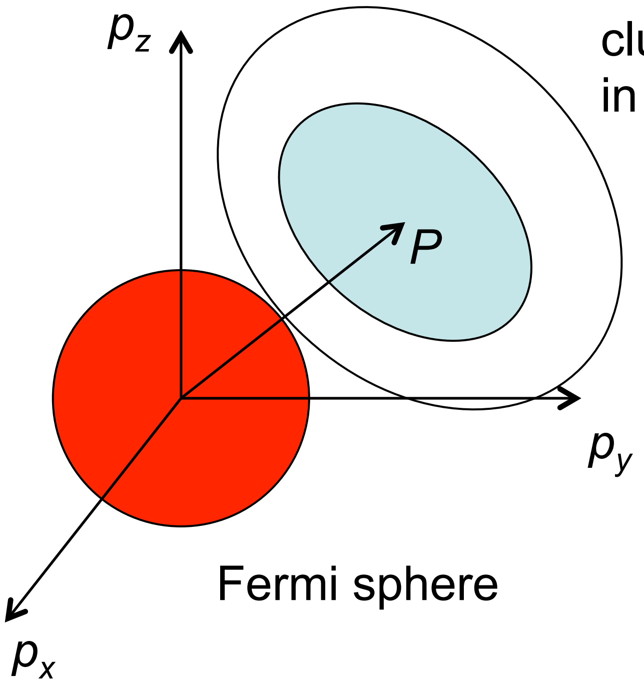
- Bose-Einstein condensation, phase instabilities

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$
\begin{aligned}
&\left(\left[E^{HF}(p_1) + E^{HF}(p_2) + E^{HF}(p_3) + E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4) \\
&+ \sum_{p_1',p_2'} (1 - f_{p_1} - f_{p_2}) V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4) \\
&+ \{permutations\} \\
&= E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)
\end{aligned}
$$

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

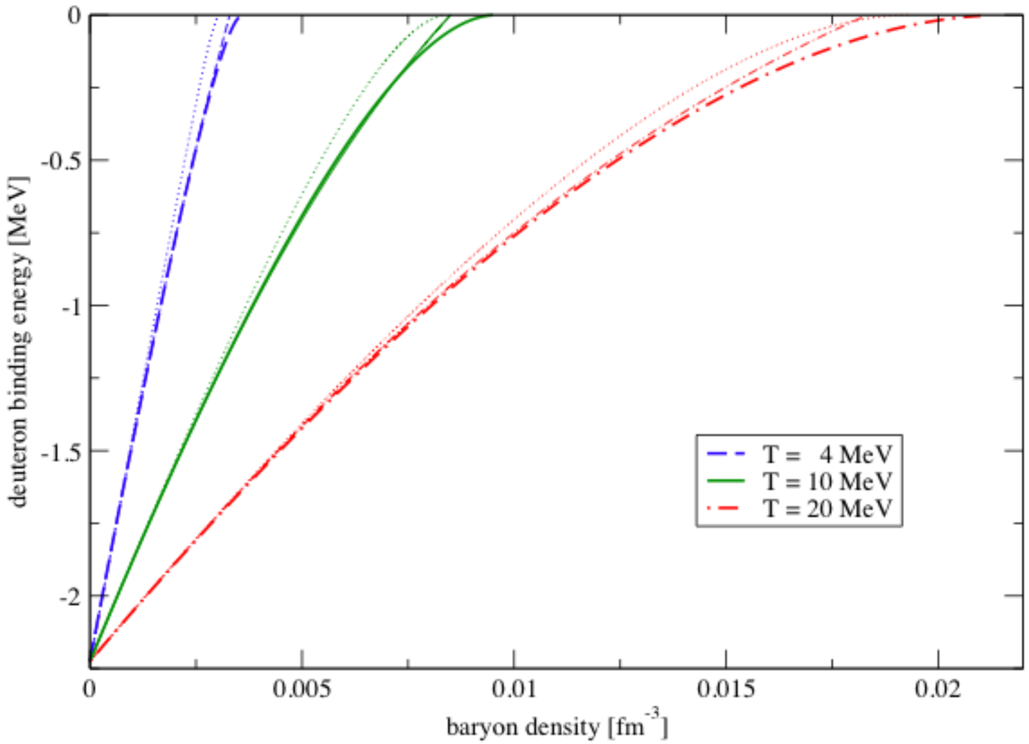
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum



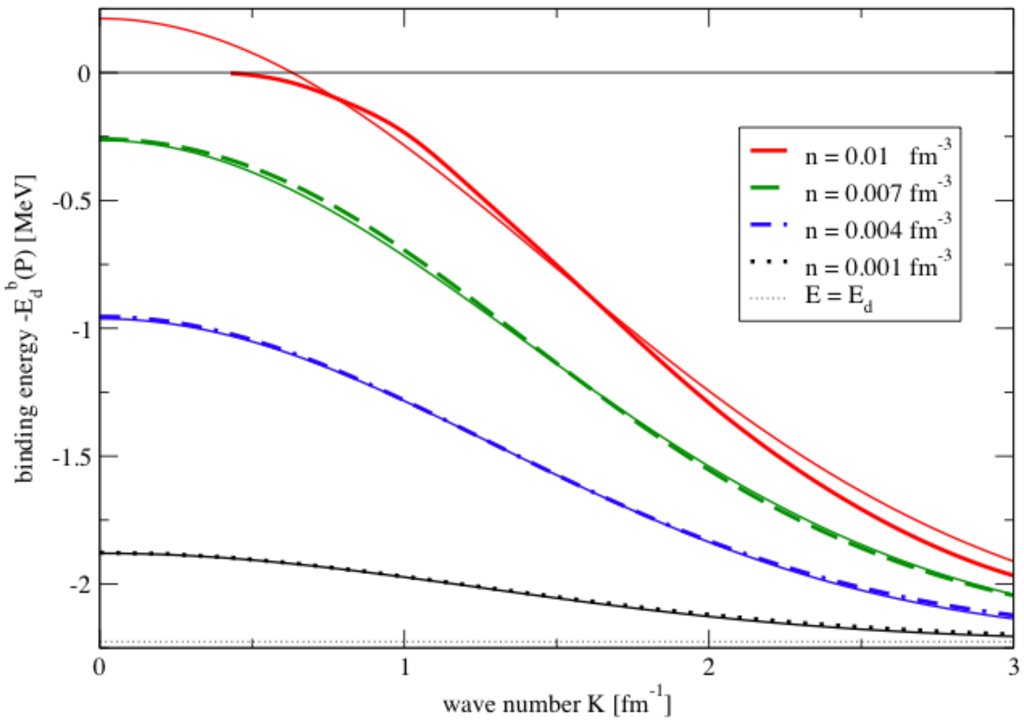
thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Shift of the deuteron bound state energy

Dependence on center of mass momentum,
various densities, T=10 MeV



thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

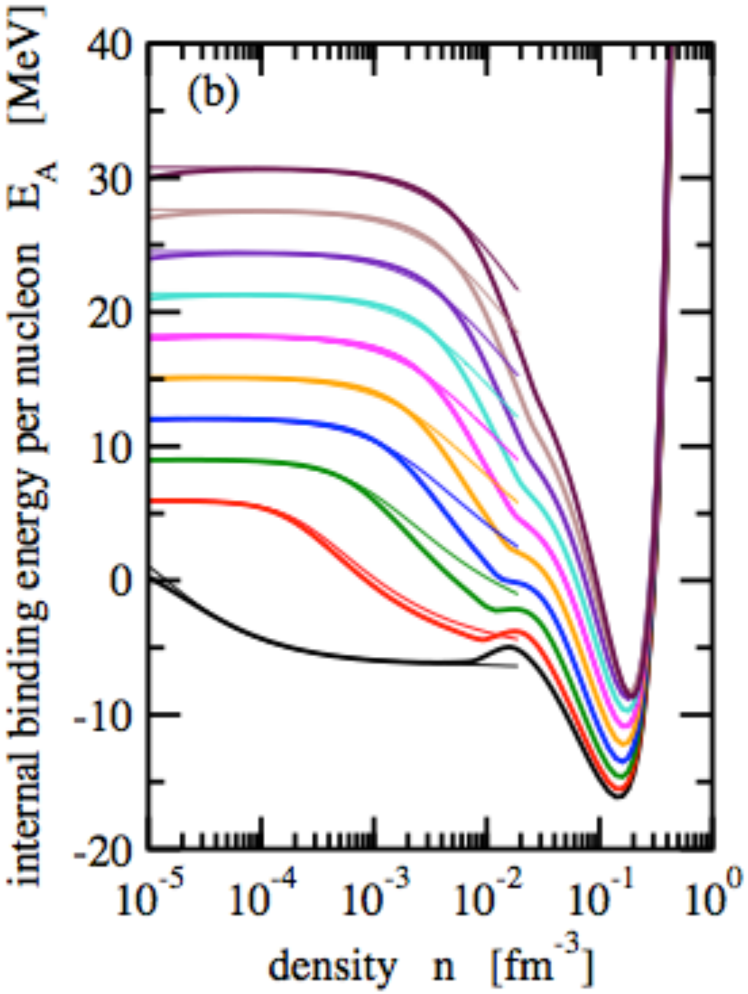
Internal energy per nucleon

Symmetric matter

Isotherms

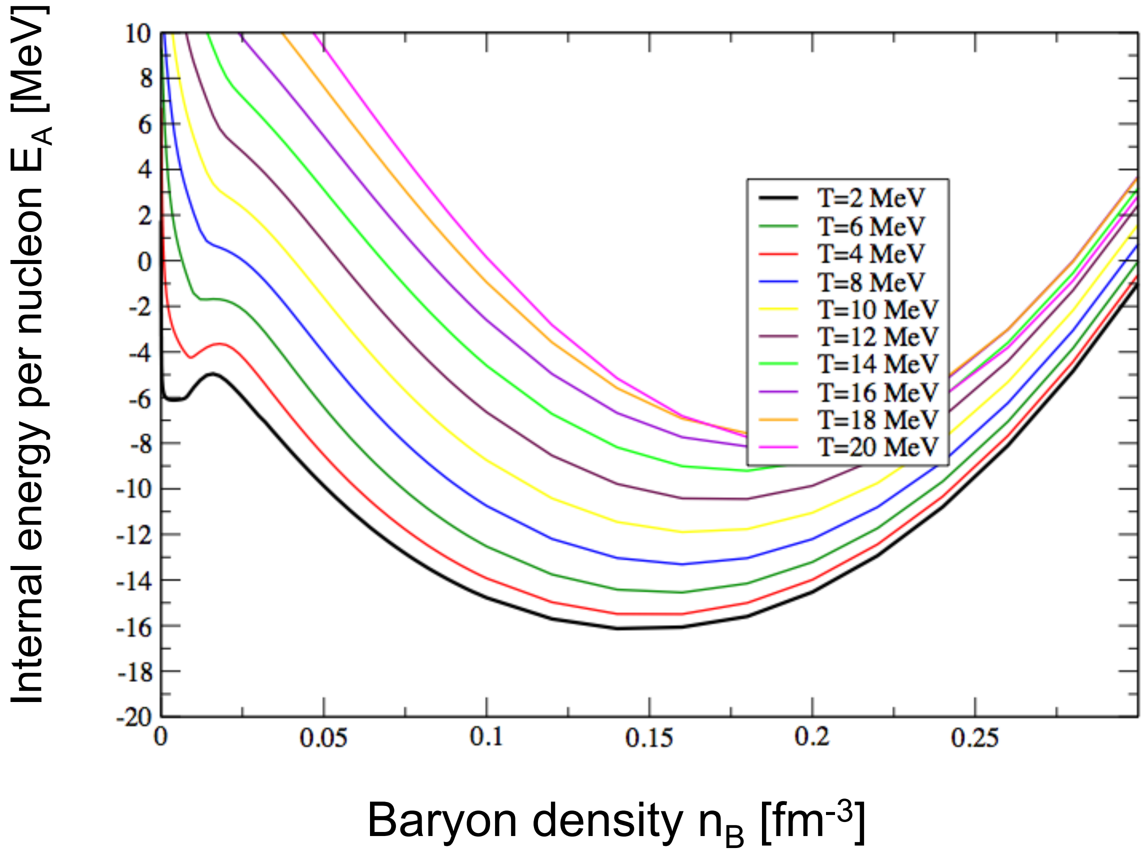
T[MeV]
20
18
16
14
12
10
8
6
4
2

thin lines: NSE



S. Typel et al., PRC **81**, 015803 (2010)

Internal energy per nucleon



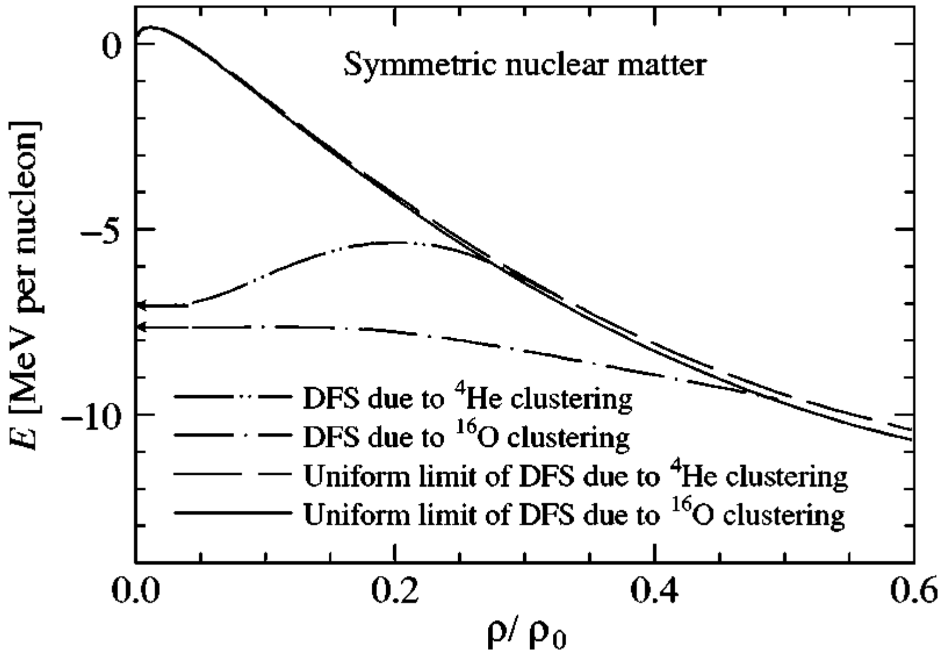
Quantum statistical approach:

Cluster ?

Condensate?

EOS for symmetric matter - low density region?

Clustering phenomena in nuclear matter below the saturation density



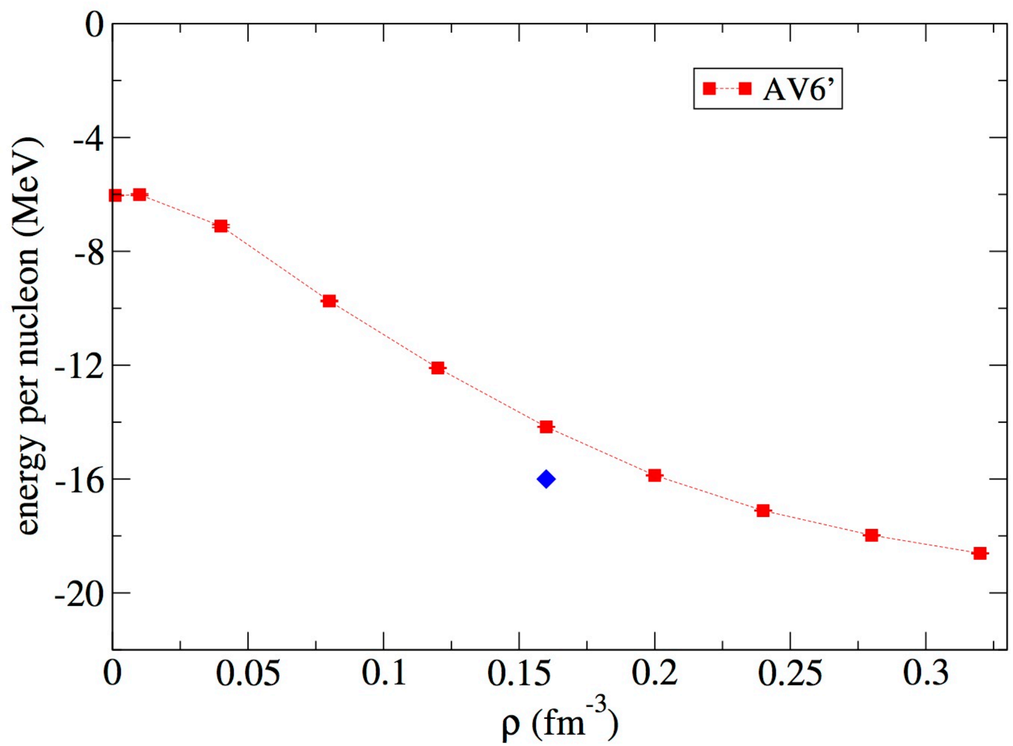
- FIG. 8. Energy curves of DFSs due to a and 16O clustering in
- the symmetric nuclear matter by the use of the BB sB4d force. The
- density of matter is normalized by the saturation density of the
- uniform matter with the Fermi sphere, r0=0.206 fm−3. The presentation
- of the curves is similar to that in Fig. 4.

Hiroki Takemoto et al.,
PR C **69**, 035802 (2004)

Stefano Gandolfi’s talk

EOS of symmetric nuclear matter using Argonne AV6’ (no three-body).

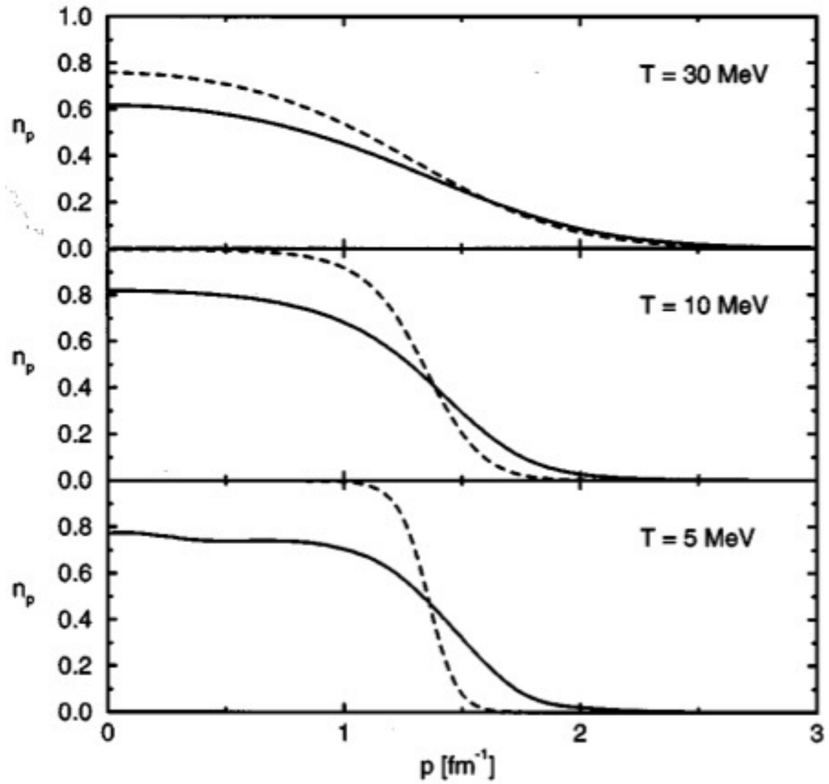
Low density VERY PRELIMINARY!!!



Gandolfi, Lovato, Carlson, Schmidt, PRC (2014)

Single nucleon distribution function

Dependence on temperature



saturation density

Alm et al., PRC 53, 2181 (1996)

Pauli blocking, correlated medium

In-medium Schroedinger equation

$$[E_{\tau_1}(\mathbf{p}_1;T,\mu_n,\mu_p)+\cdots+E_{\tau_A}(\mathbf{p}_A;T,\mu_n,\mu_p)-E_{A\nu}(\mathbf{P};T,\mu_n,\mu_p)]\psi_{A\nu\mathbf{P}}(1\dots A)$$
$$+\sum_{1'\dots A'}\sum_{i<j}[1-n(i;T,\mu_n,\mu_p)-n(j;T,\mu_n,\mu_p)]V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu\mathbf{P}}(1'\dots i'\dots j'\dots A')=0$$

effective occupation numbers

$$n(1)=f_{1,\tau_1}(1)+\sum_{B=2}^{\infty}\sum_{\bar\nu,\bar{\mathbf{P}}}\sum_{2\dots B}B f_B\left(E_{B,\bar\nu}(\bar{\mathbf{P}};T,\mu_n,\mu_p)\right)|\psi_{B\bar\nu\bar{\mathbf{P}}}(1\dots B)|^2$$

effective Fermi distribution

$$n(1;T,\mu_n,\mu_p)\approx f_{1,\tau_1}(1;T_{\rm eff},\mu_n^{\rm eff},\mu_p^{\rm eff})$$

blocking by all nucleons

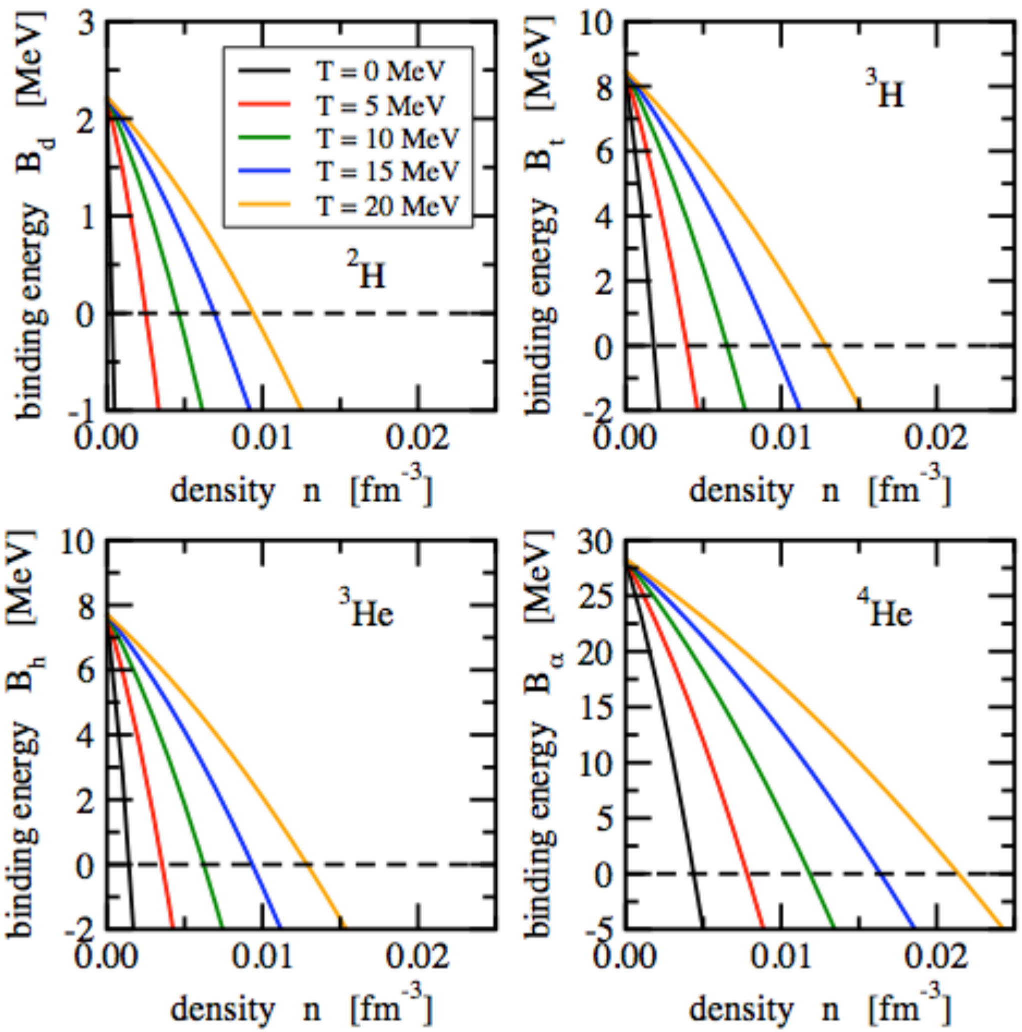
$$n(1;T,\mu_n,\mu_p)\approx \tilde f_{1,\tau_1}(1;T_{\rm eff},n_B,Y_p)$$

