

# Impact of the pion potential on pion observables in intermediate energy heavy-ion collisions

**Dan Cozma**

**IFIN-HH  
Magurele/Bucharest, Romania**

dan.cozma@theory.nipne.ro

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# Motivation & Contents

$$V(\Delta^{++}) = V_s + \frac{3}{2} V_v$$

$$V(\Delta^+) = V_s + \frac{1}{2} V_v$$

$$V(\Delta^0) = V_s - \frac{1}{2} V_v$$

$$V(\Delta^-) = V_s - \frac{3}{2} V_v$$

$$V_s = \frac{1}{2}(V_n + V_p)$$

$$\delta = \frac{1}{3}(V_n - V_p)$$

B.-A. Li, NPA 708,  
365 (2002)

## The Model

Pion Production  
Isovector Potential  
Energy Conservation

## Pion Potential

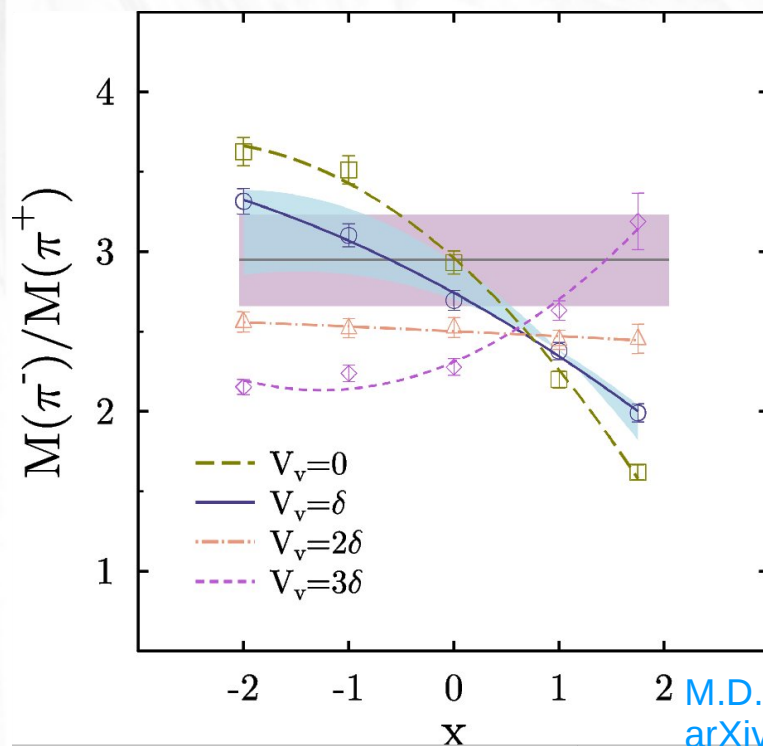
S Wave Component  
P Wave Component

## Observables

Pion Multiplicities  
Pion Average  $p_T$

## Constraints on SE Stiffness

## Summary & Conclusions

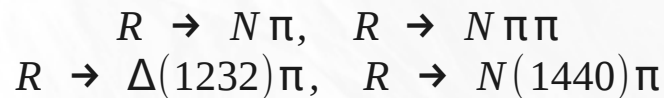


softer

# Pion production

## two step process:

- resonance excitation in baryon-baryon collisions  
parametrization of the OBE model of  
[S.Huber et al., NPA 573, 587 \(1994\)](#)
- resonance decay:  
Breit-Wigner shape of the resonance spectral function;  
parameters -> [K. Shekhter, PRC 68, 014904 \(2003\)](#)  
decay channels:

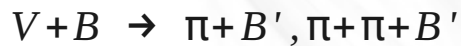


## pion absorption:

- resonance model (all 4\* resonances below 2 GeV)  
[K. Shekhter, PRC 68, 014904 \(2003\)](#)