effective temperature

$$T_{\rm eff}\approx 5.5\,{\rm MeV}+0.5\,T+60\,n_B\ {\rm MeV\ fm}^3$$

G. Roepke, arXiv: 1411.4593, submitted (PRC)

Shift of Binding Energies of Light Clusters

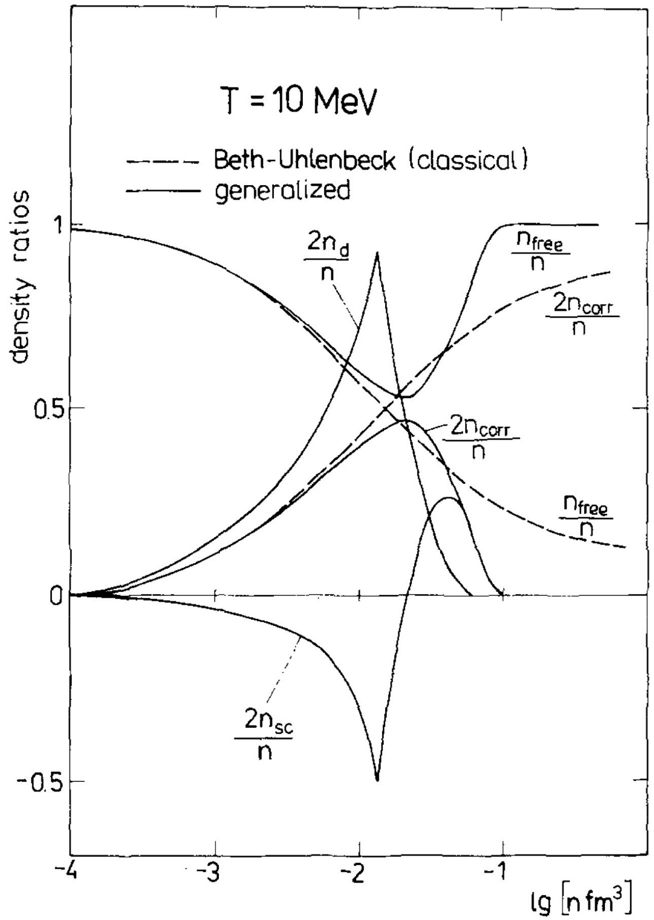


Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Two-particle correlations

Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter

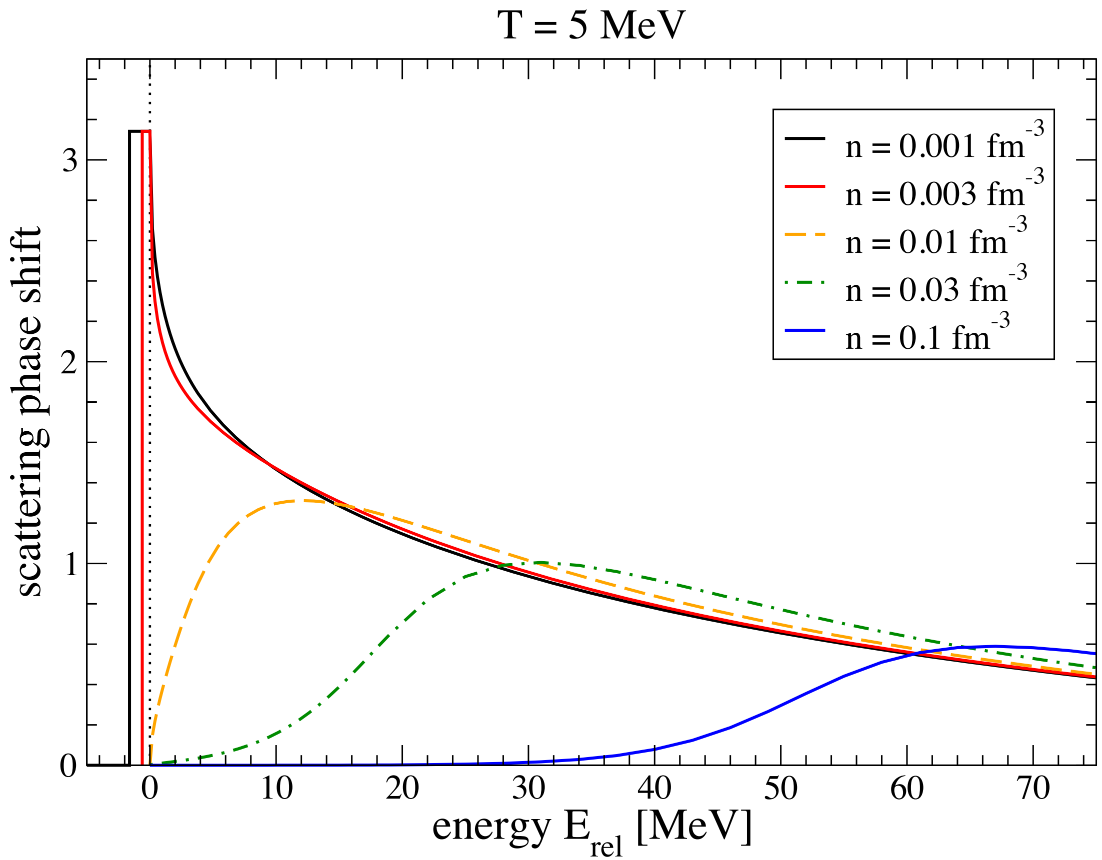


M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014).

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

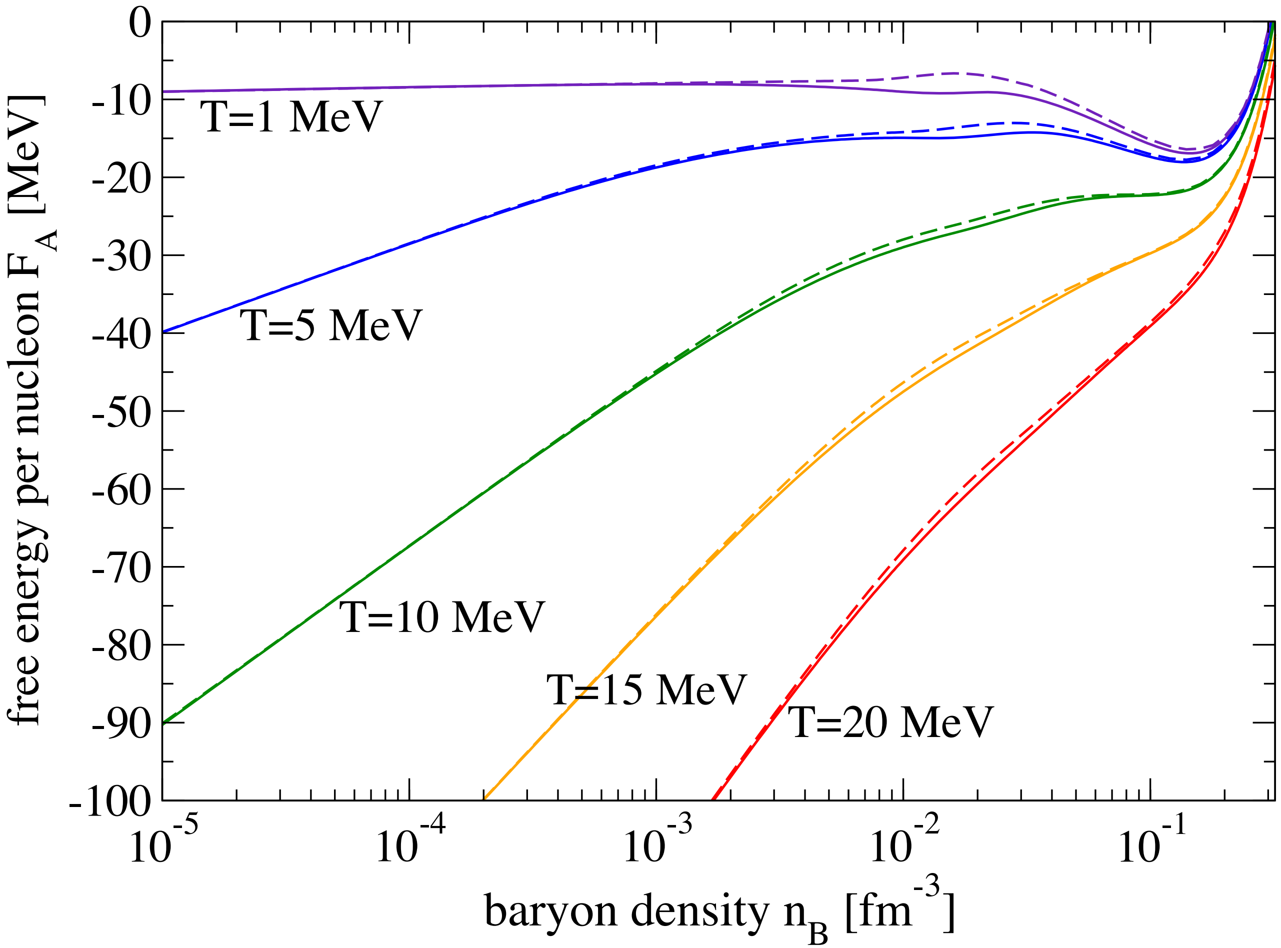
parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\; e^{-0.102424\; T/\mathrm{MeV}}$$

G. Roepke, arXiv: 1411.4593, submitted (PRC)

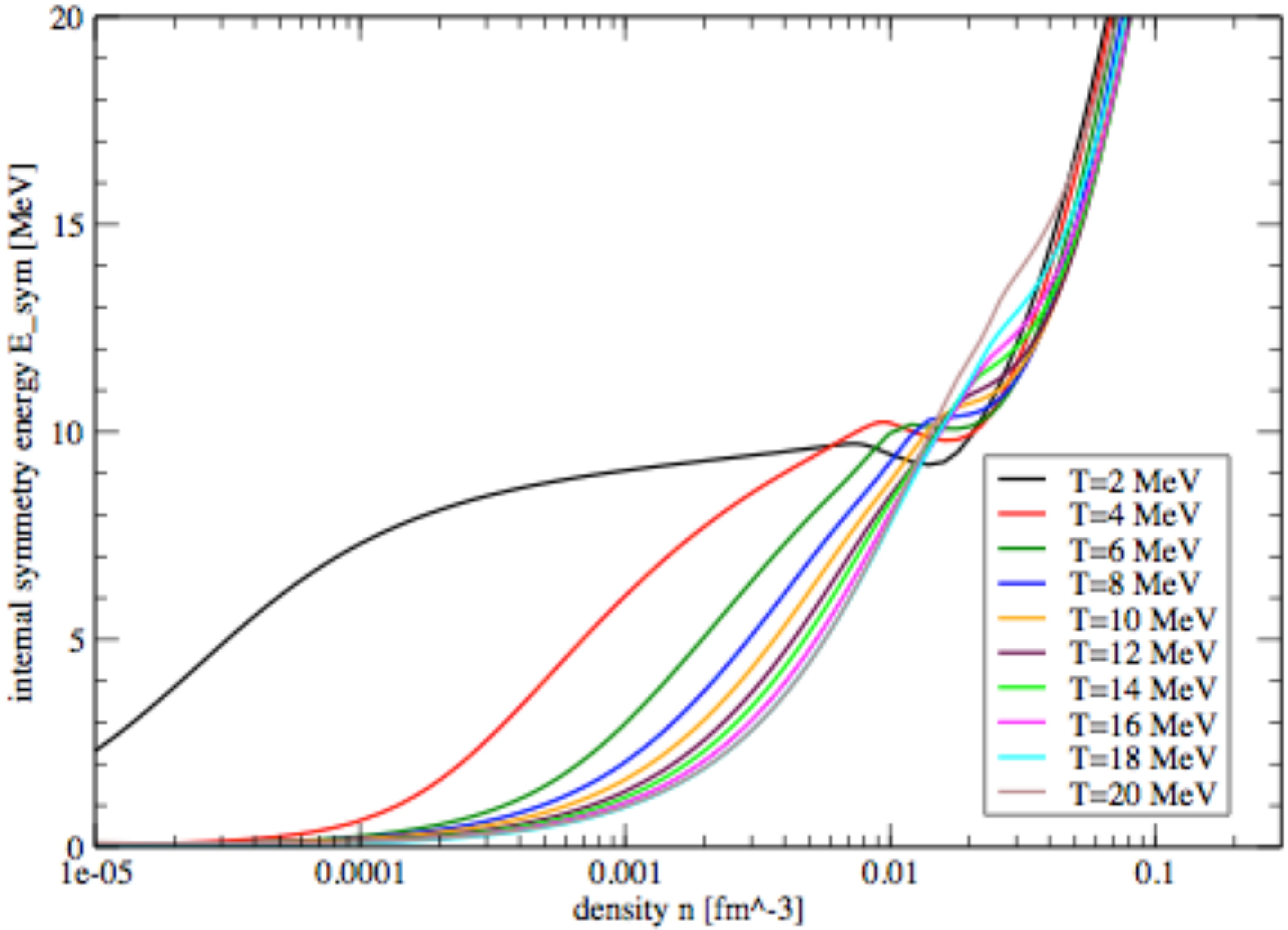
Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

G. Roepke, arXiv: 1411.4593, submitted to PRC

Internal symmetry energy

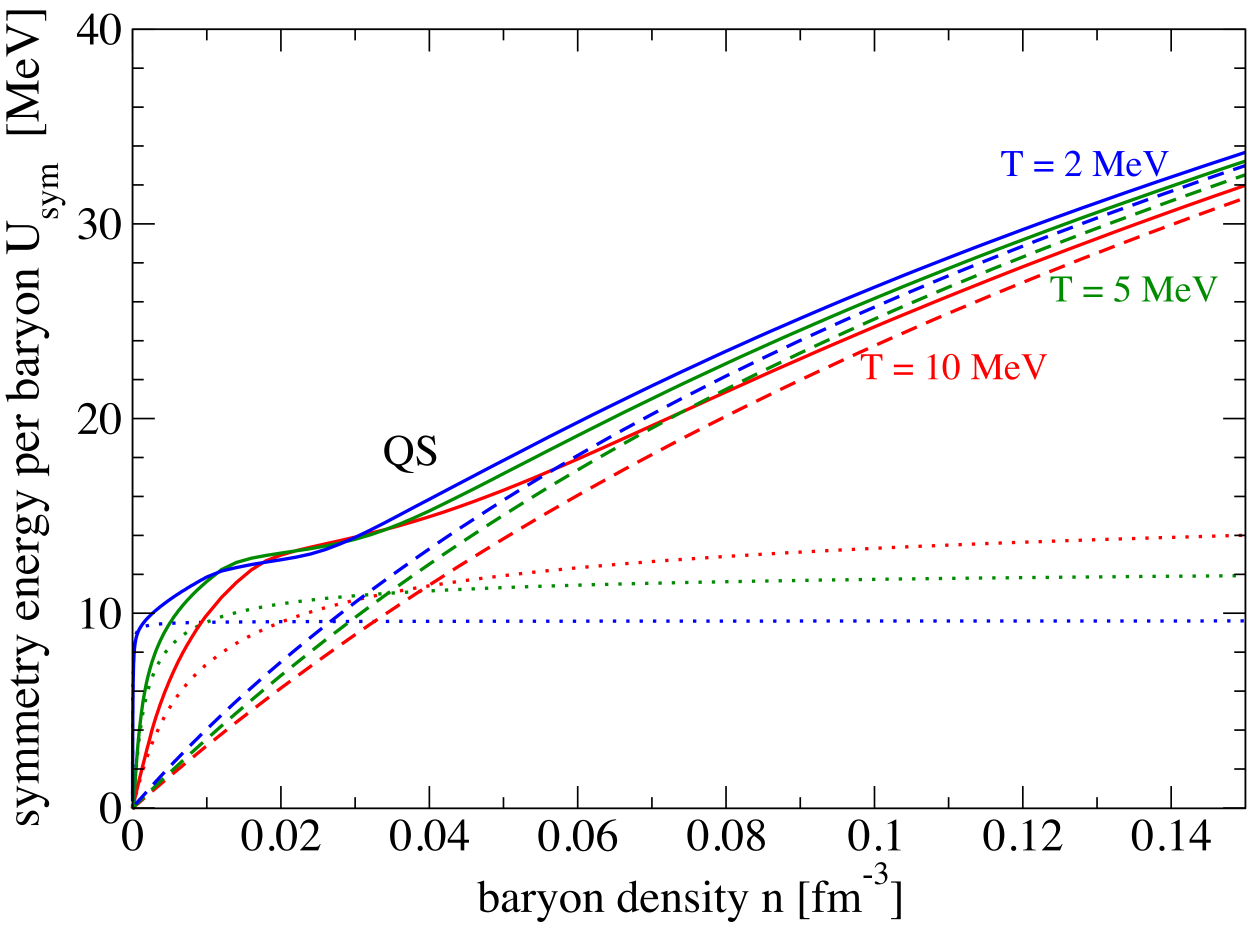


T[MeV]
2
4
6
8
10
12
14
16
18
20

QS

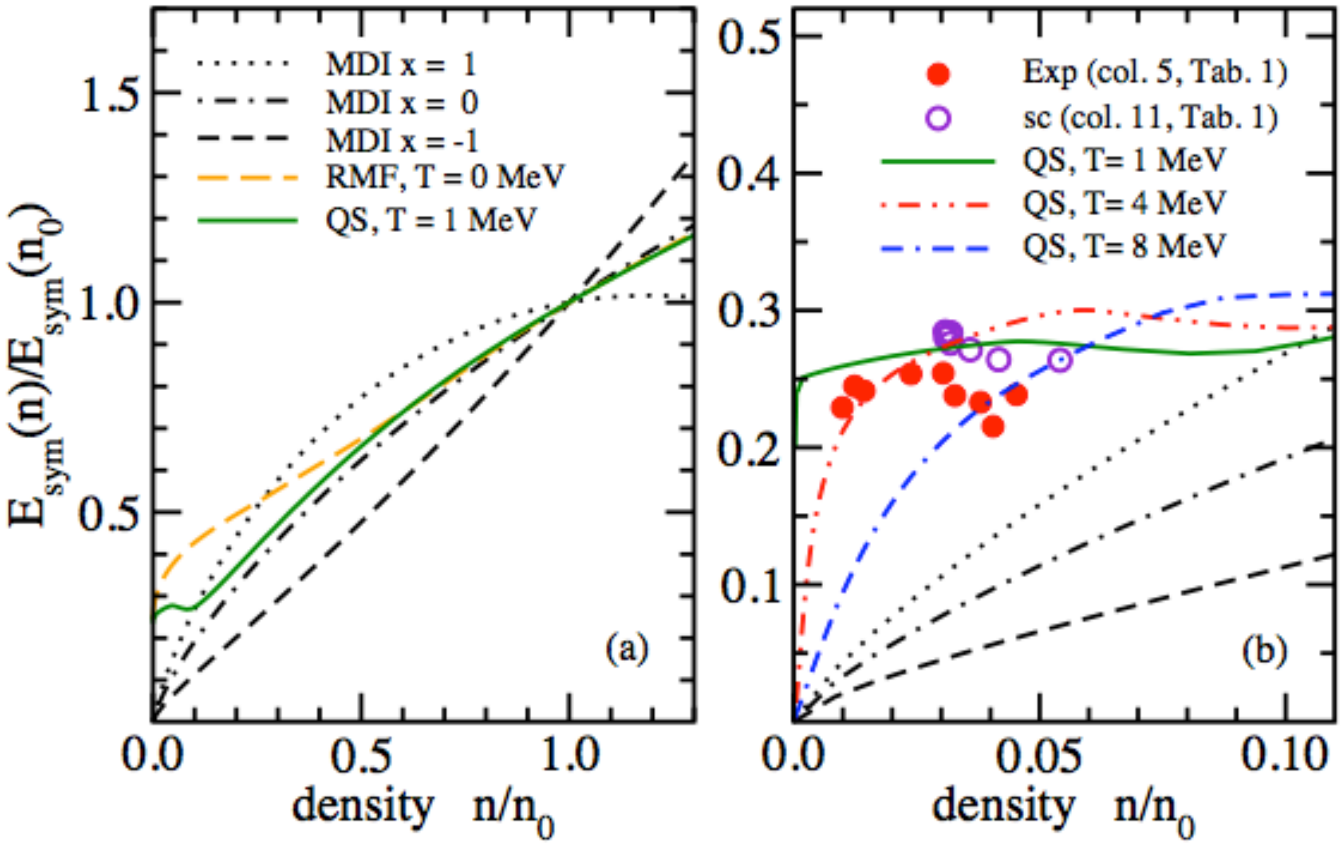
Typel et al., PRC **81**, 015803 (2010)

Light clusters and symmetry energy



K. Hagel et al.Eur. Phys. J. A (2014) 50: 39

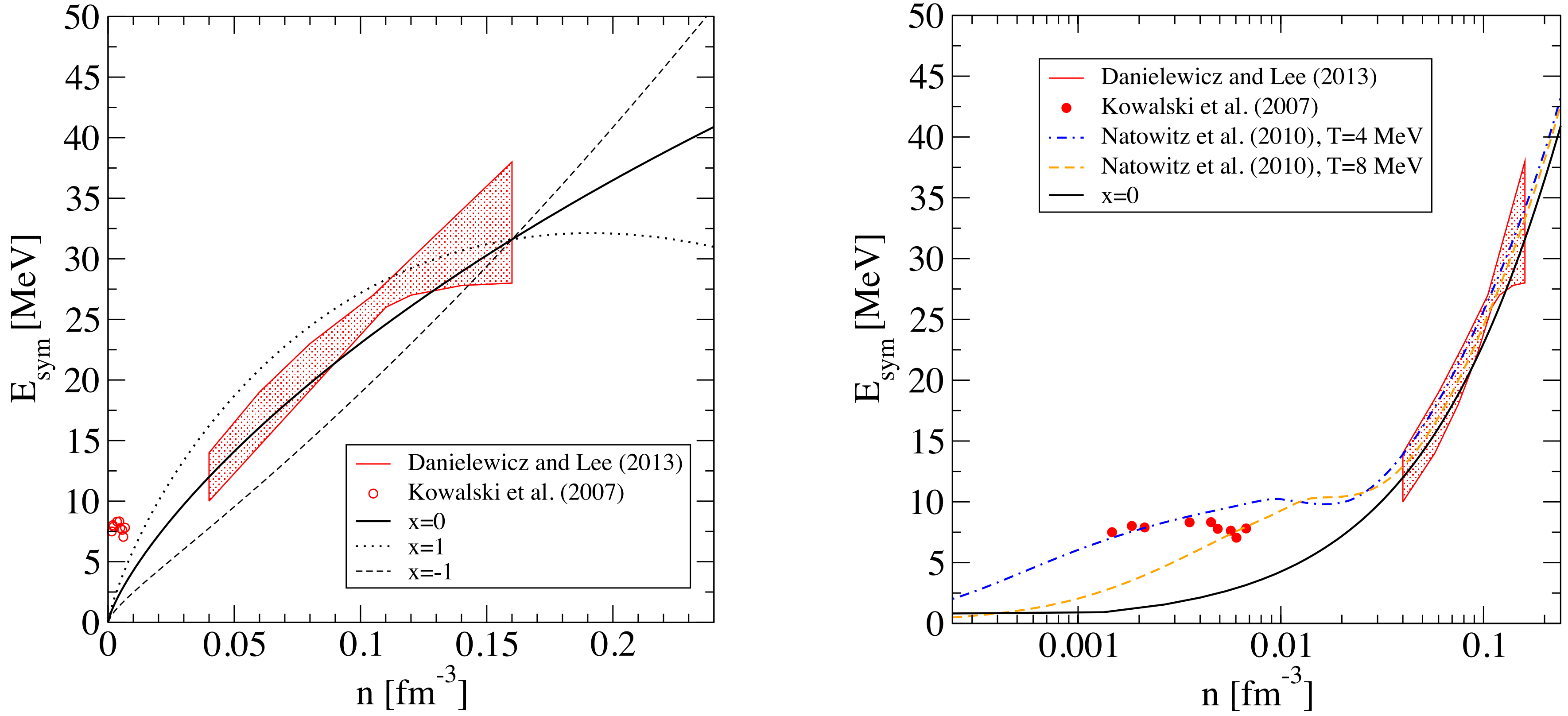
Symmetry Energy



Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Symmetry energy: low density limit



K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Summary

• The symmetry energy at subsaturation density is strongly depending on temperature T.

• The low-density limit is described by the virial expansion. High values (> 7 MeV) at low temperatures

• The influence of continuum correlations (clusters) at increasing densities requires detailed investigations.

- The blocking of bound states is modified because of correlations in the medium (α matter).
- Continuum correlations contribute to the symmetry energy (density dependent virial coefficients).