## additional channels:

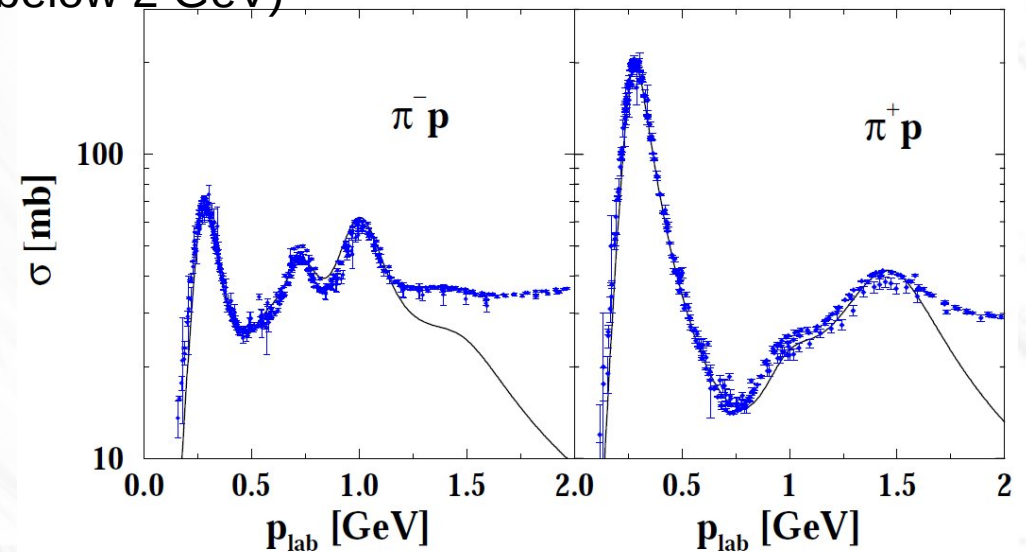
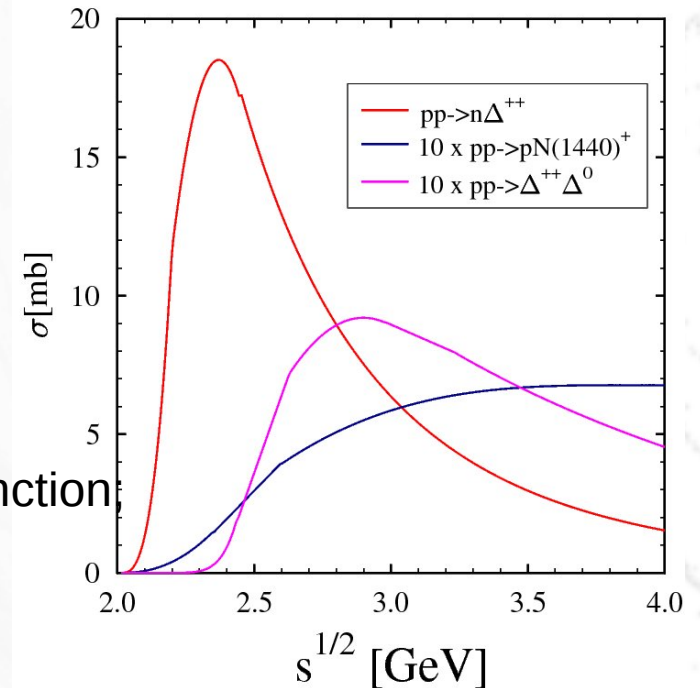
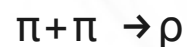
vector meson production/absorption



vector meson decay  $\rho \rightarrow \pi+\pi$



pion annihilation



# Isospin dependence of EoS

a) **momentum dependent** – generalization of the Gogny interaction:

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$U(\rho, \beta, p, \tau, x) = A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} + B(\rho/\rho_0)^{\sigma} (1 - x\beta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \beta \rho_{\tau'}$$

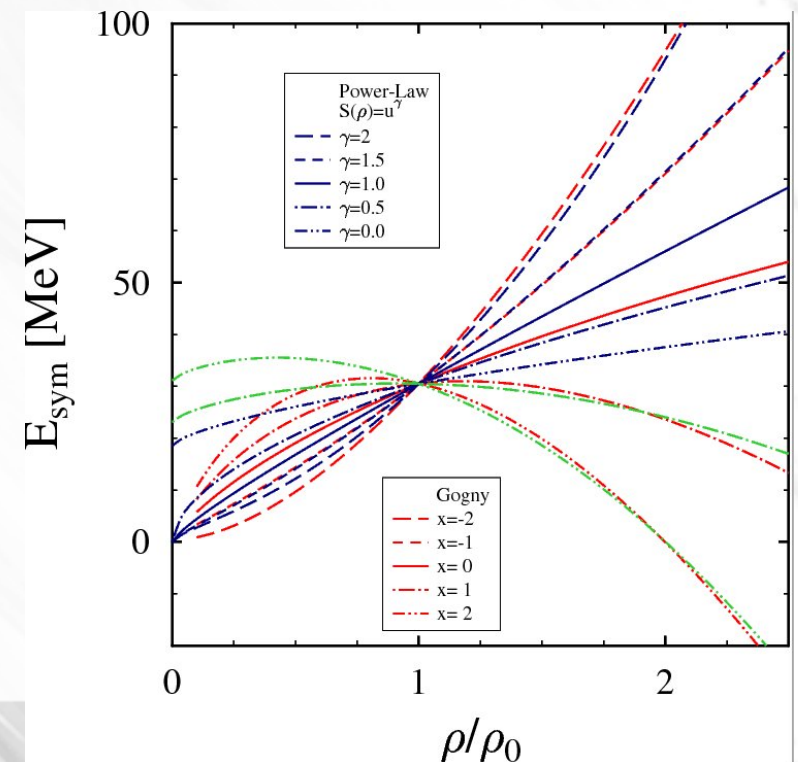
$$+ \frac{2C_{\tau\tau}}{\rho_0} \int d^3p' \frac{f_{\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} + \frac{2C_{\tau\tau'}}{\rho_0} \int d^3p' \frac{f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}$$

$$S(\rho) = S(\rho_0) + \frac{L_{\text{sym}}}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_{\text{sym}}}{18} \frac{(\rho - \rho_0)^2}{\rho_0^2}$$

x	$L_{\text{sym}}$ [MeV]	$K_{\text{sym}}$ [MeV]
-2	152	418
-1	106	127
0	61	-163
1	15	-454
2	-301	-745

b) **momentum dependent** – power law

$$U_{\text{sym}}(\rho, \beta) = \begin{cases} S_0(\rho/\rho_0)^{\gamma} - \text{linear, stiff} \\ a + (18.5 - a)(\rho/\rho_0)^{\gamma} - \text{soft, supersoft} \end{cases}$$



# Energy Conservation

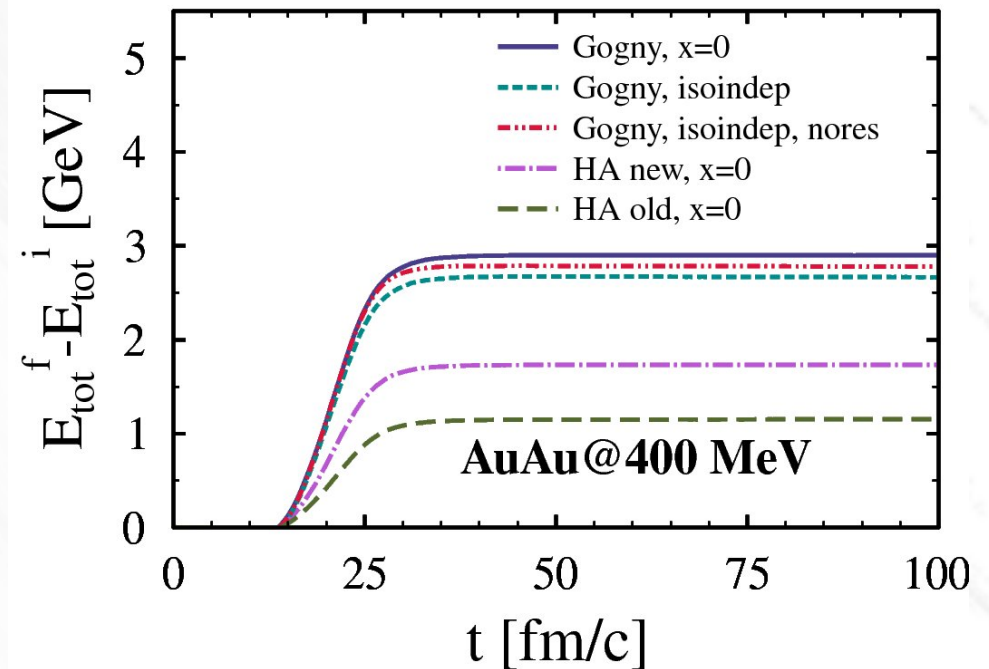
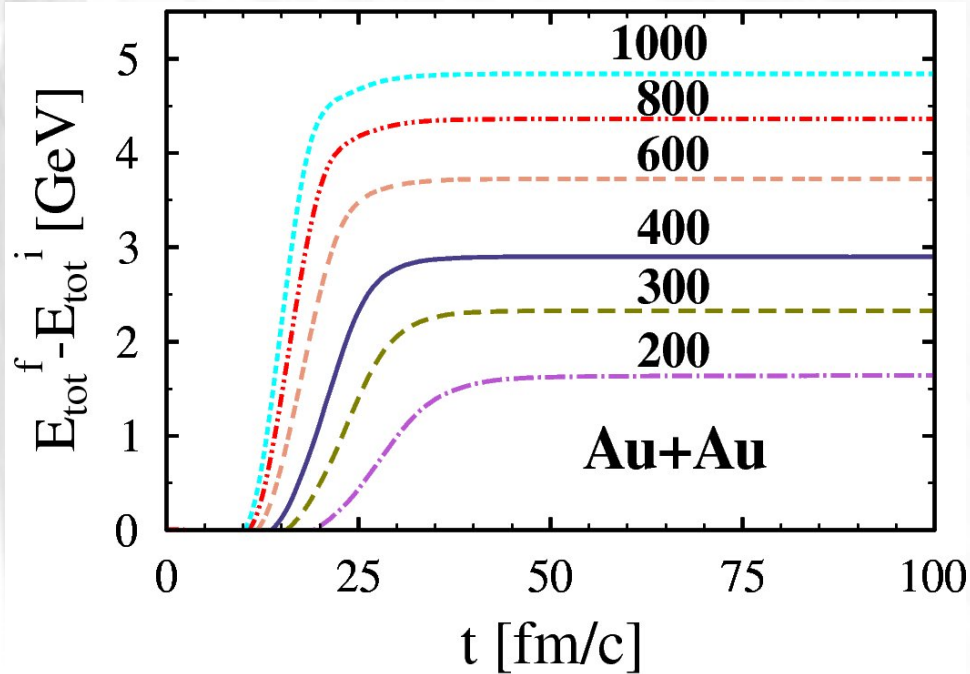
**80's transport models** – total energy conserved (potentials dependent only on density)