• relevant for HIC (freeze-out, transport theory) and astrophysics (supernova explosions)

Thanks

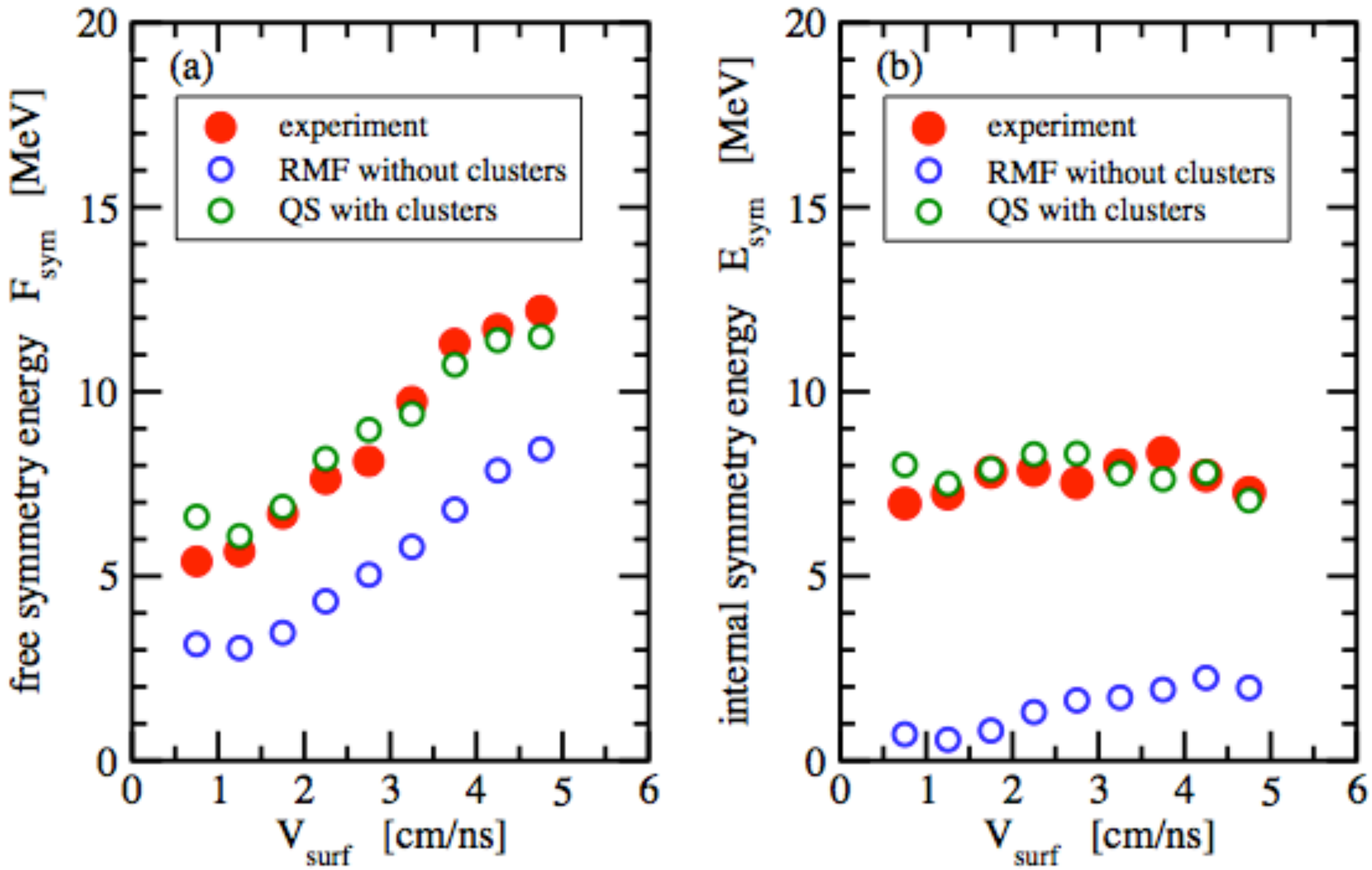
to D. Blaschke, C. Fuchs, Y. Funaki, H. Horiuchi,
J. Natowitz, T. Klaehn, Z. Ren, S. Shlomo, P. Schuck,
A. Sedrakian, K. Sumiyoshi, A. Tohsaki, S. Typel,
H. Wolter, C. Xu, T. Yamada, B. Zhou
for collaboration

to you

for attention

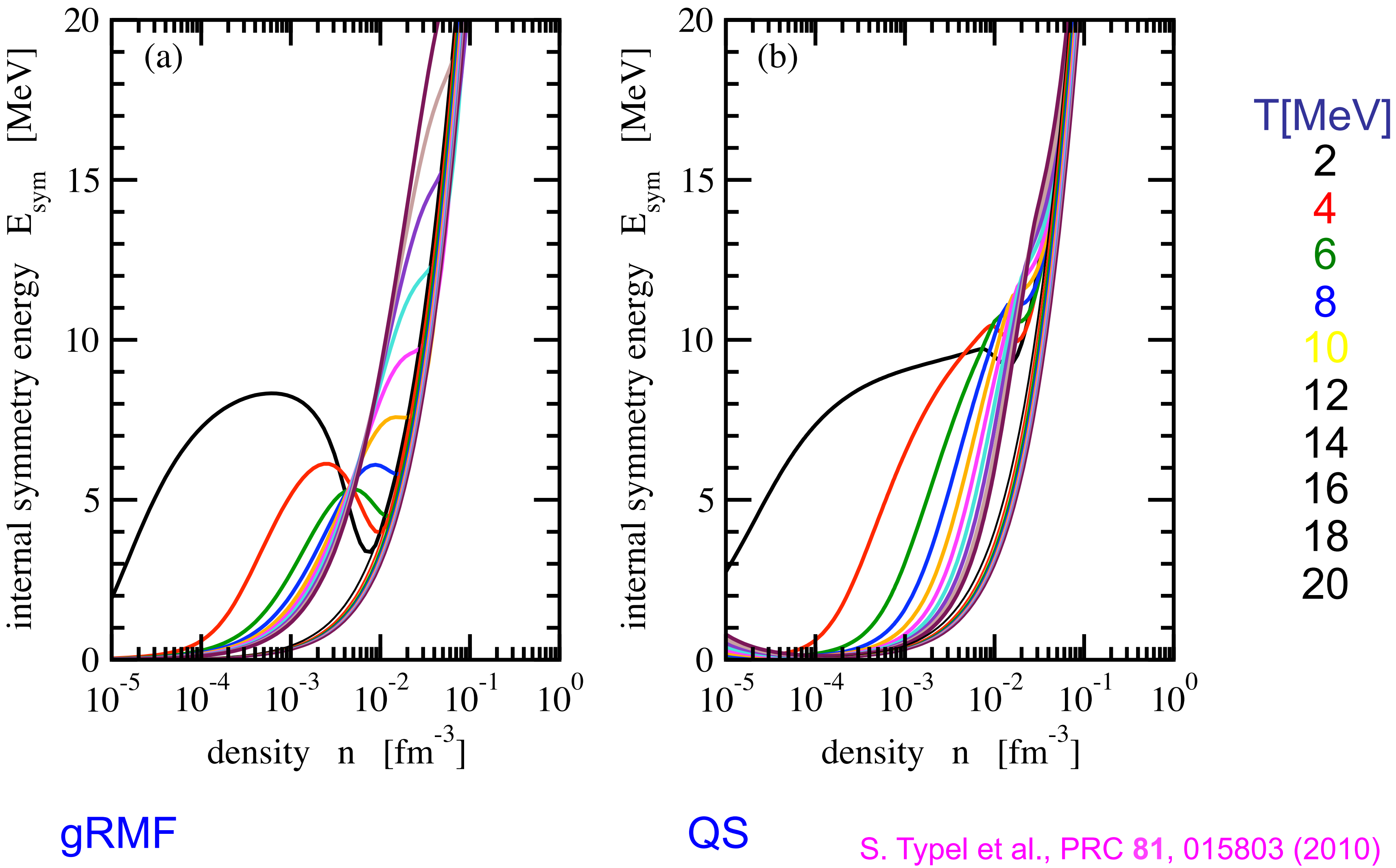
D.G.

Symmetry energy, comparison experiment with theories



J.Natowitz et al., PRL 2010

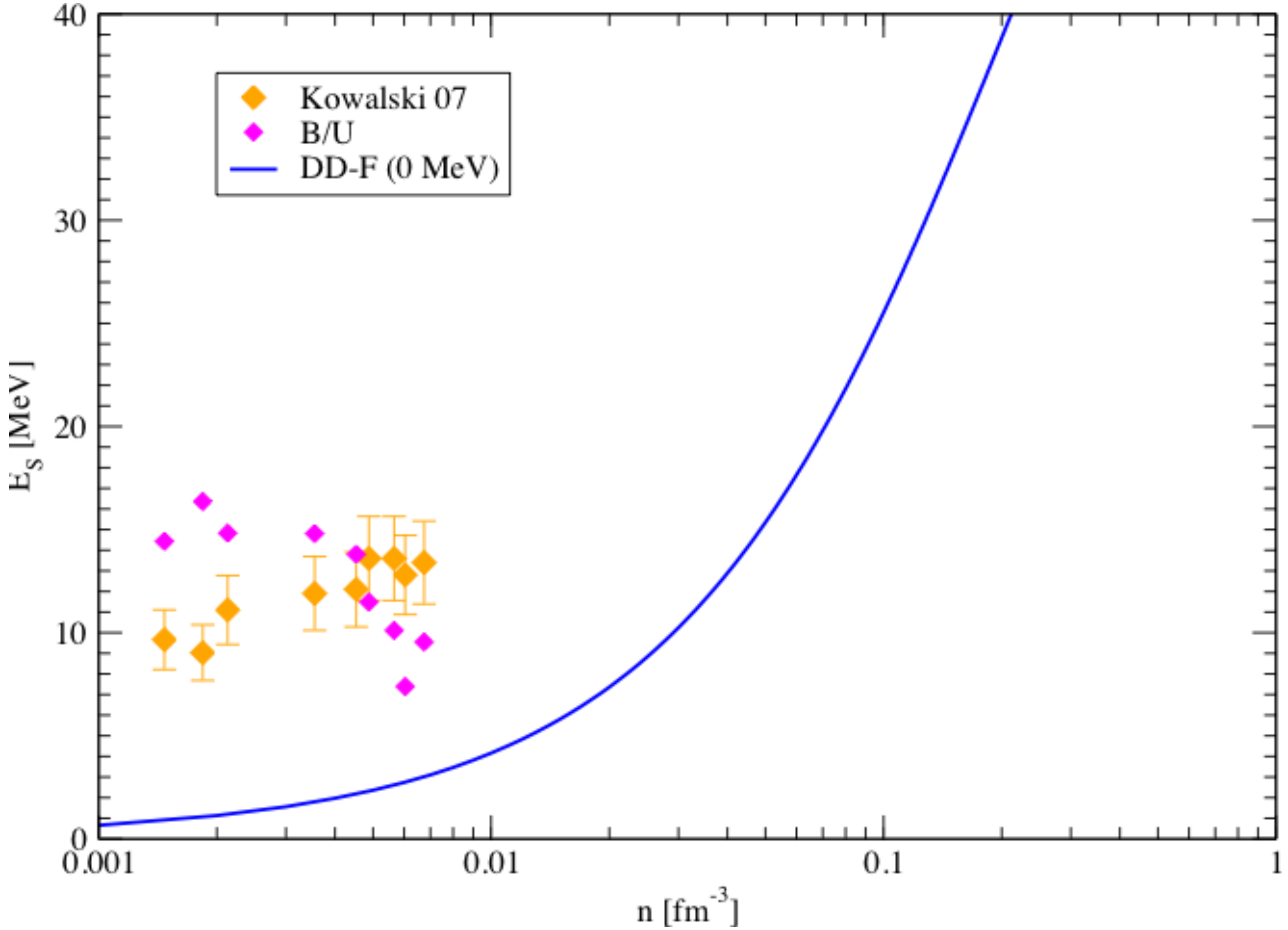
Symmetry energy



S. Typel et al., PRC **81**, 015803 (2010)

Symmetry energy

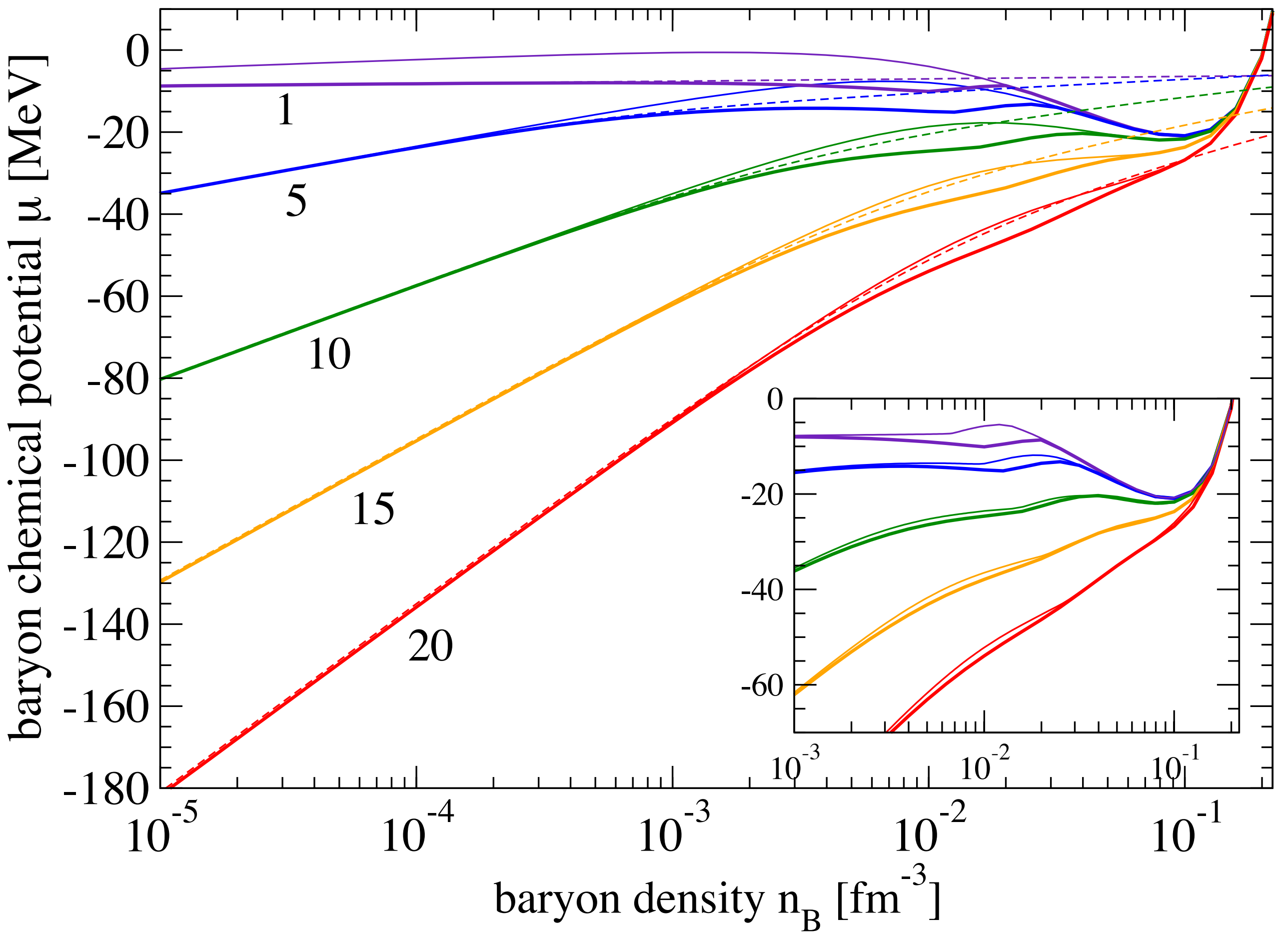
Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



S. Kowalski et al., PRC **75**, 014601 (2007)

Symmetric matter: chemical potential

QS compared with RMF (thin) and NSE (dotted)



G. Roepke, arXiv: 1411.4593, submitted to PRC

Insert: no continuum correlations (thin)

Different approximations

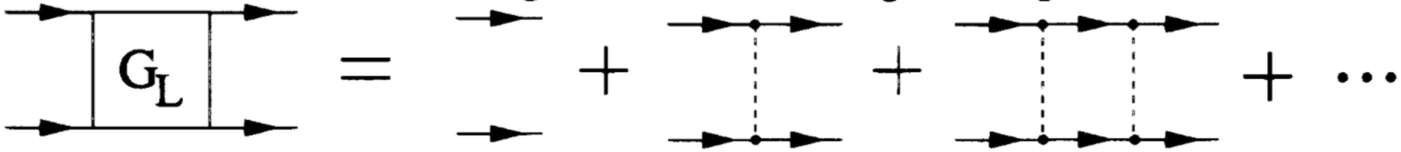
- Expansion for small $\mathrm{Im}\ \Sigma(1, \omega + i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\mathrm{quasi}}(1))}{1 - \frac{\mathrm{d}}{\mathrm{d}z}\mathrm{Re}\ \Sigma(1,z)|_{z=E^{\mathrm{quasi}}-\mu_1}}$$
$$-2\mathrm{Im}\ \Sigma(1,\omega + i\eta)\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{P}{\omega + \mu_1 - E^{\mathrm{quasi}}(1)}$$

quasiparticle energy $E^{\mathrm{quasi}}(1) = E(1) + \mathrm{Re}\ \Sigma(1,\omega)|_{\omega=E^{\mathrm{quasi}}}$

- chemical picture: bound states $\hat{=}$ new species

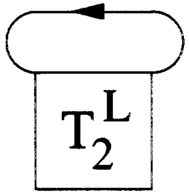
summation of ladder diagrams, Bethe-Salpeter equation



Different approximations

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$ 

$$n(\beta,\mu) = \sum_1 f_1(E^{\mathrm{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\mathrm{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k\ \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\mathrm{quasi}}(1) + E^{\mathrm{quasi}}(2)) 2\sin^2\delta_n(k) \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}k} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

M. Schmidt et al., Ann. Phys. (NY) **202**, 57 (1990)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrisierung (d – like):

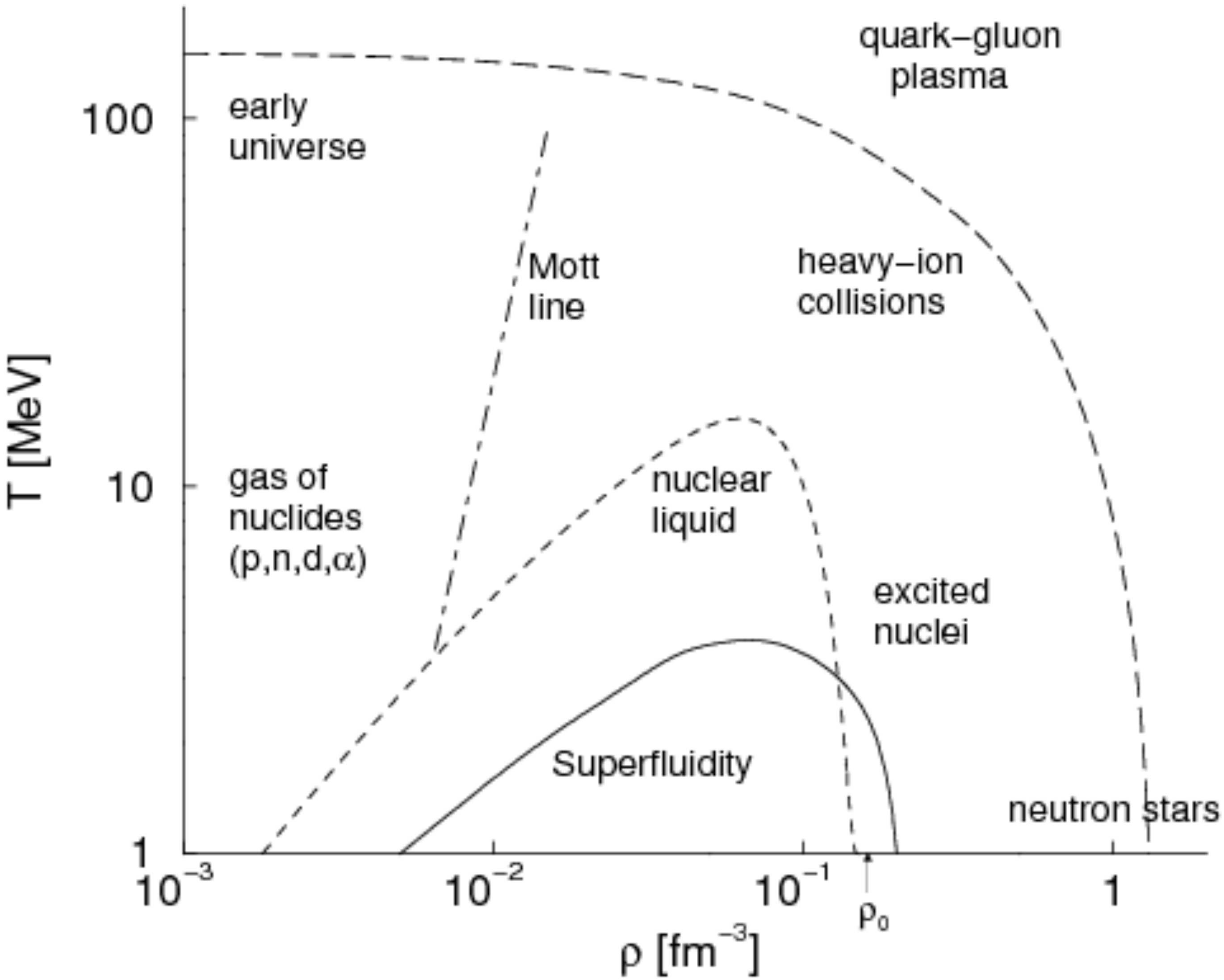
$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\ e^{-0.102424\ T/{\rm MeV}}$$

$$\gamma_d(\mathbf{P},T,n_B,Y_p)\ =\ 1873.2\ {\rm MeV\,fm^3}\ \exp\left[-P^2\ {\rm fm}^2/(1.8463+0.1617\ T\ {\rm MeV}^{-1}+0.17\ P^2\ {\rm fm}^2)\right]$$

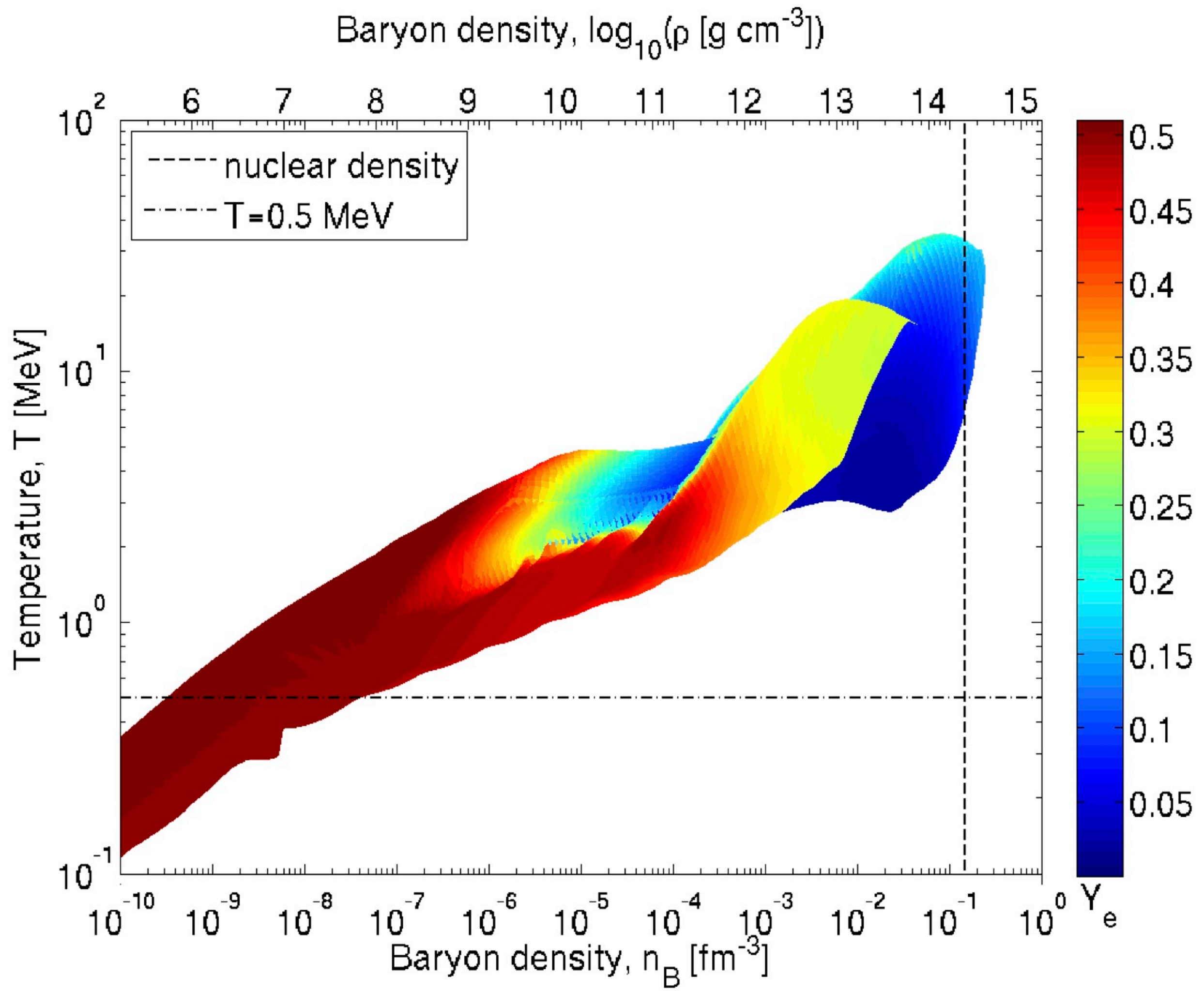
G. Roepke, arXiv: 1411.4593, submitted (PRC)