**Collective phenomena** – momentum dependence of opt.pot.

**Isospin effects** – isospin asymmetry dependence

**Violation of total energy conservation**

Determination of final state kinematics of 2-body collisions neglects medium effects



Gogny: Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

HA: Hartnack, Aichelin, PRC 49, 2901 (1994)

# Approximations

**Elastic scattering:**

$$\sqrt{s_f} \approx \sqrt{s_i}$$

$$\sqrt{s^*} = 0.5(\sqrt{s_f} + \sqrt{s_i})$$

**Resonance excitation:**

$$\sqrt{s_f} - \sqrt{s_i} \approx 25 \text{ MeV}$$

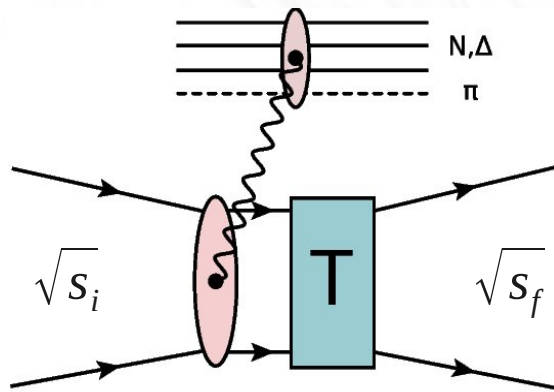
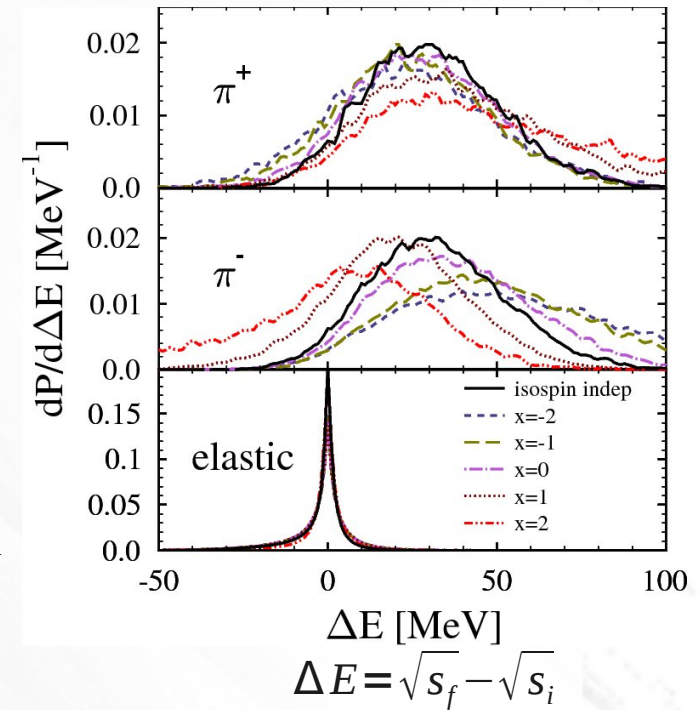
$$\sqrt{s^*} = \sqrt{s_f}$$

**Resonance absorption: detailed balance**

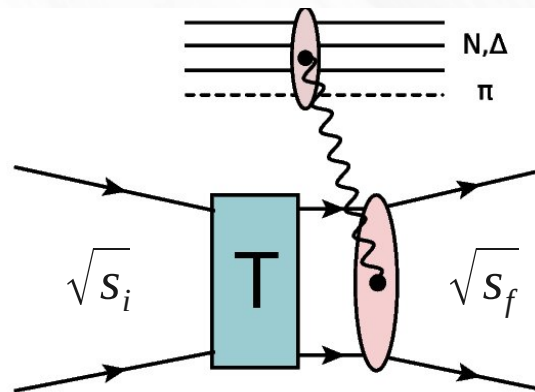
P. Danielewicz et al., NPA 533, 712 (1992)

$$\frac{d\sigma^{NR \rightarrow NN}}{d\Omega} = \frac{1}{4} \frac{m_R p_{NN}^2}{p_{NR}} \frac{d\sigma^{NN \rightarrow NR}}{d\Omega} \times \left[ \frac{1}{2\pi} \int_{m_N+m_\pi}^{\sqrt{s_i}-m_N} dM M p'_{NR} A_R(M) \right]^{-1}$$

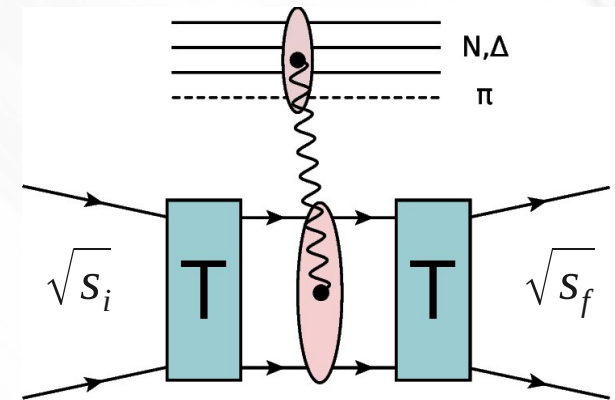
$$\sqrt{s_i} \rightarrow p_{NR}, \quad \sqrt{s_f} \rightarrow p_{NN}$$



**initial**



**final state**



**rescattering**

# Different "scenarios"

$$\sum_i \sqrt{p_i^2 + m_i^2} + U(p_i) = \sum_j \sqrt{p_j'^2 + m_j'^2} + U(p_j')$$

**VEC** – vacuum energy conservation constraint

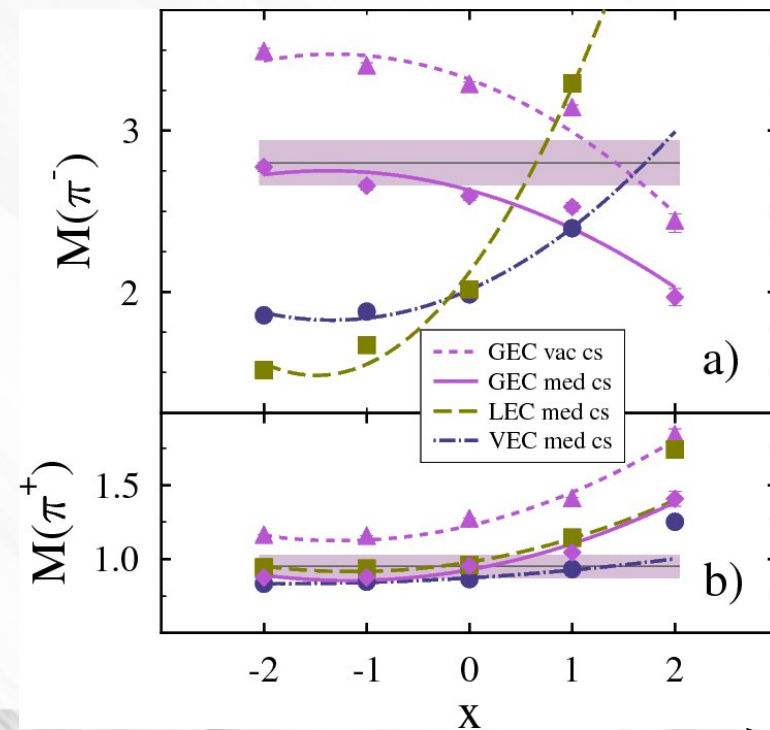
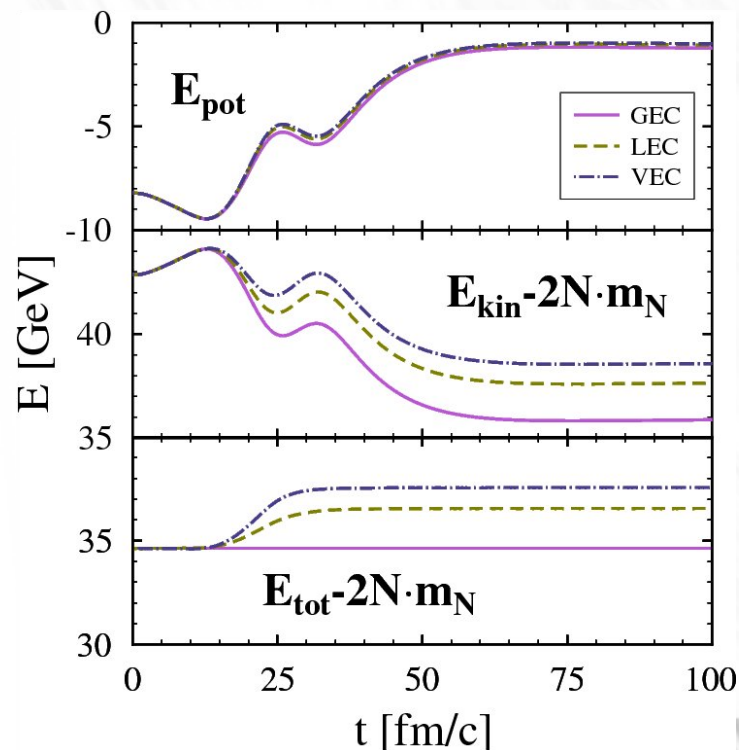
**LEC** - "local" energy conservation – limited impact on multiplicities and ratios

**GEC** - "global" energy conservation – conserve energy of the entire system  
-in-medium cross-sections for the inelastic channels

T. Song, C.M. Ko PRC 91, 014901 (2015)  
G. Ferini et al. PRL 97, 202301 (2006)  
S. Teis et al. Z.Phys. A356, 421 (1997)  
C.Fuchs et al. PRC 55, 411 (1997)

$$\sigma^{*(12 \rightarrow 23)} = \left[ \frac{m_1^* m_2^* m_3^* m_4^*}{m_1 m_2 m_3 m_4} \right]^{1/2} \sigma^{(12 \rightarrow 34)}$$