Symmetric nuclear matter: Phase diagram



Nuclear matter phase diagram

Exploding supernova



T. Fischer et al., arXiv 1307.6190

Cluster - mean field approximation

Cluster (A) interacting with a distribution of clusters (B) in the medium,
fully antisymmetrized

$$\sum_{1'\ldots A'} \{H_A^0(1\ldots A, 1'\ldots A') + \sum_i \Delta_i^{A,mf} \delta_{k,k'} + \frac{1}{2}\sum_{i,j} \Delta V_{ij}^{A,mf} \delta_{l,l'} - E_{A\nu P}\delta_{k,k'}\} \psi_{A\nu P}(1'\ldots A') = 0$$

self-energy

$$\Delta_1^{A,mf}(1) = \sum_2 V(12,12)_{ex} f^*(2) + \sum_{B\nu P}\sum_{2\ldots B'} f_B(E_{B\nu P}) \sum_i V_{1i}(1i,1'i') \psi^*_{B\nu P}(1\ldots B) \psi_{B\nu P}(1'\ldots B')$$

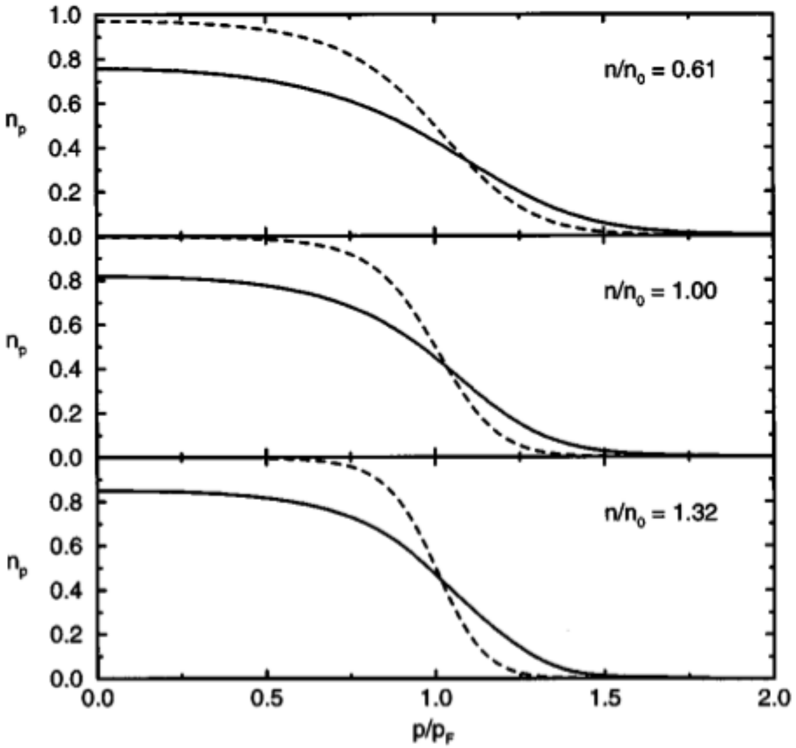
effective interaction

$$\Delta V_{12}^{A,mf} = -\frac{1}{2}[f^*(1) + f^*(2)] V(12,1'2') - \sum_{B\nu P}\sum_{2^*\ldots B''} f_B(E_{B\nu P}) \sum_i V_{1i} \psi^*_{B\nu P}(22^*\ldots B^*) \psi_{B\nu P}(2'2''\ldots B'')$$

phase space occupation $\quad f^*(1) = f_1(1) + \sum_{B\nu P}\sum_{2\ldots B} f_B(E_{B\nu P}) |\psi_{B\nu P}(1\ldots B)|^2$

Single nucleon distribution function

Dependence on density



T = 10 MeV

Alm et al., PRC 53, 2181 (1996)

Different approximations

bound state formation

continuum contribution

chemical & physical picture

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

medium effects

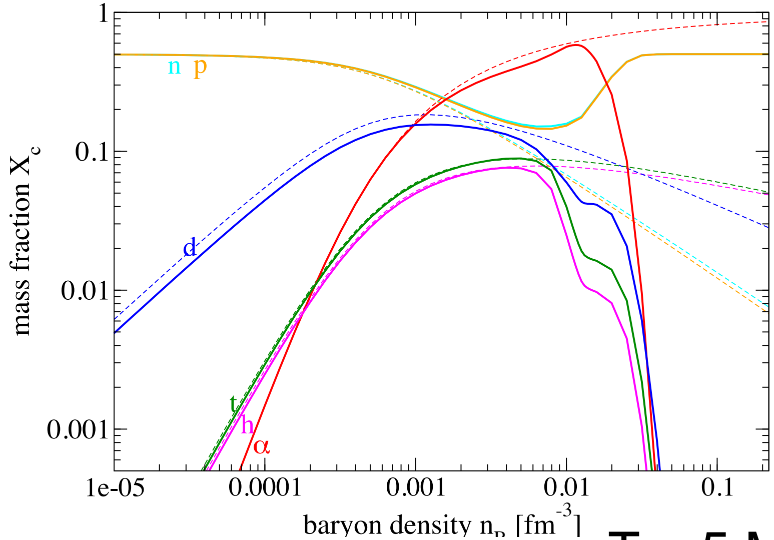
Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

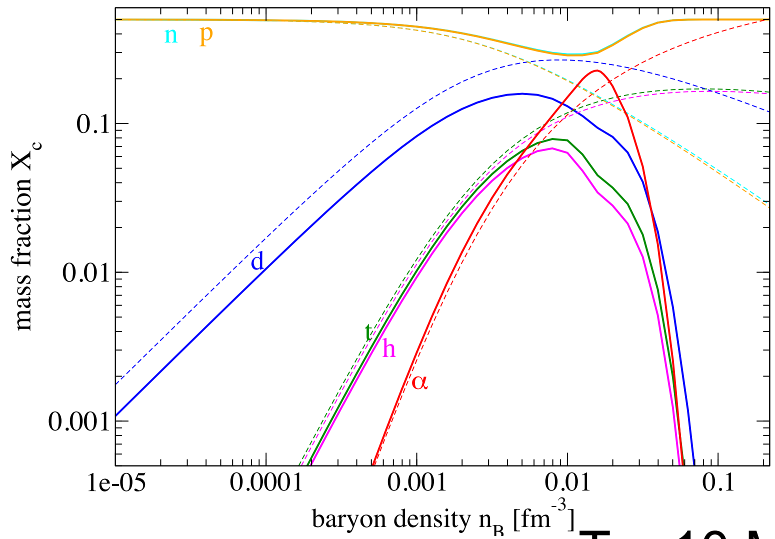
Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Correlated medium

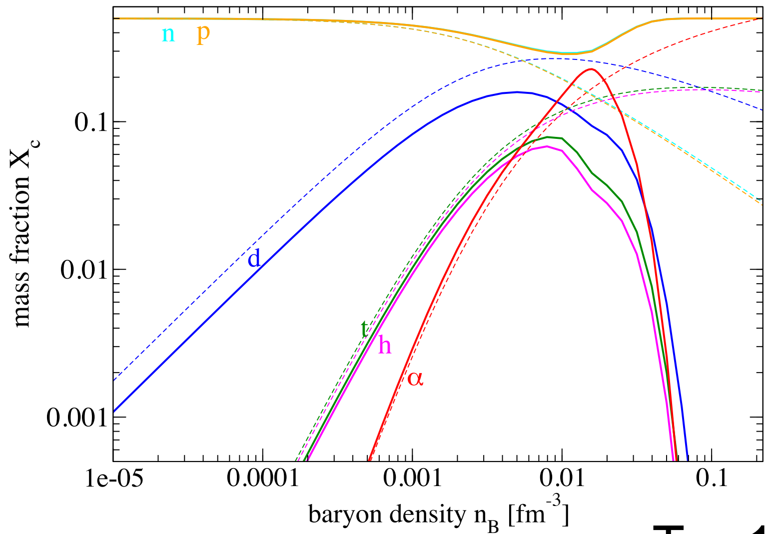
Symmetric matter: composition



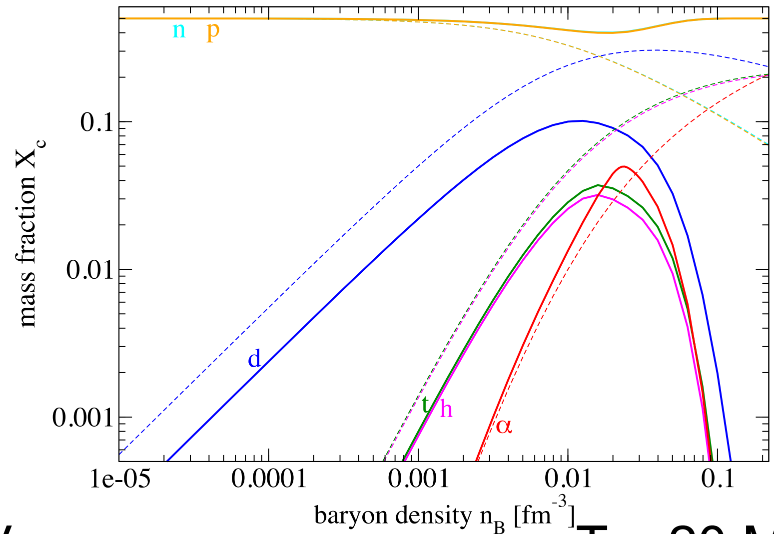
T = 5 MeV



T = 10 MeV



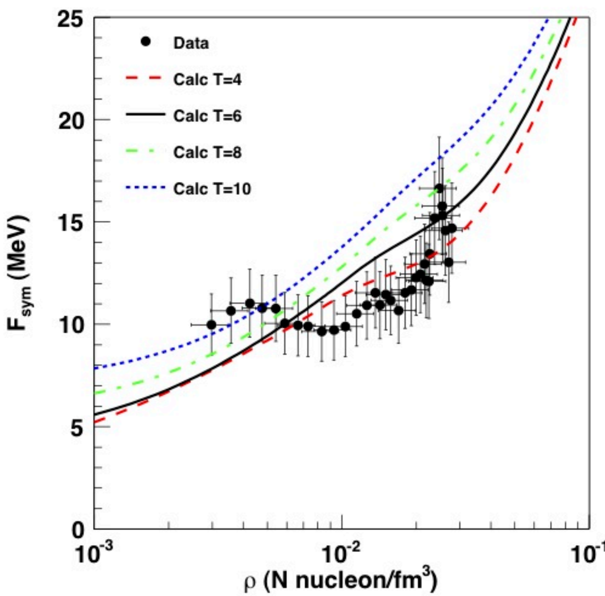
T = 15 MeV

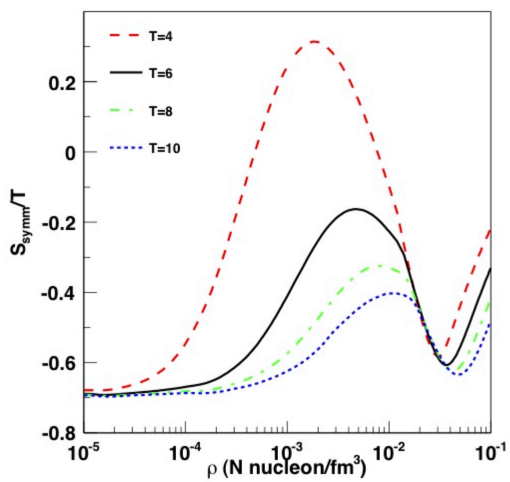


T = 20 MeV

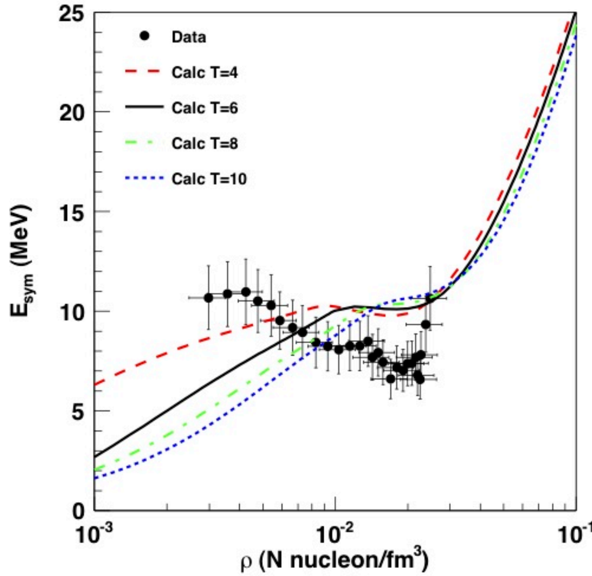
G. Roepke, arXiv: 1411.4593, submitted to PRC

Free symmetry energy





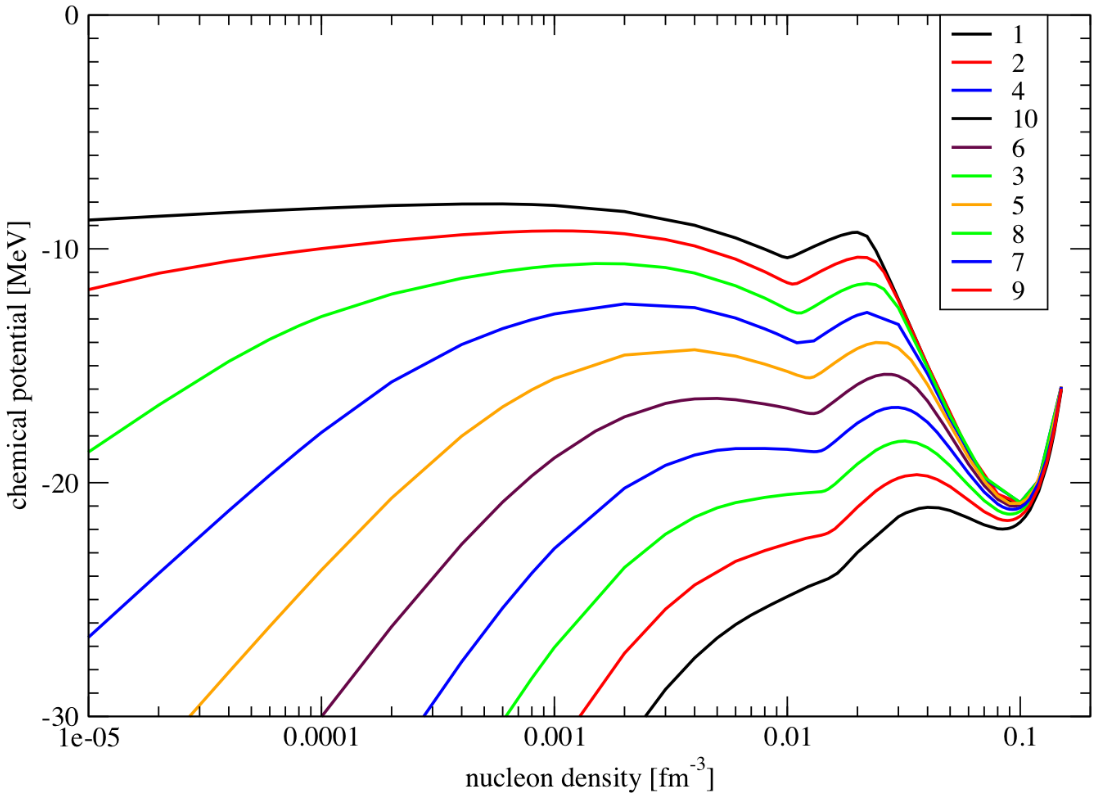
symmetry entropy



Internal symmetry energy

R. Wada et al., Phys. Rev. C 85, 064618 (2012).

Chemical potential



preliminary

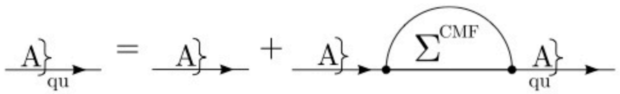
Cluster virial expansion for nuclear matter within a quasiparticle approach

Generalized Beth-Uhlenbeck approach

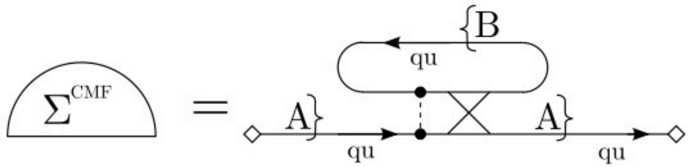
$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n)=\sum_{A,Z,\nu}\frac{A}{\Omega}\sum_{\substack{\vec{P}\\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$n_2^{\mathrm{qu}}(T,\mu_p,\mu_n)=\sum_{A,Z,\nu}\sum_{A',Z',\nu'}\frac{A+A'}{\Omega}\sum_{\vec{P}}\sum_{c} g_c\frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi}\times\int_0^\infty dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right)2\sin^2(\delta_c)\frac{d\delta_c}{dE}$$

Avoid double counting



Generating functional



G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Chemical picture and medium corrections

Nuclear matter at given temperature T,
baryon density n_B,
proton fraction (asymmetry) $Y_e = n_p/n_B$: equation of state

- low-density limit: ideal mixture of reacting components:
  Nuclear statistical equilibrium (NSE)
- interactions: virial expansion (cluster virial expansion)
- higher densities: quasiparticle concept,
  medium modification of components (cluster mean-field approximation)

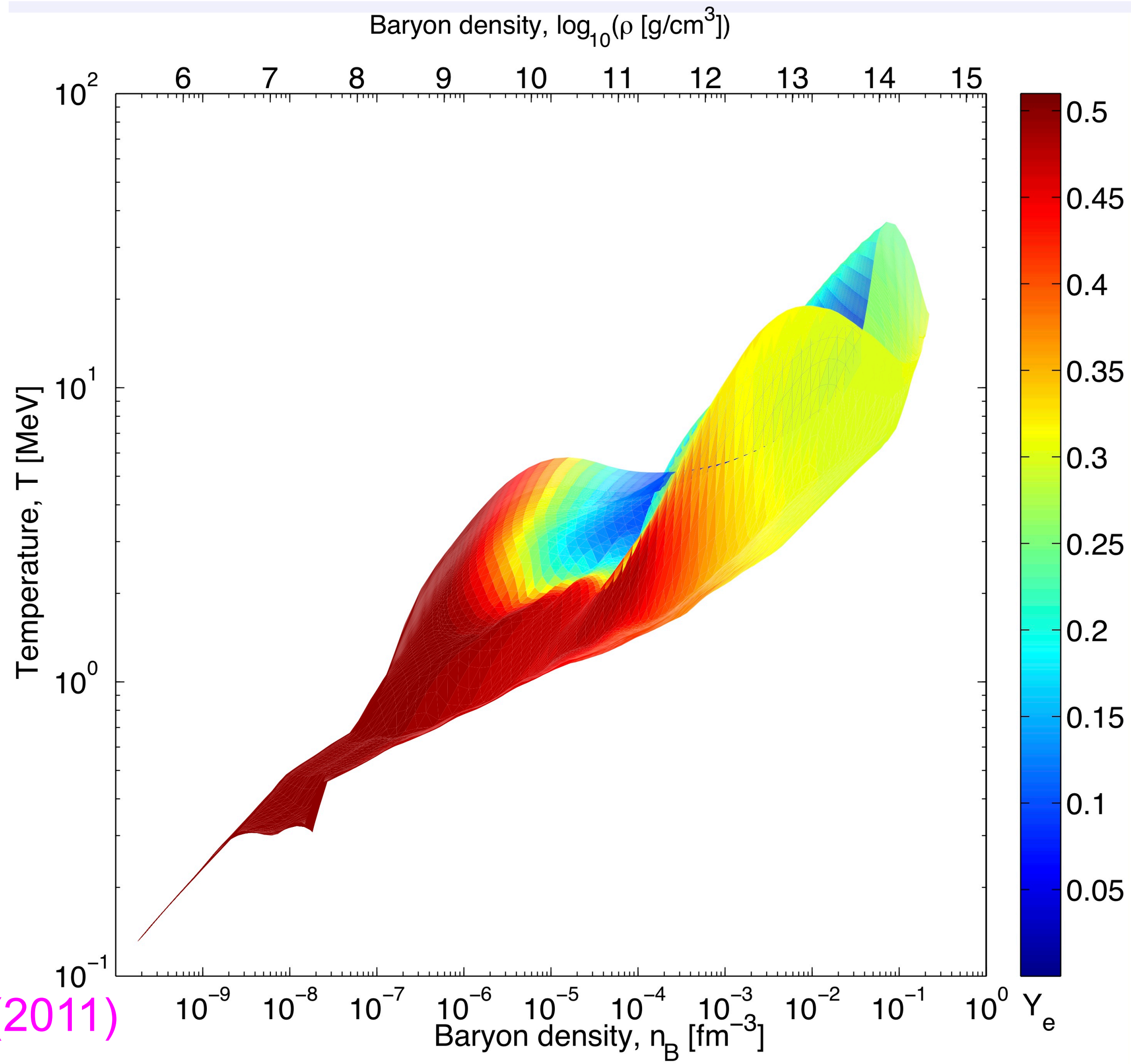
  - nucleons as quasiparticles:
    Skyrme, relativistic mean-field (RMF), Dirac Brueckner Hartree-Fock
  - light elements (d, t, h, α) as quasiparticles:
    shift of energy (self-energy, Pauli blocking), Mott effect.
    - excluded volume
    - quantum statistical approach (QS): $E_{A,Z}(p;T,n_B,Y_e)$

Nuclear matter phase diagram

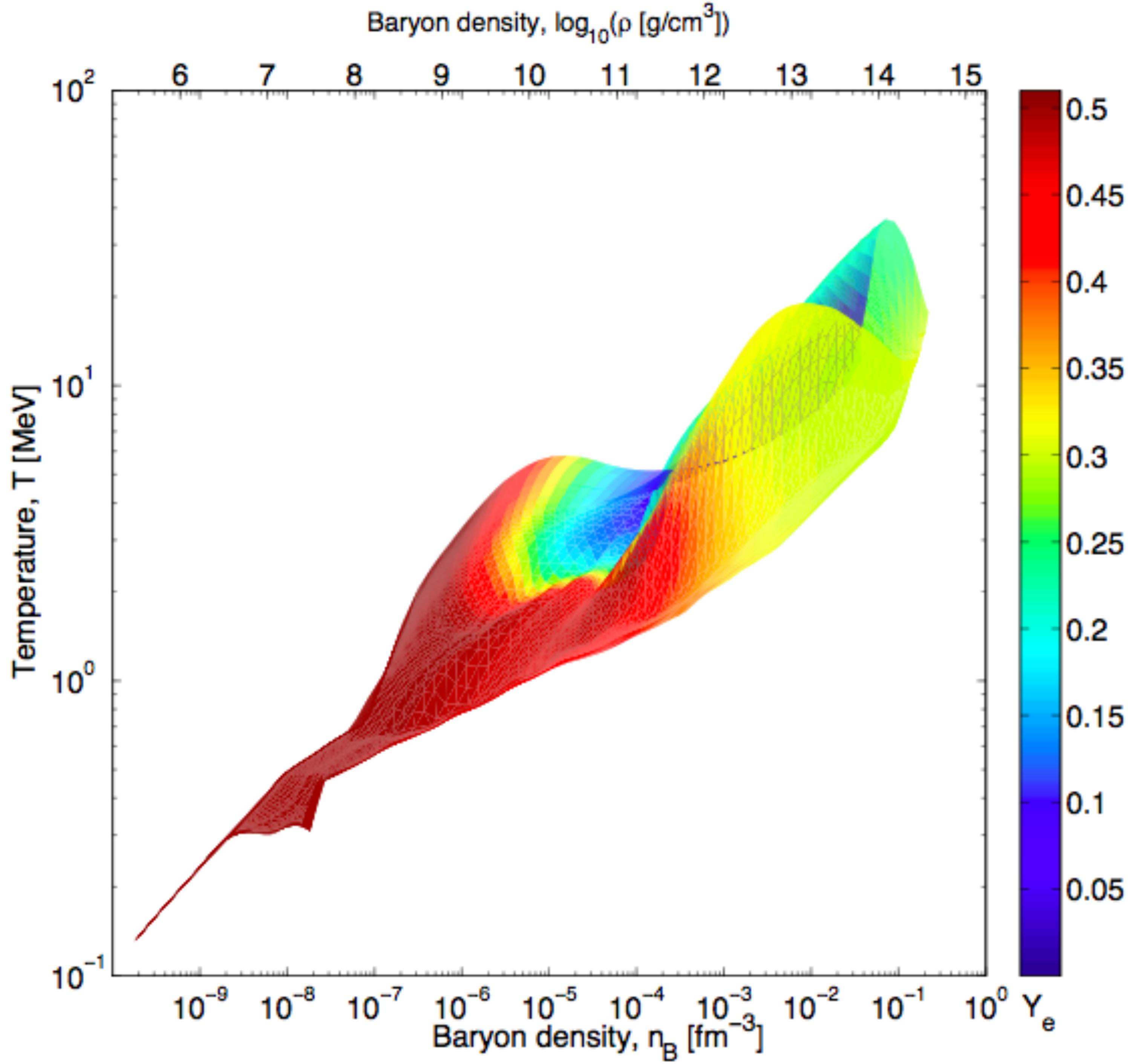
Core collapse supernovae

Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$

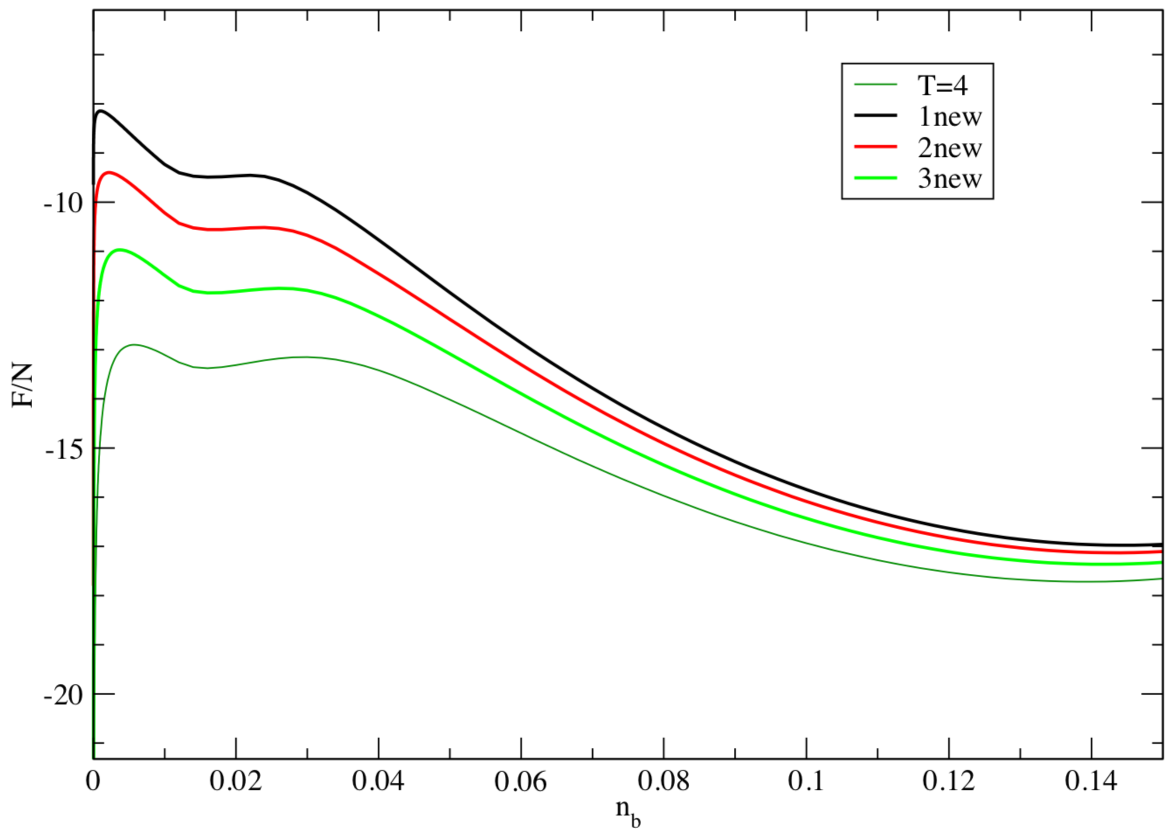


T. Fischer et al., ApJS 194, 39 (2011)



Baryon density, $\log_{10}(\rho\ [\mathrm{g/cm^3}])$
6
7
8
9
10
11
12
13
14
15
10^2
10^1
10^0
10^{-1}
Temperature, T [MeV]
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
Baryon density, n_B [fm^{-3}]
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
Y_e