AuAu@400 MeV, b=0 fm

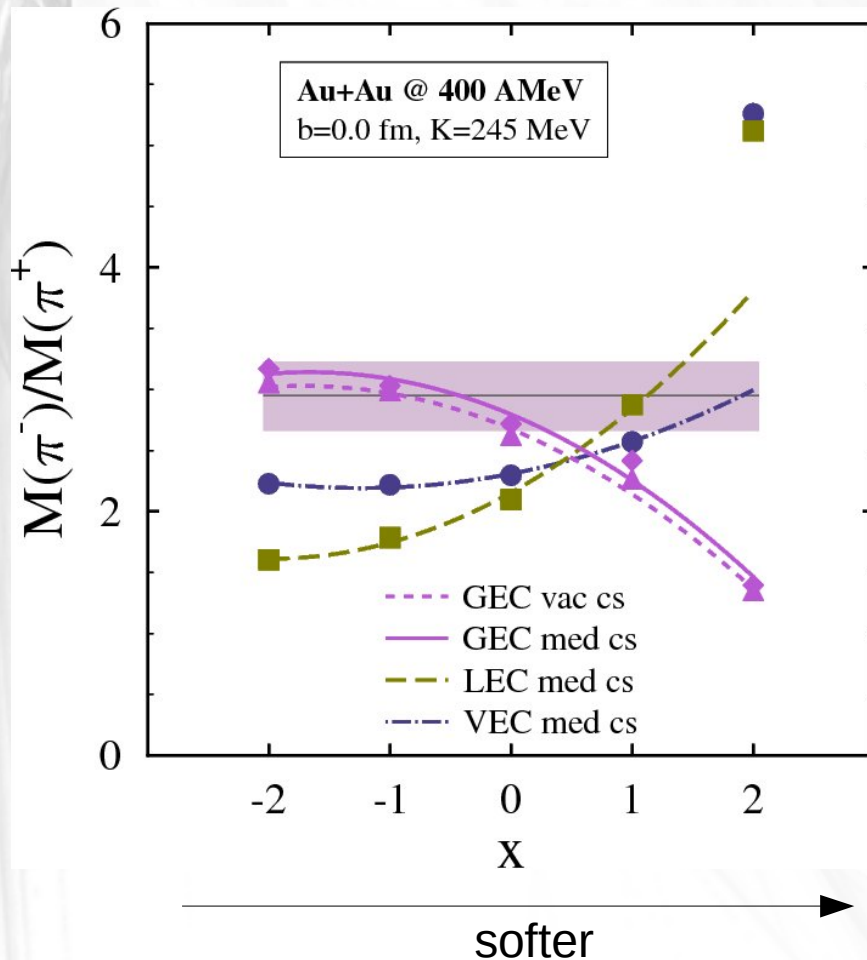


Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

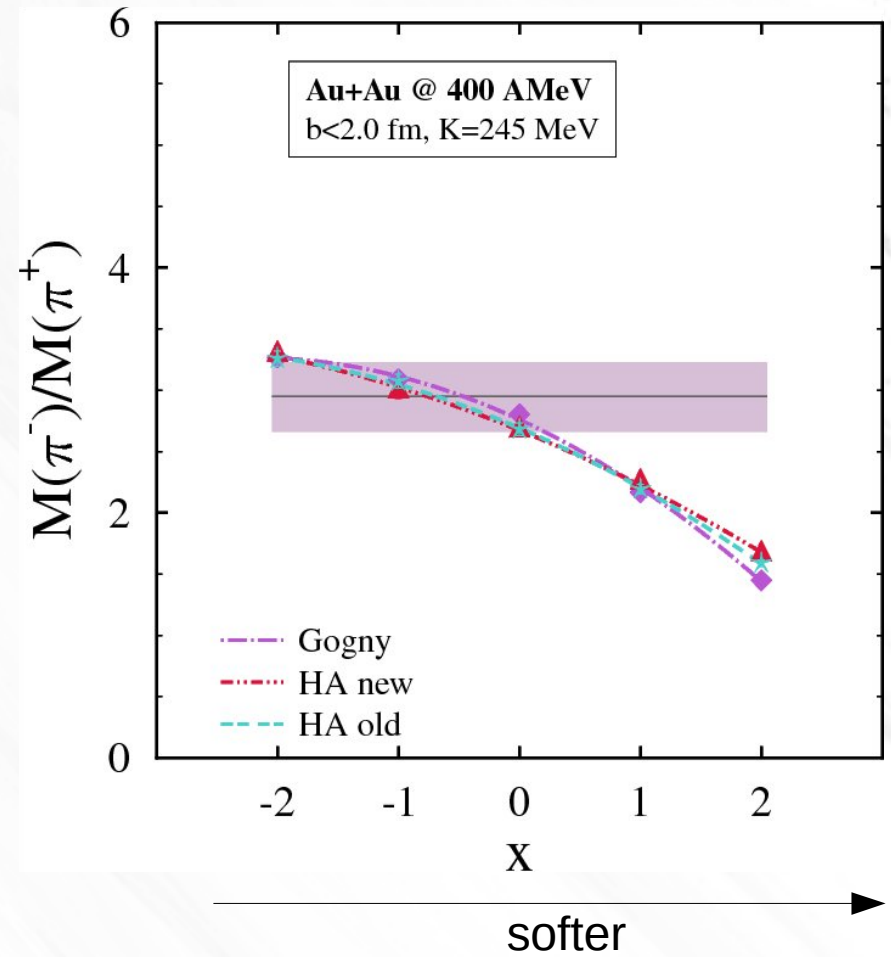
softer

# Multiplicity Ratio

## Energy Conservation Scenario



## Optical Potential Dependence





# Pion S wave potential

**Microscopical approaches: Hadronic Models** J.Nieves et al., NPA 554, 509 (1993)

$$V(\pi^-)=26.3 \text{ MeV} \quad V(\pi^0)=16.2 \text{ MeV} \quad V(\pi^+)=6.2 \text{ MeV}$$

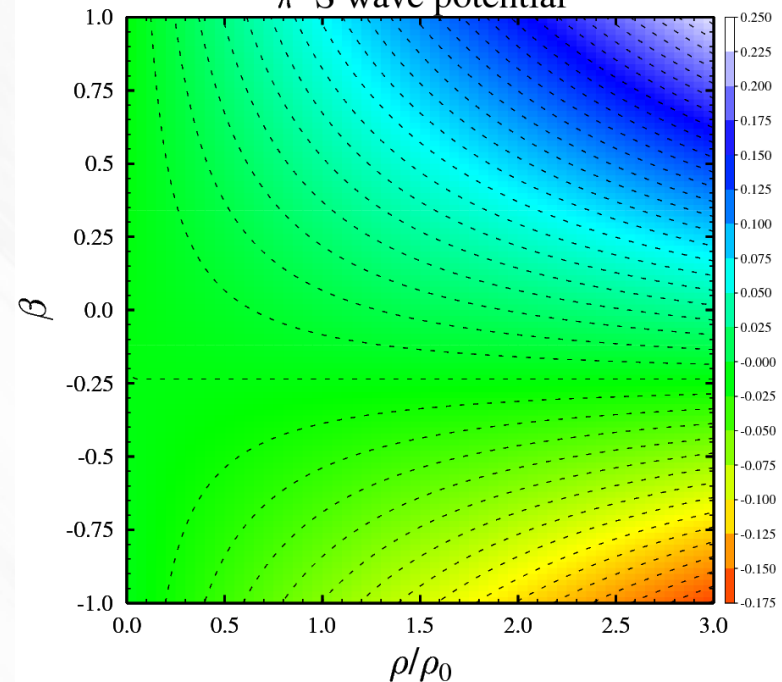
M.Doring et al, PRC 77, 024602 (2008)

can accommodate repulsion needed by phenomenological fits; large theoretical uncertainties

**ChPT:** two loop approximation N. Kaiser et al. PLB 512, 283 (2001)

$$V(\pi^-)=13.8 \text{ MeV} \quad V(\pi^0)=6.1 \text{ MeV} \quad V(\pi^+)=-1.2 \text{ MeV}$$

$\pi^-$  S wave potential



## Parametrization:

$$V_S(r) = -4\pi [b(r) + \epsilon_2 B_0 \rho^2(r)]$$

$$b(r) = \epsilon_1 [b_0 \rho(r) + b_1 (\rho_n(r) - \rho_p(r))]$$

$$\epsilon_1 = \frac{1}{2\mu} + \frac{1}{2M}$$