Free energy per nucleon



(preliminary)

correlated medium

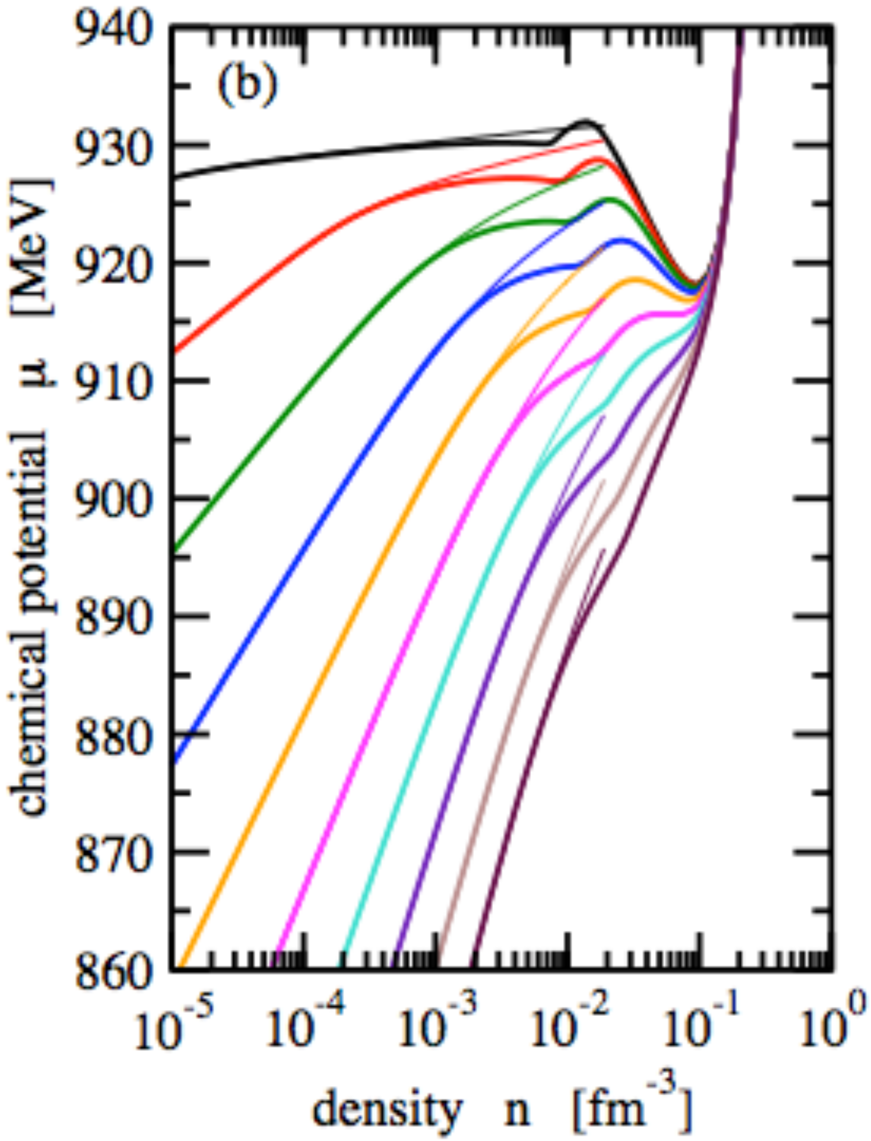
Constrained THSR calculations as function of the c.o.m. width B?

Chemical potential of symmetric matter

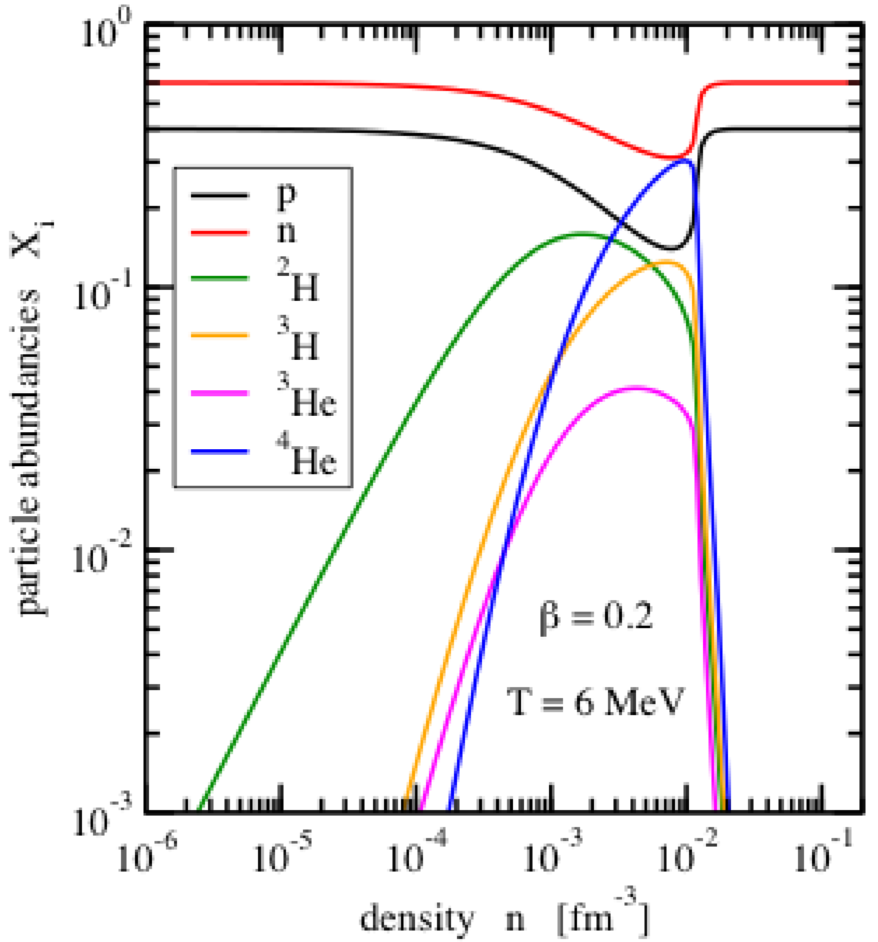
Isotherms

T[MeV]
2
4
6
8
10
12
14
16
18
20

thin lines: NSE



Light Cluster Abundances



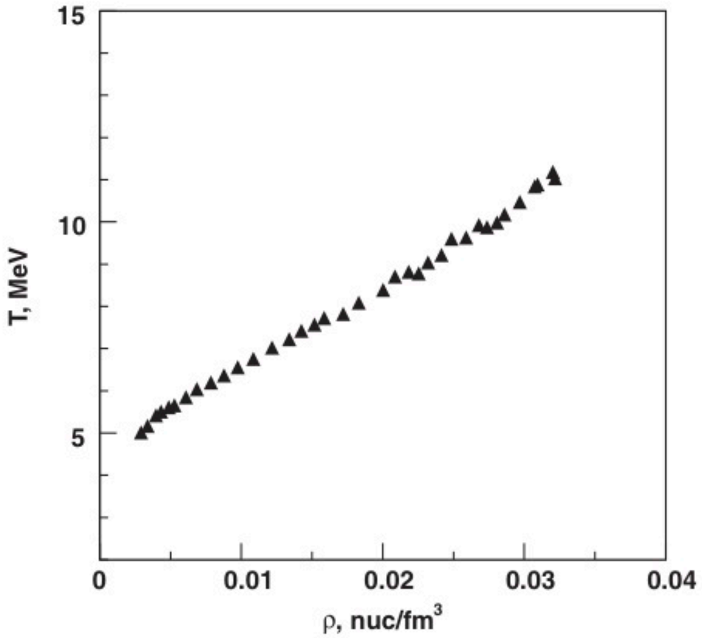
S. Typel et al.,
PRC **81,** 015803 (2010)

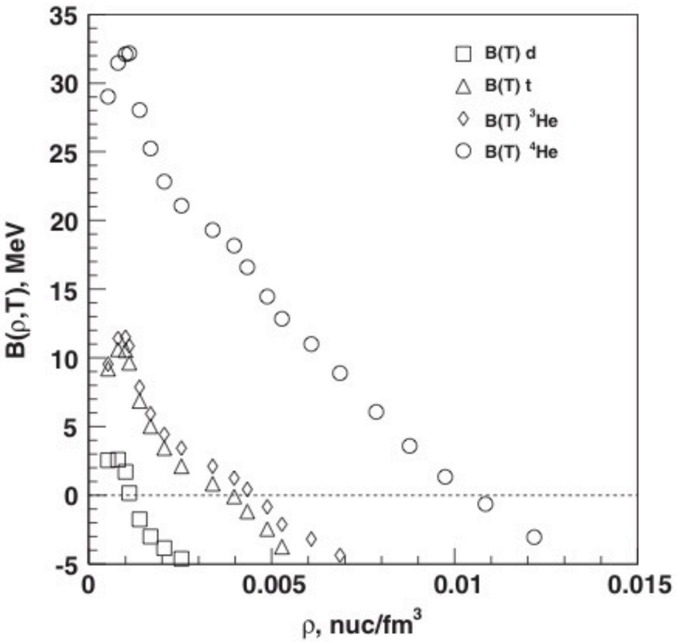
Cluster yields in HIC

PRL 108, 062702 (2012) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2012

Experimental Determination of In-Medium Cluster Binding Energies and Mott Points in Nuclear Matter

K. Hagel,[1] R. Wada,[2,1] L. Qin,[1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[2] M. Huang,[2] J. Wang,[2] H. Zheng,[1] S. Kowalski,[6] C. Bottosso,[1] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[7] M. Lunardon,[7] S. Moretto,[7] G. Nebbia,[7] S. Pesente,[7] V. Rizzi,[7] G. Viesti,[7] M. Cinausero,[8] G. Prete,[8] T. Keutgen,[9] Y. El Masri,[9] and Z. Majka[10]





in-medium binding energies

Mott points from cluster yields

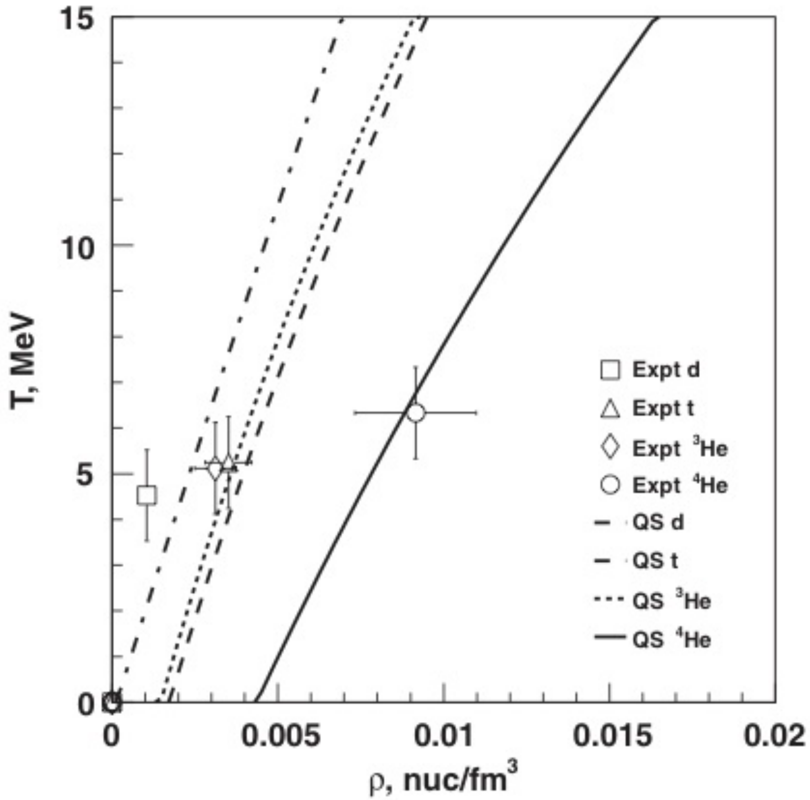


FIG. 3. Comparison of experimentally derived Mott point densities and temperatures with theoretical values. Symbols represent the experimental data. Estimated errors on the temperatures are 10% and on the densities 20%. Lines show polynomial fits to the Mott points presented in Ref. [1].

K. Hagel et al., PRL **108**, 062702 (2012)

Different approximations

bound state formation

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

continuum contribution

Second virial coefficient:

account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

chemical & physical picture

Cluster virial approach:

all bound states (clusters)
scattering phase shifts of all pairs

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium with quasiparticle clusters:

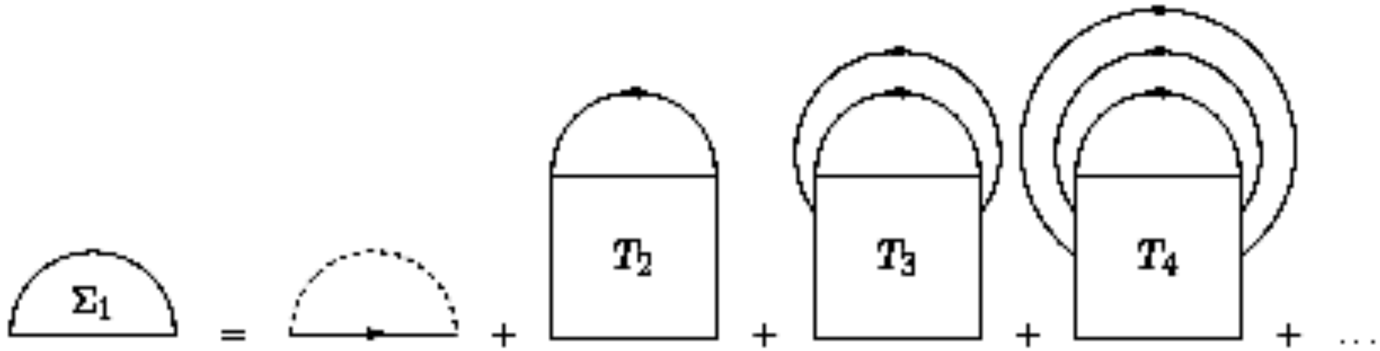
self-energy and Pauli blocking

Generalized Beth-Uhlenbeck formula:

medium modified binding energies,
medium modified scattering phase shifts

Correlated medium

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

Beth-Uhlenbeck formula

$$
\begin{aligned}
n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\
&+ \dots
\end{aligned}
$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Quantum condensates in nuclei?

Lot of semantics – my position

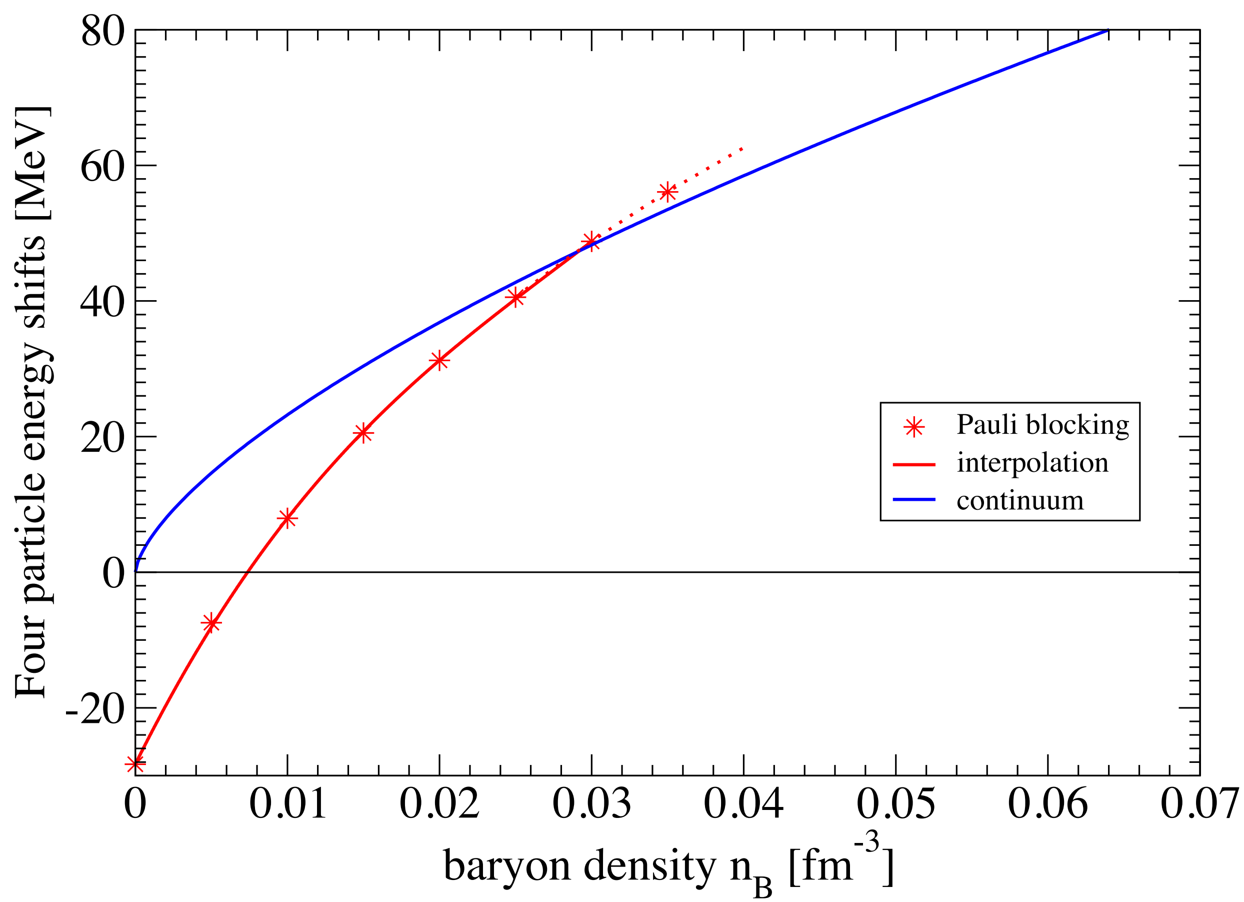
- Pairing is well accepted.
- Quartetting is not very well-known and simple.
- The main point is the formation of clusters (correlations) in low-density matter.
- We are interested in an efficient description (optimal wave function) for the cluster state.
- The center of mass motion has to be considered as new (collective) degree of freedom.

Four-nucleon energies at finite density

Solution of the in-medium wave equation, T = 0

4 free nucleons
at the Fermi energy
(continuum)

bound state
(α particle)
with Pauli blocking



Pauli blocking and Mott effect

Two different fermions (a,b: proton,neutron) form a bound state (c: deuteron).

$$c_q = \sum_p F(q,p) a_p b_{q-p}$$

Is the bound state a boson? Commutator relation

$$\left[c_q, c_{q'}^+\right]_- = \sum_{p,p'} F(q,p) F^*(q',p') \left[a_p b_{q-p}, b_{q'-p'}^+ a_{p'}^+\right]_-$$

$$\underline{a_p b_{q-p} b_{q'-p'}^+ a_{p'}^+} + a_p b_{q'-p'}^+ b_{q-p} a_{p'}^+ - a_p b_{q'-p'}^+ b_{q-p} a_{p'}^+ - b_{q'-p'}^+ a_p b_{q-p} a_{p'}^+$$

$$+ b_{q'-p'}^+ a_p b_{q-p} a_{p'}^+ + b_{q'-p'}^+ a_p a_{p'}^+ b_{q-p} - b_{q'-p'}^+ a_p a_{p'}^+ b_{q-p} \underline{- b_{q'-p'}^+ a_{p'}^+ a_p b_{q-p}}$$

$$= a_p a_{p'}^+ \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'} = \left(\delta_{p,p'} - a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'}$$

$$\left[c_q, c_{q'}^+\right]_- = \sum_{p,p'} F(q,p) F^*(q',p') \left[\left(\delta_{p,p'} - a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} - b_{q'-p'}^+ b_{q-p} \delta_{p,p'}\right]$$

$$= \sum_p F(q,p) F^*(q,p) \delta_{q,q'} - \sum_{p,p'} F(q,p) F^*(q',p') \left[\left(a_{p'}^+ a_p\right) \delta_{q-p,q'-p'} + \left(b_{q'-p'}^+ b_{q-p}\right) \delta_{p,p'}\right]$$

averaging

$$\left\langle \left[c_q, c_{q'}^+\right]_- \right\rangle = \delta_{q,q'} \left[1 - \sum_p F(q,p) F^*(q,p) \left(\left\langle a_p^+ a_p \right\rangle + \left\langle b_{q-p}^+ b_{q-p} \right\rangle\right)\right]$$

Fermionic substructure: phase space occupation, “excluded volume”

Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

mass number *A*,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
K: center of mass momentum

Nuclear Statistical Equilibrium (NSE)

Parametrization

- Single-nucleon quasiparticle energies $E_{\tau}(p, T, n_{B,} Y_e)$ (DBHF, Skyrme, RMF,…)
- Bound state energies $E_{A,Z,\nu}(p, T, n_{B,} Y_e)$

G.R., NP A **867**, 66 (2011)