$$\epsilon_1 = \frac{1}{2\mu} + \frac{1}{4M}$$

$$b_0 = 0.0283 m_\pi^{-1}$$

$$b_1 = -0.120 m_\pi^{-1}$$

$$B_0 = 0.042 i m_\pi^{-4}$$

R. Seki, K. Masutani, PRC 27, 2799 (1983)

$$V(\pi^-)=28.8 \text{ MeV} \quad V(\pi^0)=15.6 \text{ MeV} \quad V(\pi^+)=2.4 \text{ MeV}$$

(at  $\rho = \rho_0$ ,  $\beta = 0.20$ )

# Pion P wave potential

**Three level model (3LM):** allows analytical calculation (self-energies)

M.Urban et al.  
NPA 641, 433 (1988)

**Approximations:** only  $\Delta$  and non-resonant scattering  
perform a non-relativistic reduction  
pion momentum larger than Fermi momentum

$$\Pi(k) = \frac{\Pi_{Nh} + \Pi_{\Delta h} - (g_{11} - 2g_{12} + g_{22}) \Pi_{Nh} \Pi_{\Delta h}}{1 - g_{11} \Pi_{Nh} - g_{22} \Pi_{\Delta h} + (g_{11} g_{22} - g_{12}^2) \Pi_{Nh} \Pi_{\Delta h}}$$

$$\Delta_{\pi}(k) = \frac{1}{k^2 - m_{\pi}^2 - \vec{k}^2 \Pi(k)}$$

$$\Delta_{\pi}(k) \stackrel{3LM}{=} \frac{S_1(\vec{k})}{k_0^2 - \omega_1^2(\vec{k})} + \frac{S_2(\vec{k})}{k_0^2 - \omega_2^2(\vec{k})} + \frac{S_3(\vec{k})}{k_0^2 - \omega_3^2(\vec{k})}$$