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

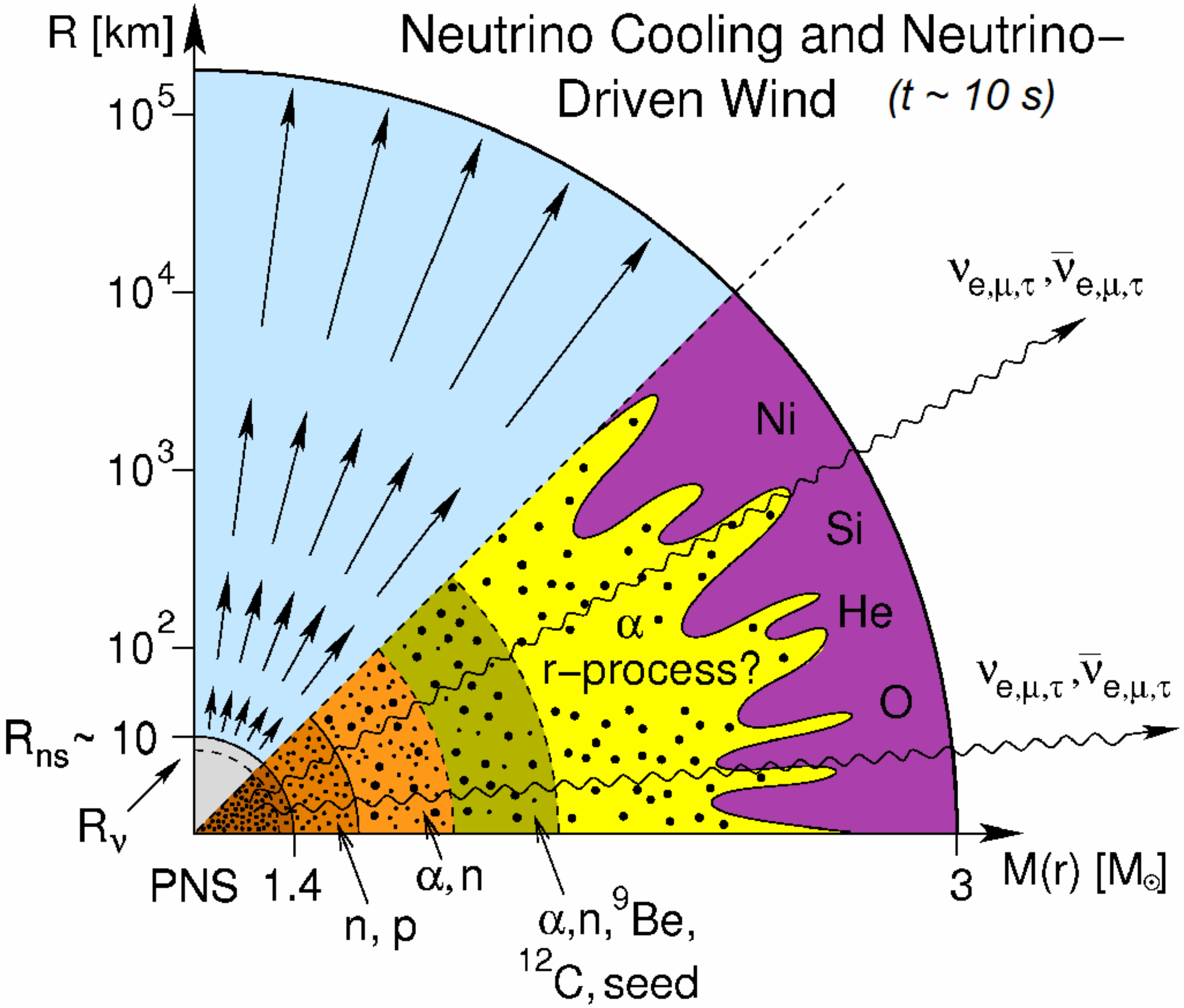
mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν: internal quantum number,

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations

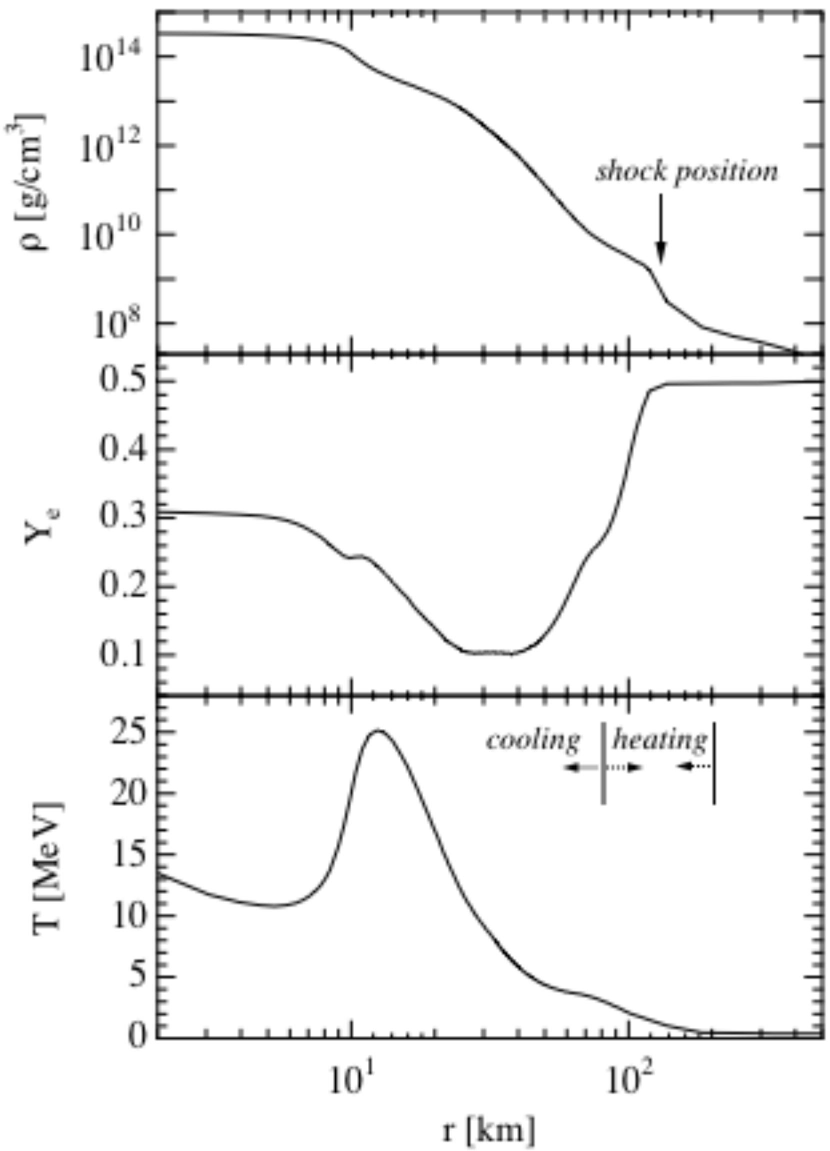
- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

Supernova explosion



T.Janka

Core-collapse supernovae



Density.

electron fraction, and

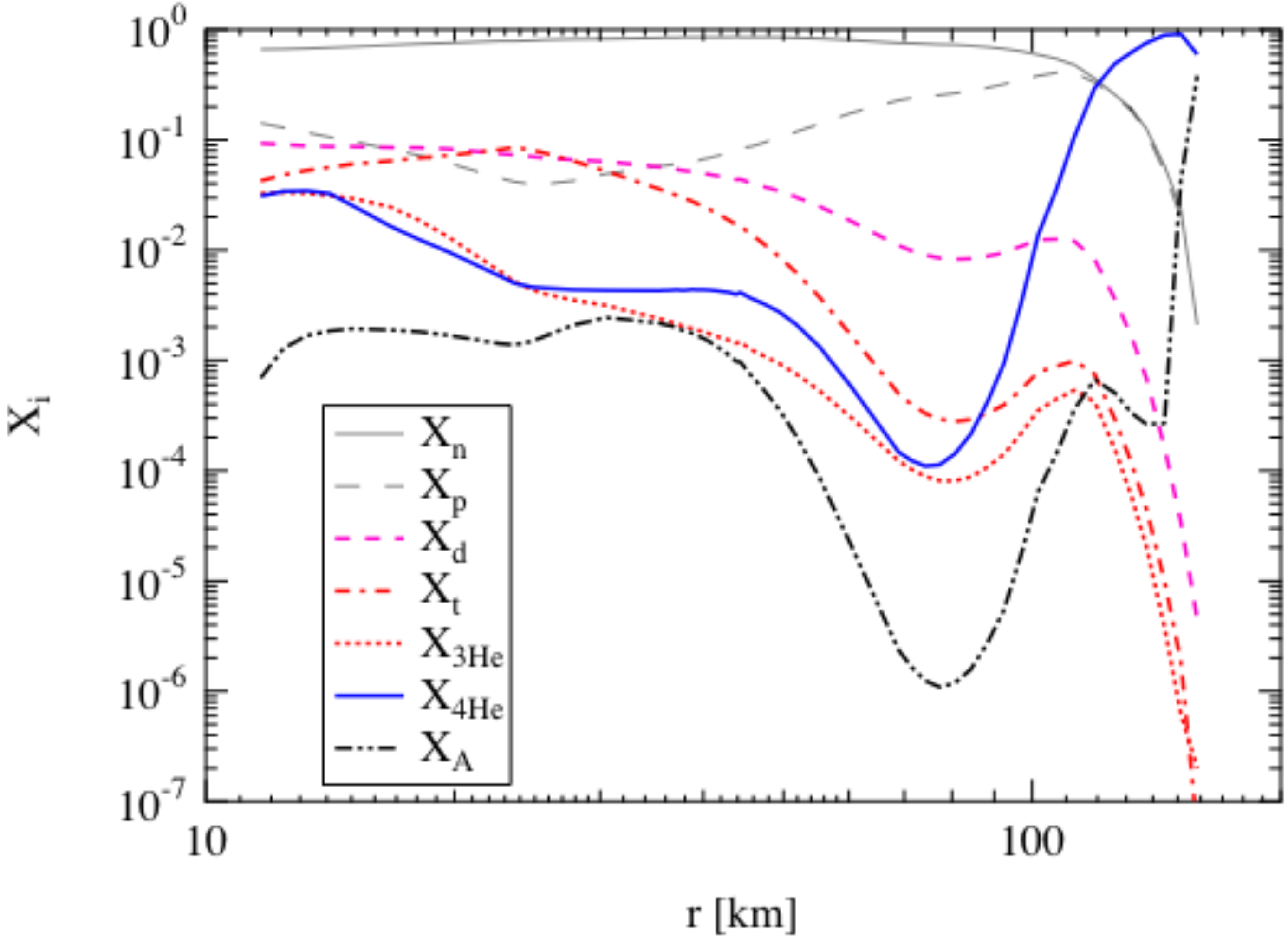
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

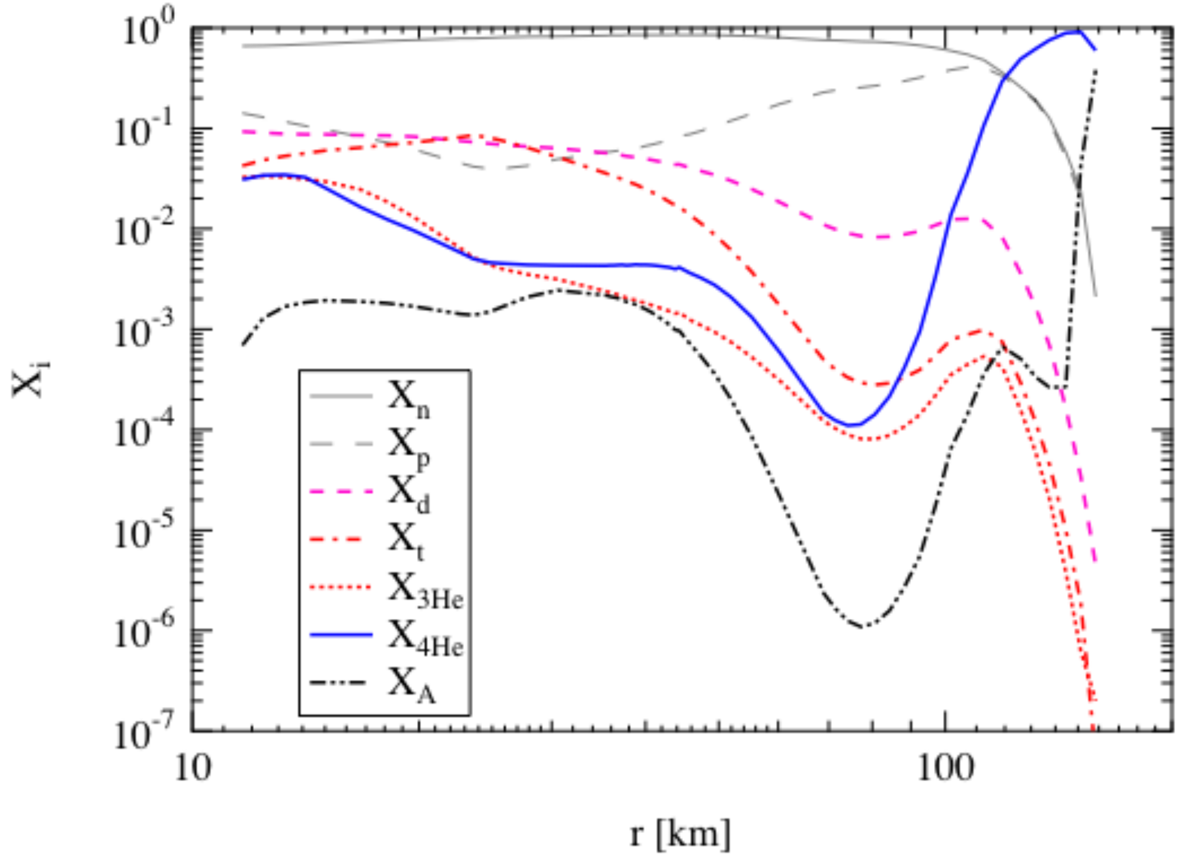
Composition of supernova core



Mass fraction X of light clusters for a post-bounce supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

Composition of supernova core

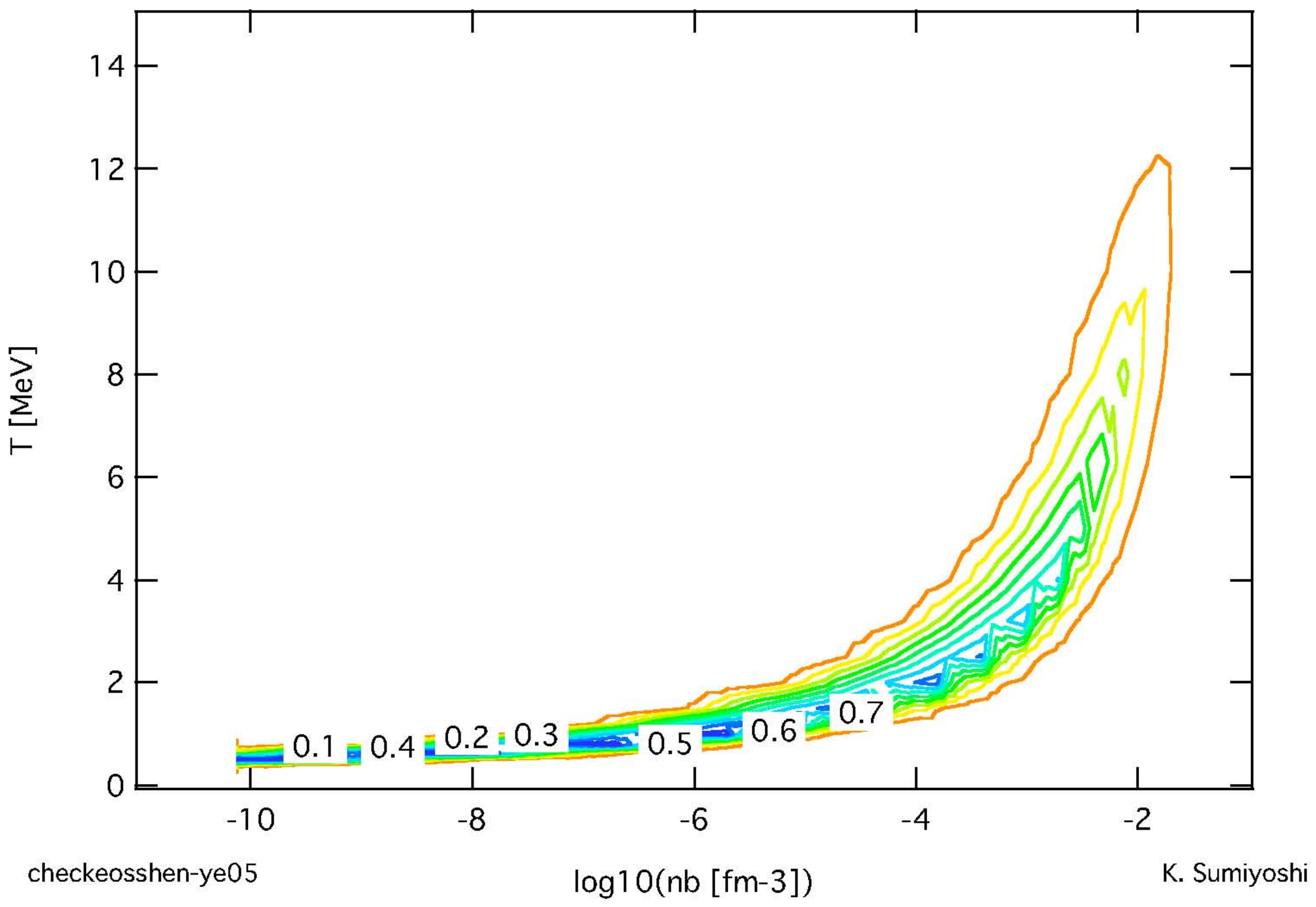


Mass fraction X
of light clusters
for a post-bounce
supernova core

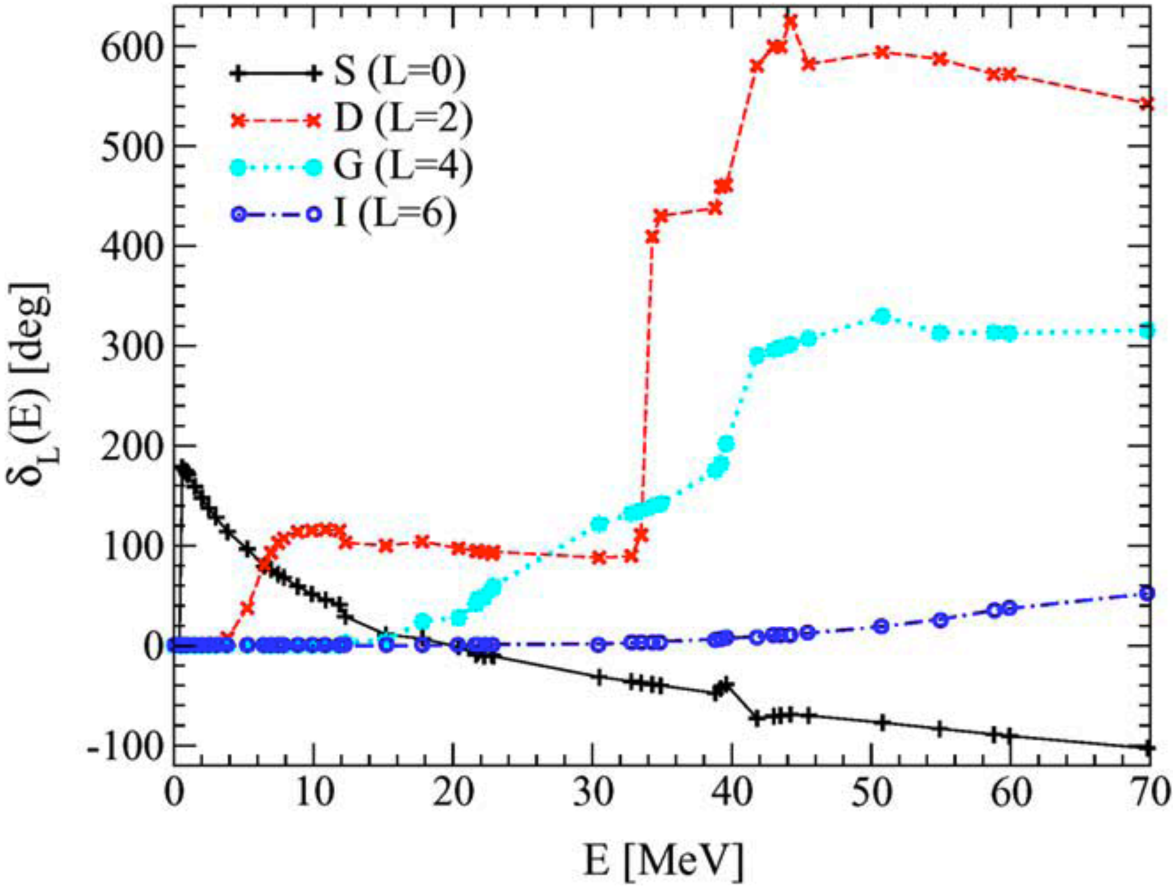
K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

S. Heckel, P. P. Schneider and A. Sedrakian,
Light nuclei in supernova envelopes: a quasiparticle gas model
Phys. Rev. C **80**, 015805 (2009).

alpha-fraction in symmetric matter



α-α scattering phase shifts



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

α-n scattering phase shifts

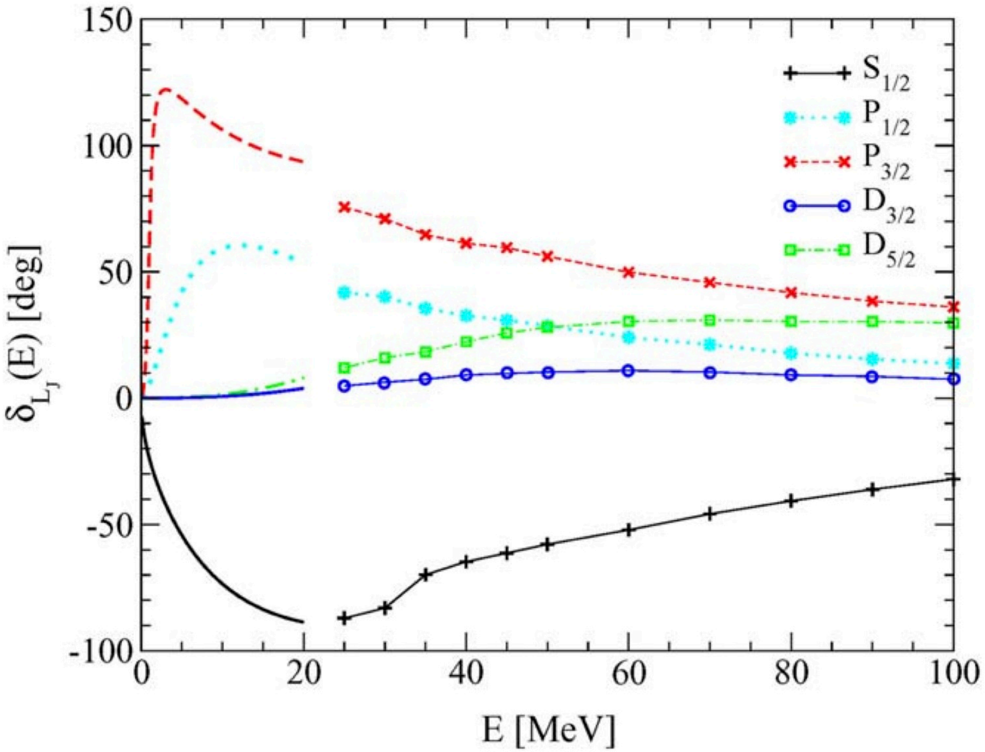
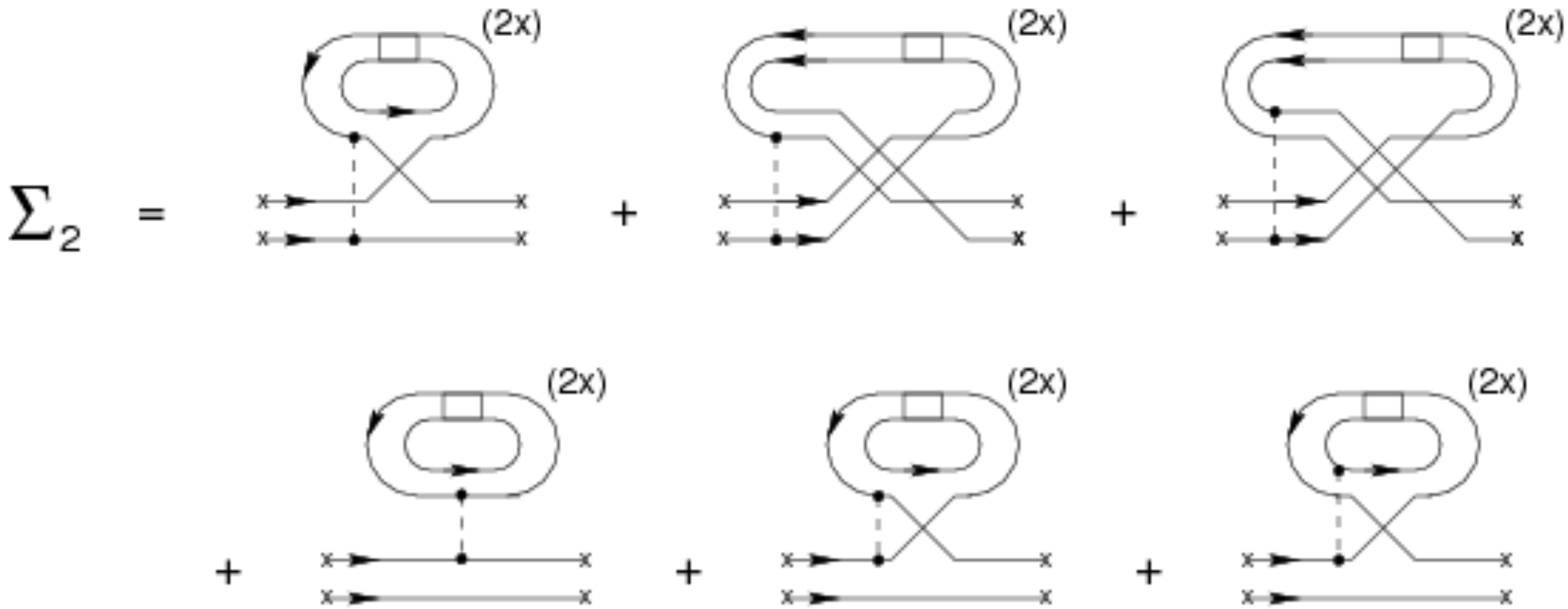


Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{L_J}(E)$ versus laboratory energy E. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F-waves included in our results for $b_{\alpha n}$.

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Correlations in the medium



cluster mean-field approximation

EOS at low densities from HIC

PRL **108**, 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

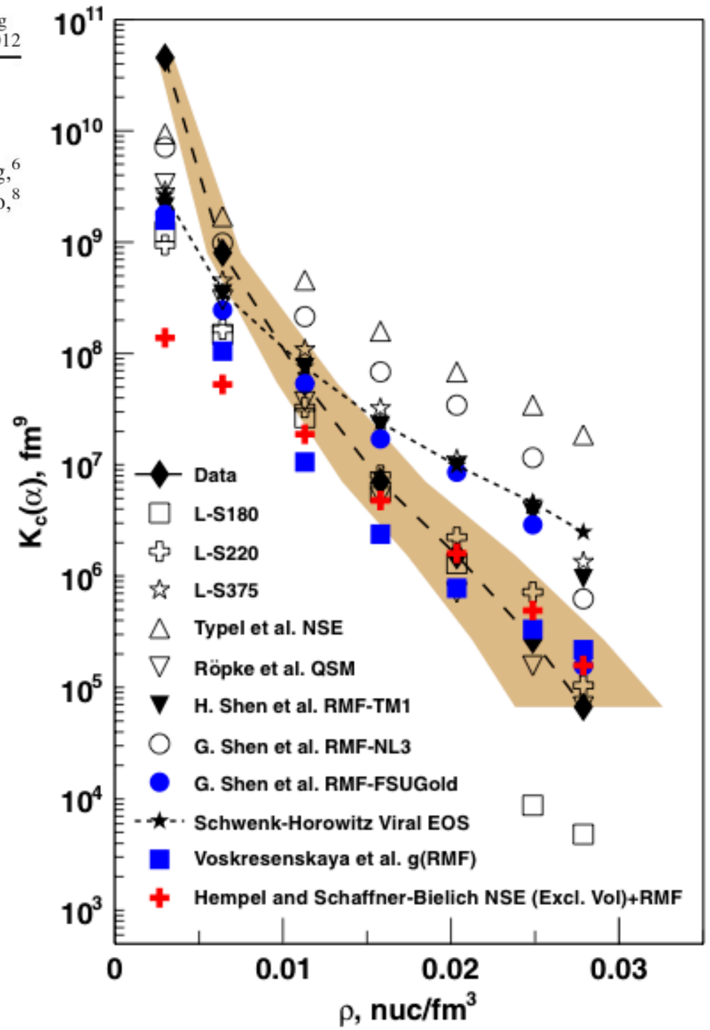
Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

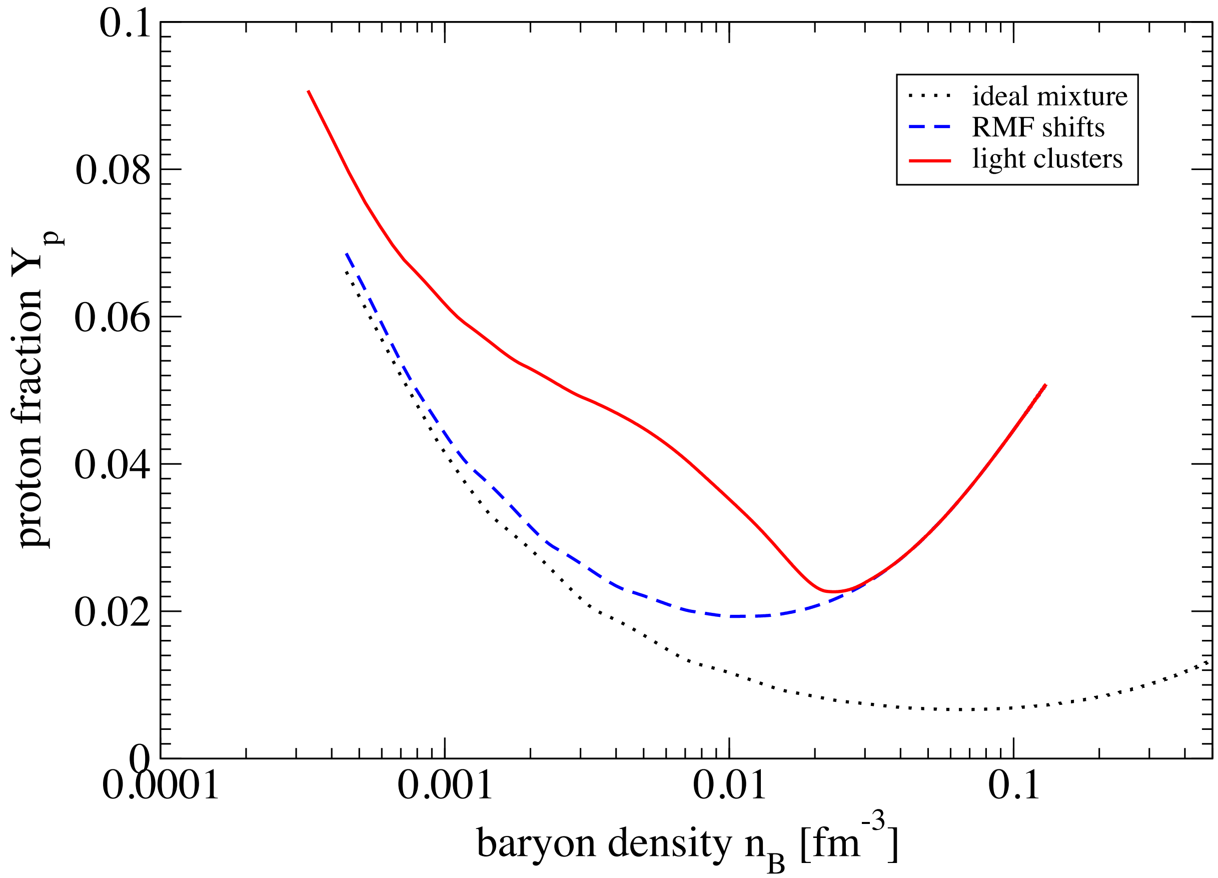
$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$



Influence of cluster formation on β equilibrium

T = 5 MeV
$\mu_\nu = 0$
(no neutrinos)

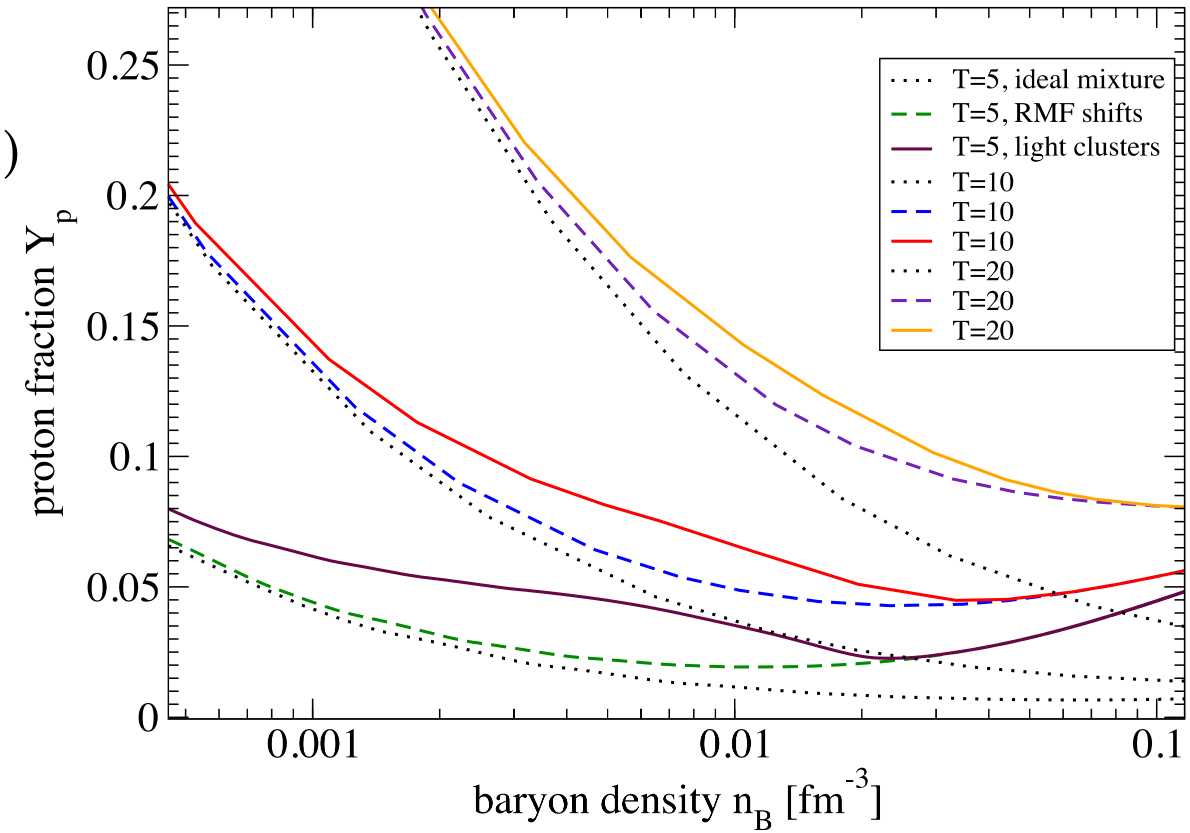
- Ideal mixture
- RMF shifts
- Including light elements



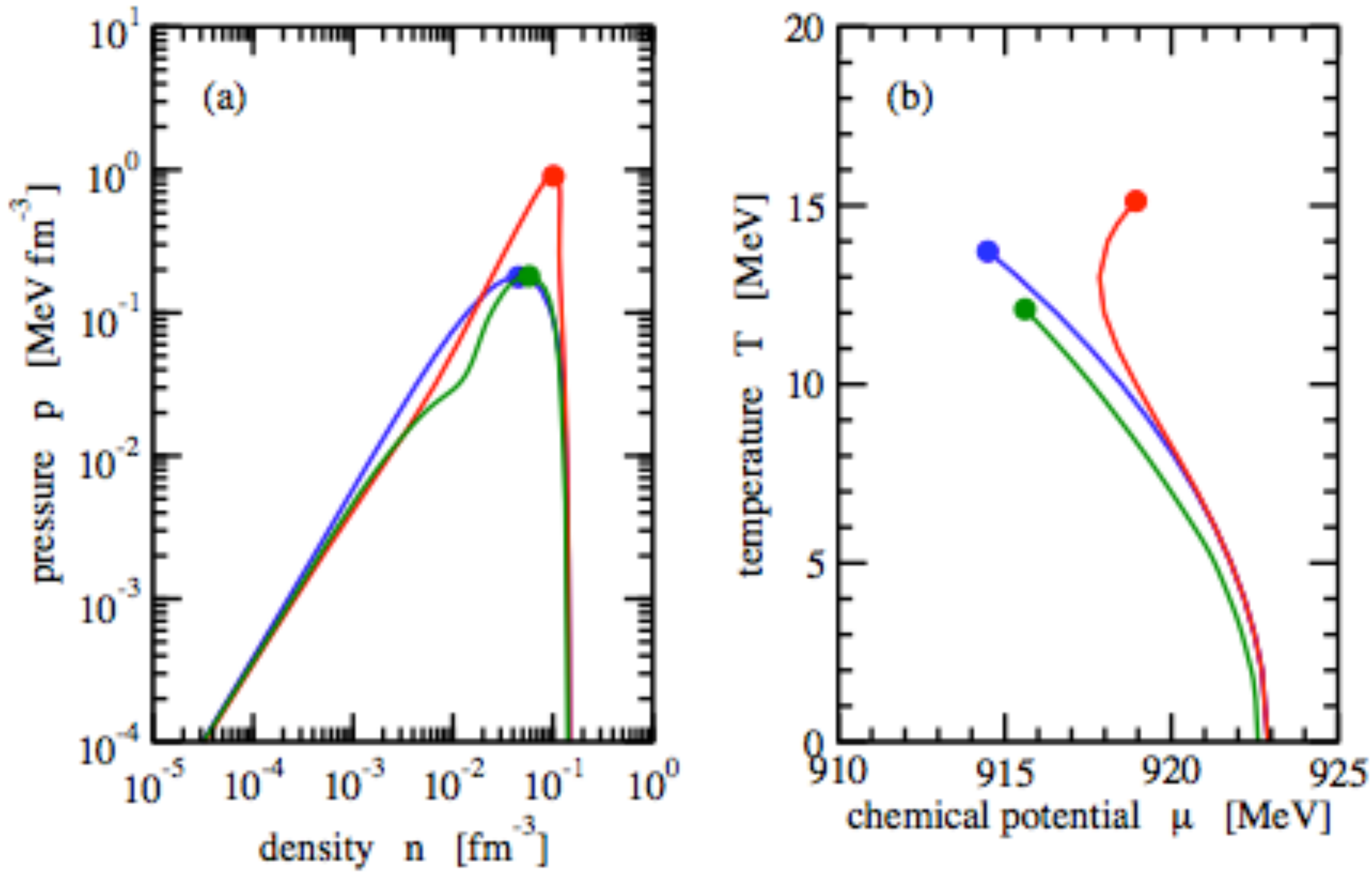
Influence of cluster formation on β equilibrium

Dependence on T
$\mu_\nu = 0$ (no neutrinos)

- Ideal mixture
- RMF shifts
- Including light elements



Liquid-vapor phase transition



blue: no light cluster, green: with light clusters, QS, red: cluster-RMF

S. Typel et al., PRC **81**, 015803 (2010)

α−matter and cluster-virial expansion

- Bound states may become dominant:
  Treat it like new species (chemical picture)

- Cluster expansion:
  Introduce corresponding fugacities (chemical potentials)

- Inclusion of continuum states:
  scattering phase shifts, Beth-Uhlenbeck formula

- Introduce quasiparticles and avoid double counting;
  transition to the high-density region

G. Roepke et al., Nucl. Phys. A, 897, 20 (2013)

Bound state formation in many-fermion systems

Examples:

Quark-gluon plasma: hadrons; deconfinement phase transition
Nucleon systems: nuclei; nuclear matter
Electron-ion plasma: atoms; metals
Electron-hole plasma: exciton; electron-hole droplets

Properties:

Equation of state, transport coefficient, dynamical structure factor

Theoretical approaches:

Simulations, Path integral, Quantum statistics of many-particle systems:

- Lagrangian/Hamiltonian
- Green functions (Feynman diagrams, partial summations, chemical picture)
- Self-energy, spectral function
- Correlation functions, physical properties

Nucleon-nucleon interaction

QCD? Effective Lagrangians, interaction potentials (PEST)

singlet (nn, pp): a = -23.678 fm, r = 1.726 fm

triplet (pn): a = 5.396 fm, r = 2.729 fm, E = -2.225 MeV

$$k \cot \delta = -\frac{1}{a} + r_0 \frac{k^2}{2}$$

Separable interaction

- **general form:**

$$V_\alpha(p, p') = \sum_{i,j=1}^{N} w_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}(p') \qquad \text{uncoupled}$$

and

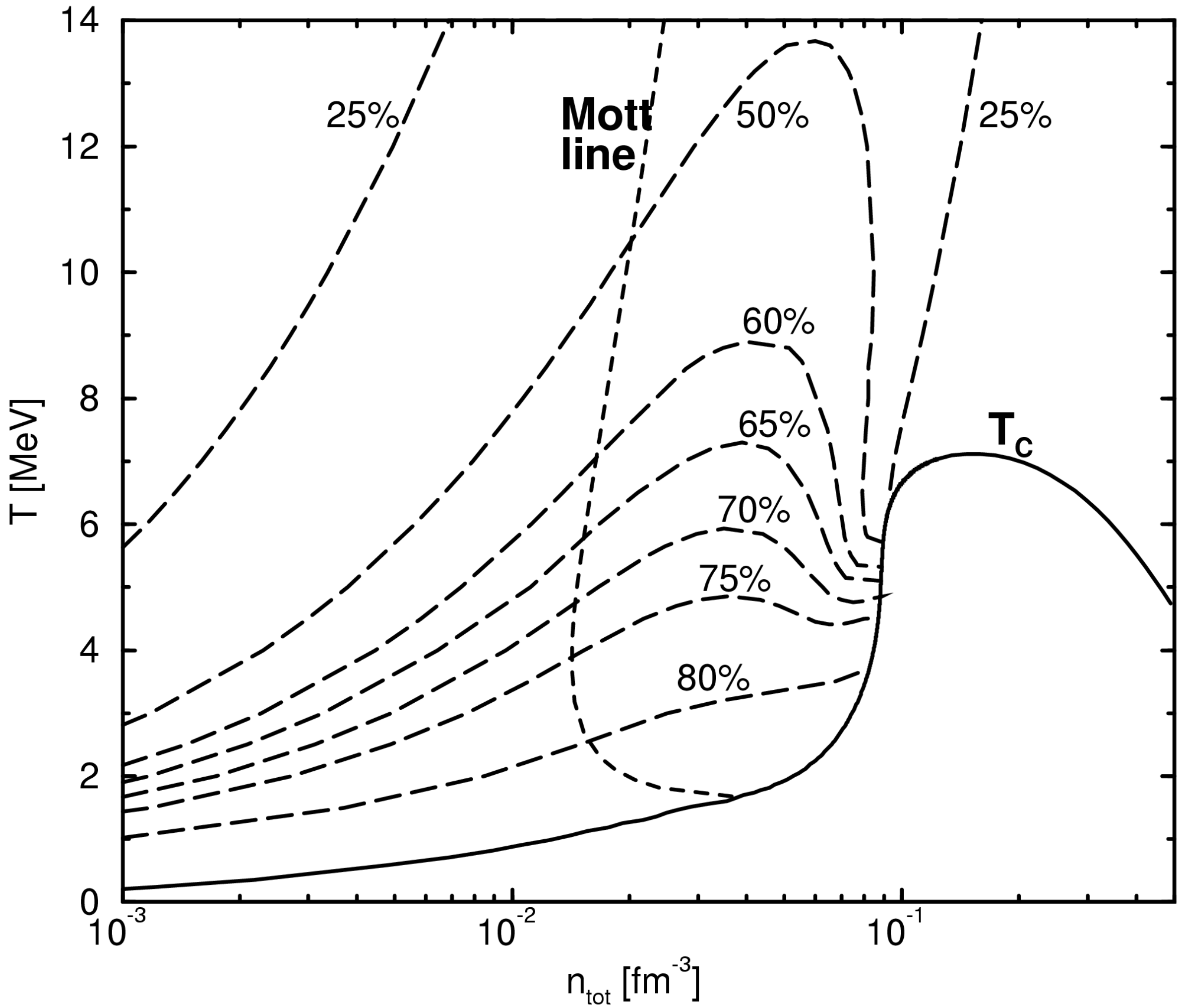
$$V_\alpha^{LL'}(p, p') = \sum_{i,j=1}^{N} w_{\alpha i}^{L}(p) \lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \qquad \text{coupled}$$

$p,\ p'$	in- and outgoing relative momentum
α ...	channel
N ...	rank
$\lambda_{\alpha ij}$.	coupling parameter
$L,\ L'$	orbital angular momentum

Weak interaction - beta equilibrium? Coulomb interaction?

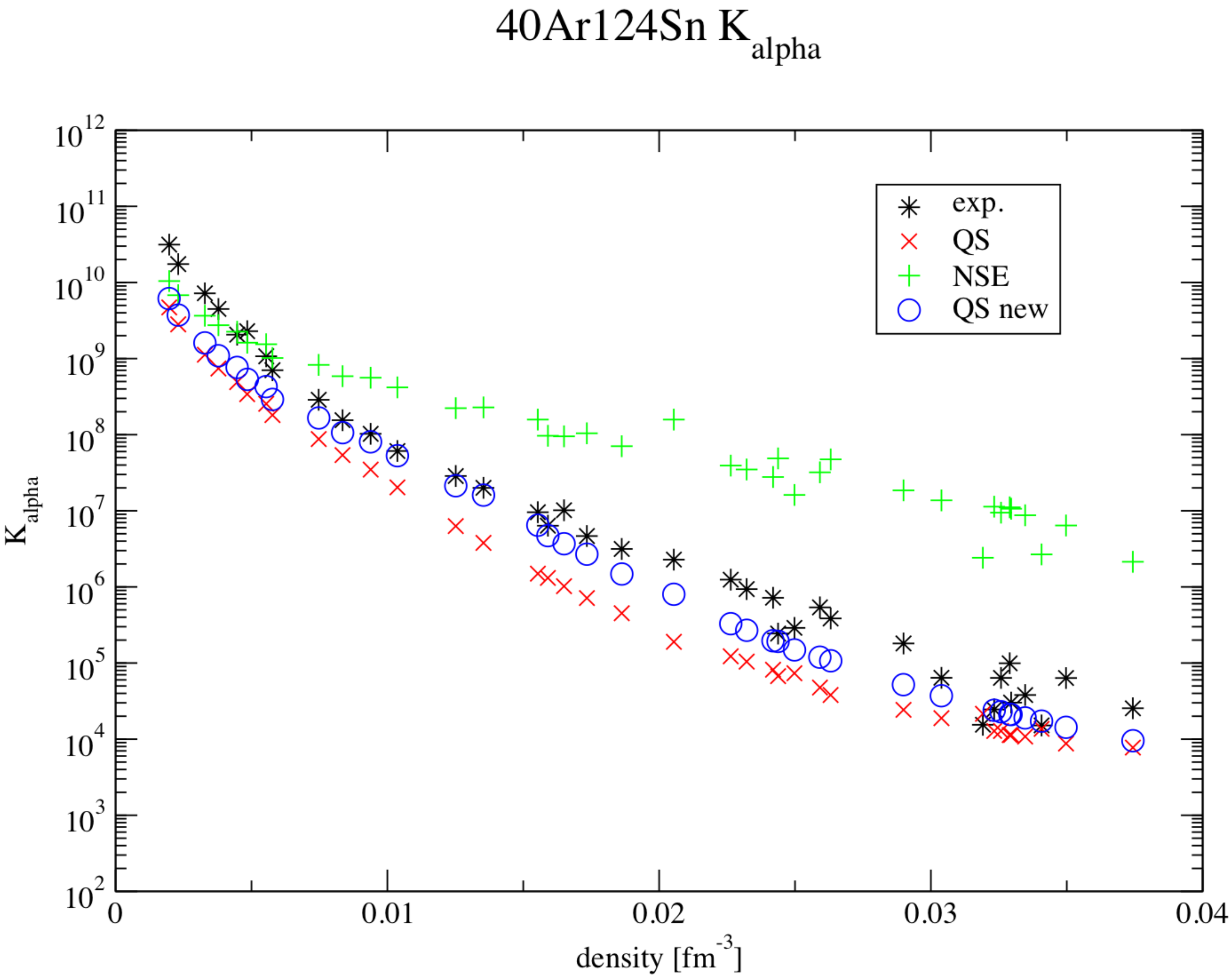
Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

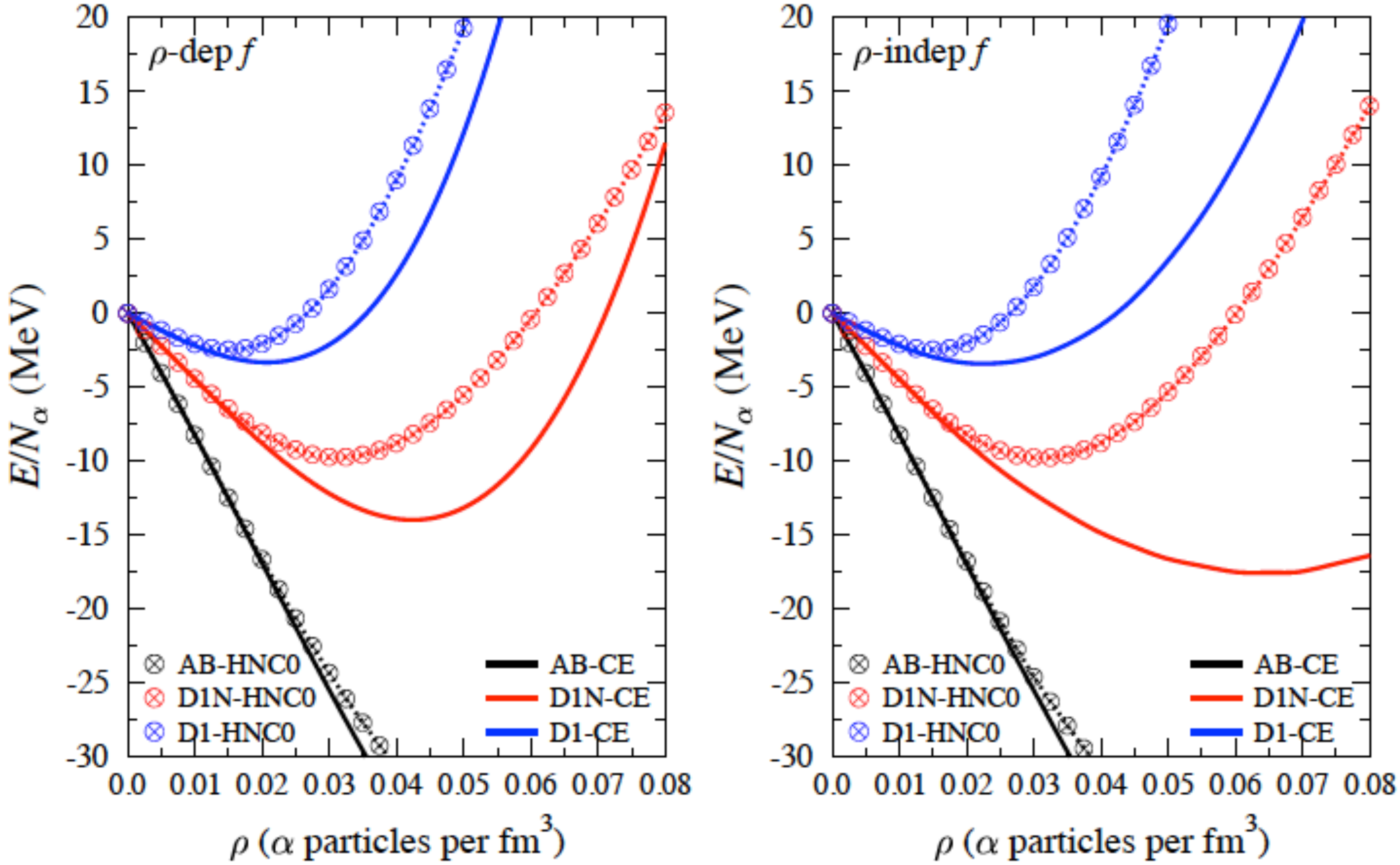


H. Stein et al.,
Z. Phys. **A351**, 259 (1995)

QS versus NSE: comparison with data



Energy of α-Matter at T=0



Total energy calculated with the cluster expansion
within the HNC/0 (circles) and HNC/4 (solid lines) approximation.
Different interaction potentials

F.Carstoiu, S.Misicu, PLB, 2009

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

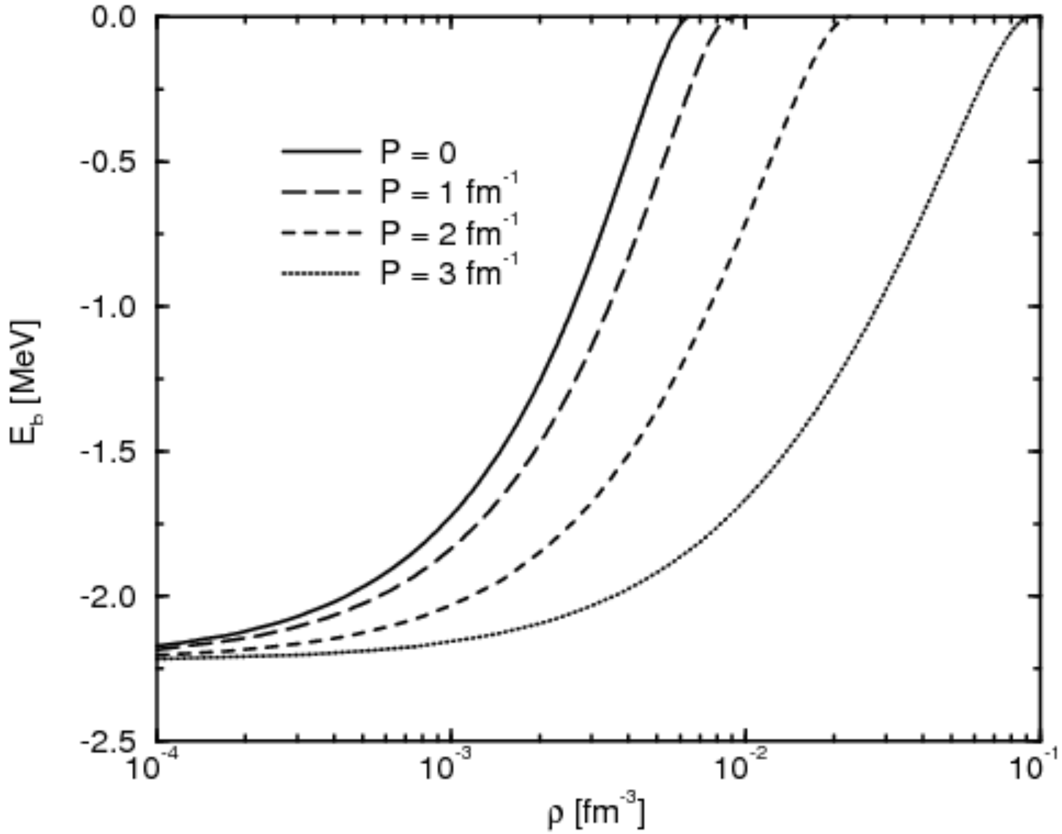
$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

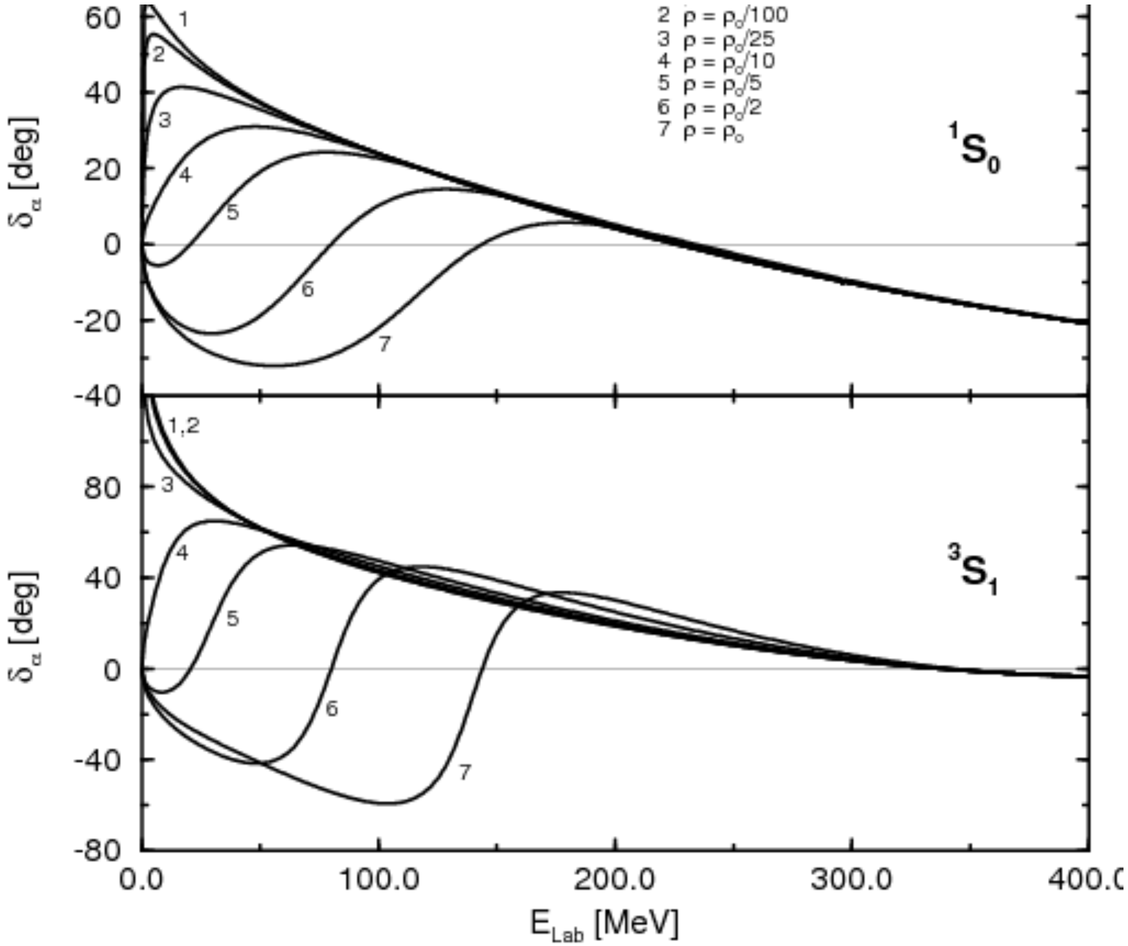
- Bose-Einstein condensation

Deuterons in nuclear matter



T=10 MeV, P: center of mass momentum

Scattering phase shifts in matter



Outline

- Correlations and bound state formation in fermion systems

  The chemical picture, Green functions, spectral functions

- Correlations in nuclear systems

  where it occurs, what do we know: nuclei, stars, HIC; effective interactions

- Many-particle theory: Equation of state, generalized Beth-Uhlenbeck eq.

  Low-density limit: Mass action law, nuclear statistical equilibrium, virial expansion

  Near saturation: quasiparticles, dissolution of bound states, correlated mean field

- Applications:

  Equation of state in SN explosions, HIC, Hoyle-like states, surface correlations

- Quantum condensates in nuclear matter:

  Pairing vs. quartetting, BEC-BCS crossover

- Bose condensates in finite systems:

  Suppression of condensate, HFB and cluster formation (pairing and quartetting)

- Applications

Signatures of clustering

- Nuclear structure: Hoyle state, states near the threshold n-alpha decay
- rms radii: comparatively large, low-density matter, skin?
- Nuclear reactions, alpha decay (preformation at the surface)
- Yields of light elements in HIC, equation of state and symmetry energy
- Excited states

Clustering is not “exotic”.
It is a general feature of (low density) nuclear systems.

Mott points from cluster yields

PRL **108,** 062702 (2012) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2012

Experimental Determination of In-Medium Cluster Binding Energies and Mott Points in Nuclear Matter

K. Hagel,[1] R. Wada,[2,1] L. Qin,[1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[2] M. Huang,[2] J. Wang,[2] H. Zheng,[1] S. Kowalski,[6] C. Bottosso,[1] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[7] M. Lunardon,[7] S. Moretto,[7] G. Nebbia,[7] S. Pesente,[7] V. Rizzi,[7] G. Viesti,[7] M. Cinausero,[8] G. Prete,[8] T. Keutgen,[9] Y. El Masri,[9] and Z. Majka[10]

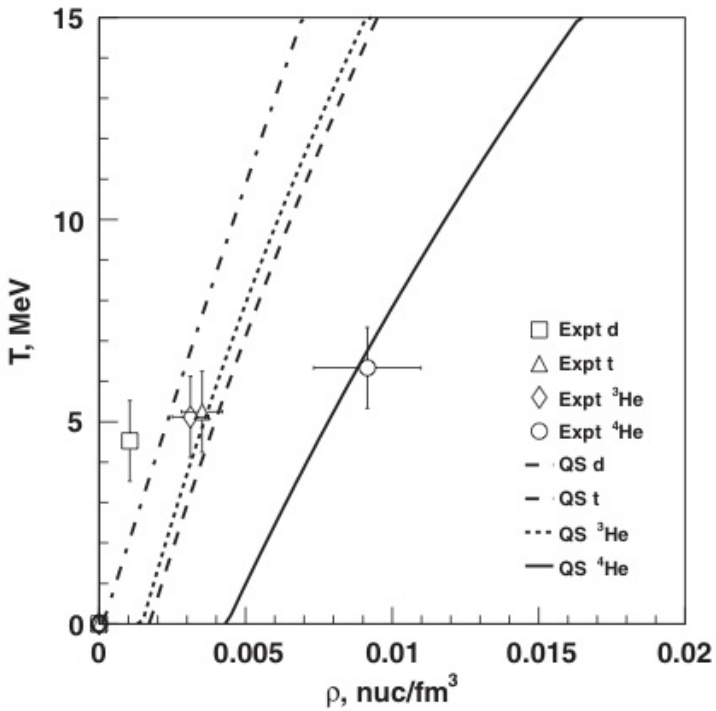
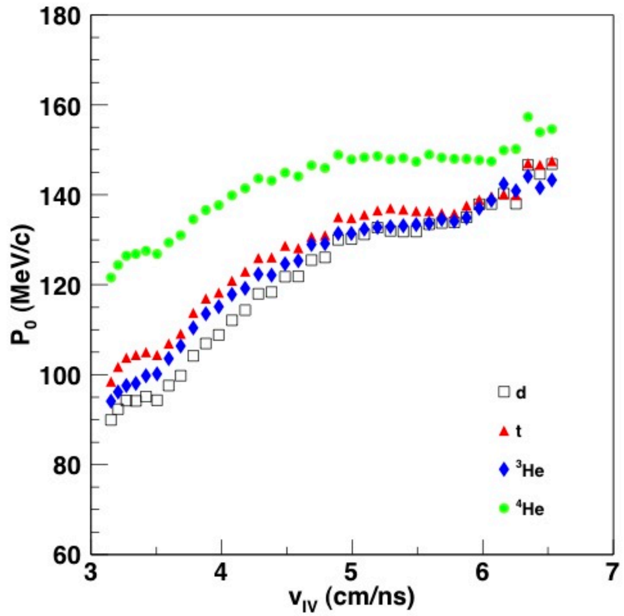


FIG. 3. Comparison of experimentally derived Mott point densities and temperatures with theoretical values. Symbols represent the experimental data. Estimated errors on the temperatures are 10% and on the densities 20%. Lines show polynomial fits to the Mott points presented in Ref. [1].

Symmetry energy at medium densities

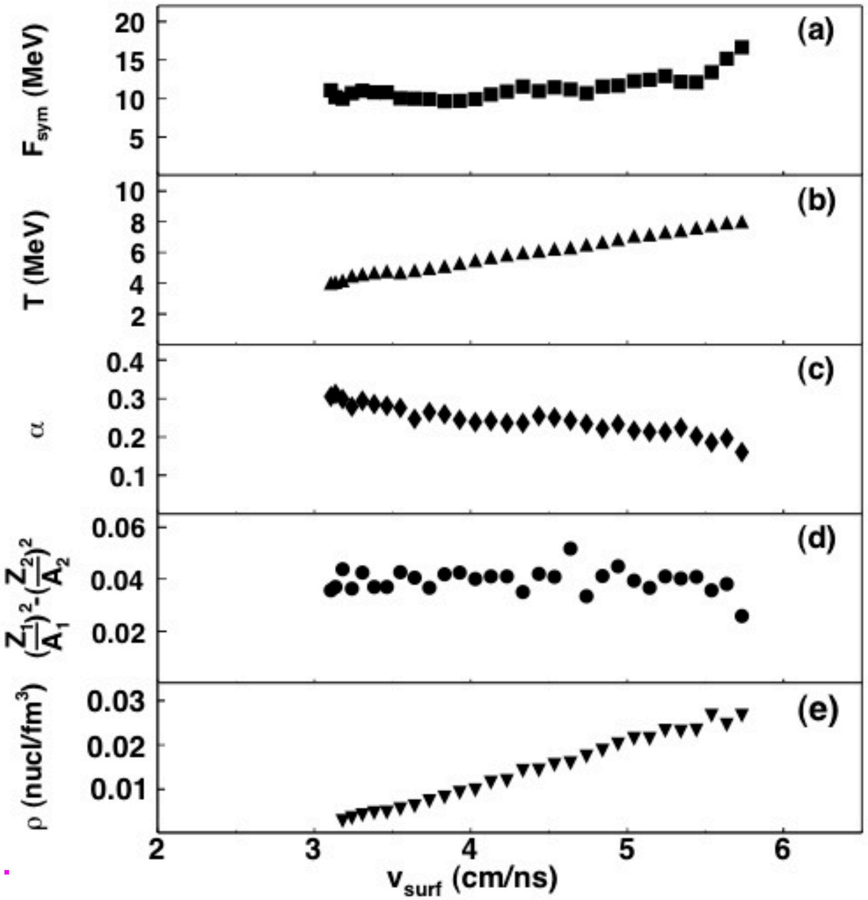
The Nuclear Matter Symmetry Energy at $0.03 \leq \rho/\rho_0 \leq 0.2$

R. Wada,[1,2] K. Hagel,[2] L. Qin,[2] J. B. Natowitz,[2] Y. G. Ma,[3] G. Röpke,[4] S. Shlomo,[2] A. Bonasera,[2,5] S. Typel,[6] Z. Chen,[2,1] M. Huang,[2,1] J. Wang,[2,1] H. Zheng,[2] S. Kowalski,[7] C. Bottosso,[2] M. Barbui,[2] M. R. D. Rodrigues,[2] K. Schmidt,[2] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] and Z. Majka[11]



Coalescence parameter,
Mekjian model

R. Wada et al., Phys. Rev. C 85, 064618 (2012).



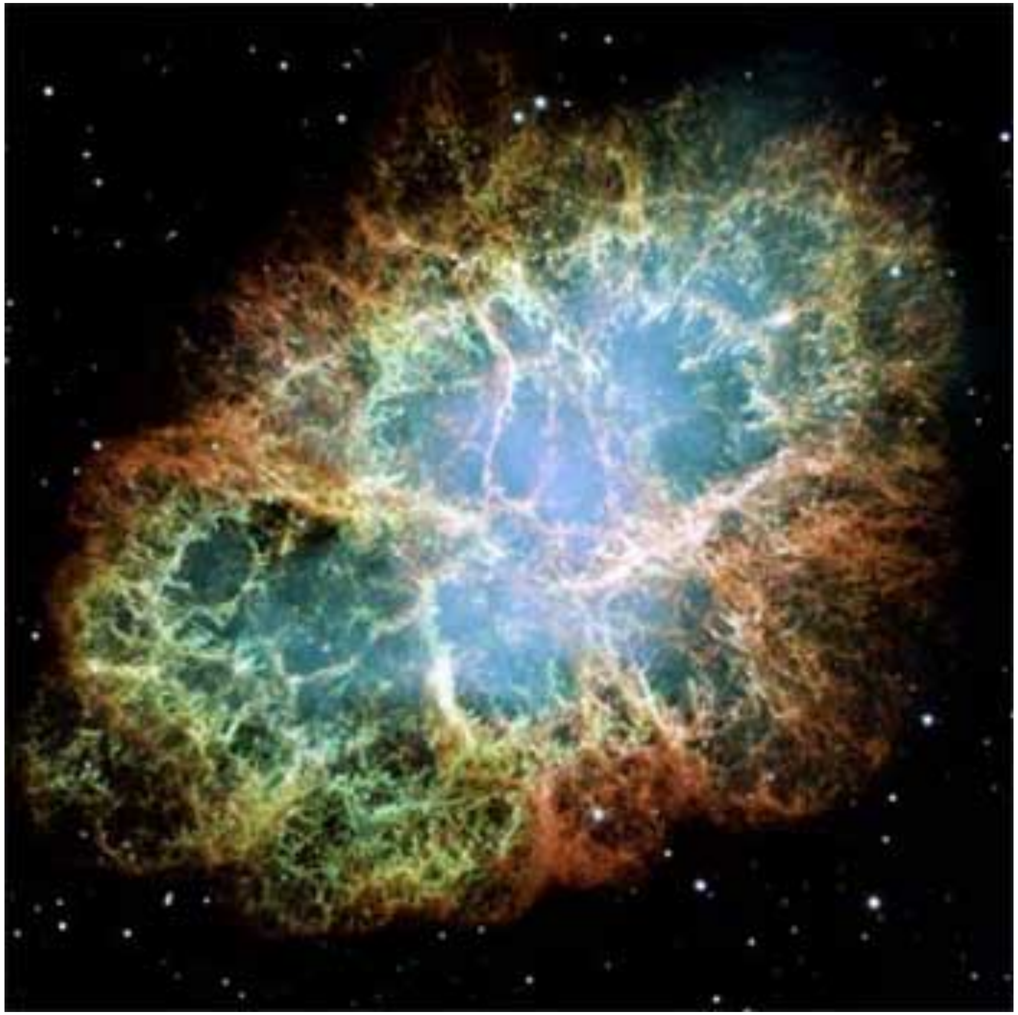
In-medium modification of transport properties of dense matter

- D. Blaschke, G. Röpke, H. Schulz, A.D. Sedrakian,
  Nuclear in-medium effect on the thermal conductivity and viscosity of neutron star matter
  PL B 338, **111** (1994)

- D. Blaschke, G. Röpke, H. Schulz, A.D. Sedrakian, D. Voskresensky,
  Nuclear in-medium effects and neutrino emissivity of neutron stars.
  M. N. R. A. S. **273**, 596 (1995)

Supernova

Crab nebula, 1054 China, PSR 0531+21

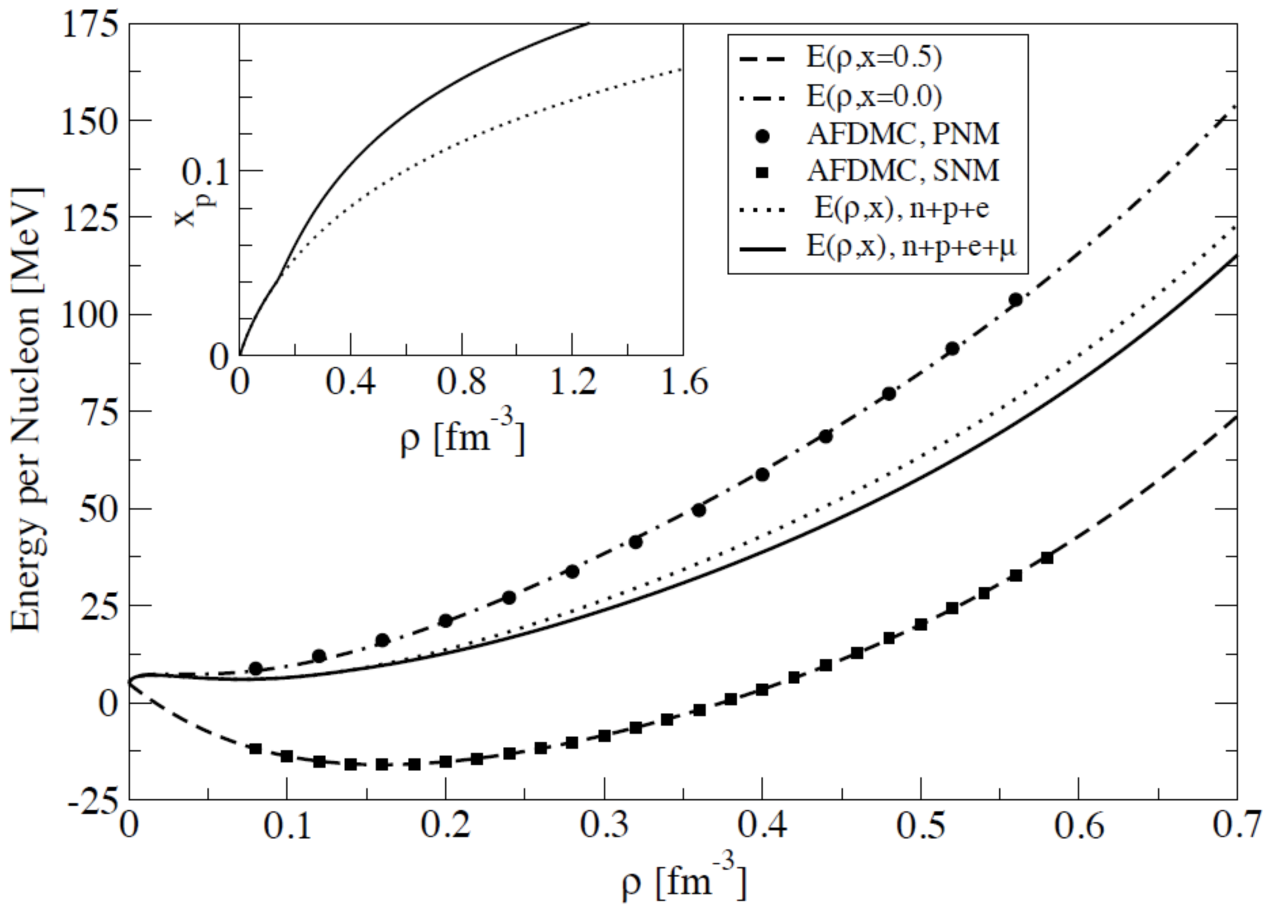


M1, the Crab Nebula. Courtesy of NASA/ESA

Nuclear matter equation of state

- Nuclear systems: Quasiparticle approach
  Brueckner, HFB; Skyrme, Relativistic Mean Field (RMF)
- Account of correlations in warm dense matter:
  two-particle (deuteron, pairing),
  four-particle (alpha-like) correlations, light elements
- Low-density regions: Nuclear Statistical Equilibrium (NSE)
  Hoyle-like states in light expanded nuclei,
  surface of nuclei, neck emission, alpha matter…
- Quantum statistical approach ($n < 0.15$ fm^{-3}, $T < 20$ MeV)
  Equation of state, Beth-Uhlenbeck formula
  disappearance of clusters at high densities, Pauli blocking
- Experimental signatures
  Heavy Ion Collisions (HIC), Symmetry energy, SN explosions, …

Diffusion Monte Carlo EOS calculation



S. Gandolfi, A. Yu. Illarionov, et al., Mon.Not.R.Astron.Soc., 2010