**Effective dispersion relation:**

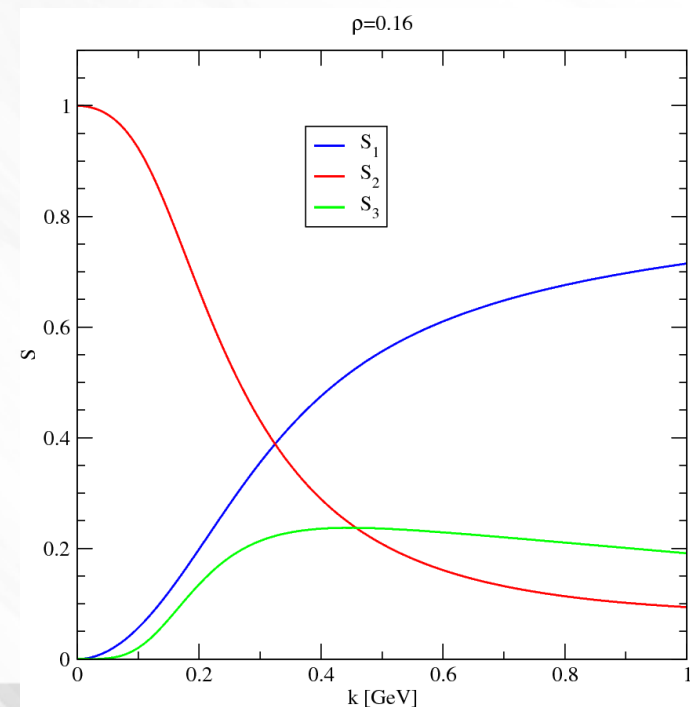
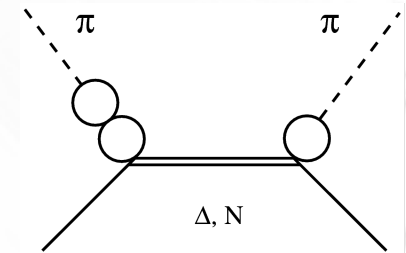
$$\omega_{eff}(\vec{k}) = S_1(\vec{k}) \omega_1(\vec{k}) + S_2(\vec{k}) \omega_2(\vec{k}) + S_3(\vec{k}) \omega_3(\vec{k})$$

W. Ekehalt et al., PLB 298, 31 (1993)

C. Fuchs et al., PRC 55, 411 (1997)

**Effective potential:**

$$V_{\pi}^{eff} = \omega^{eff} - \sqrt{m_{\pi}^2 + \vec{k}^2}$$



# Pion Potential

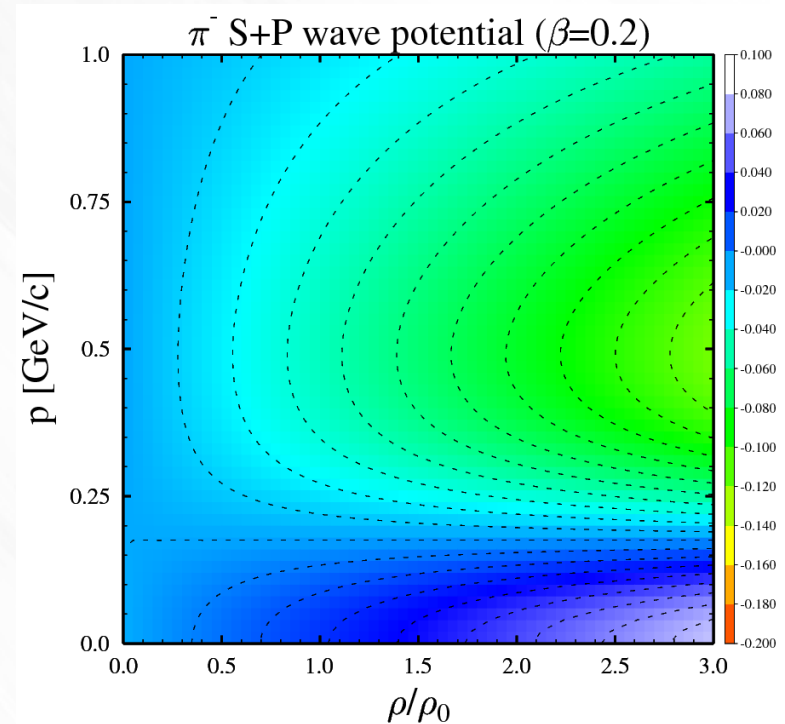
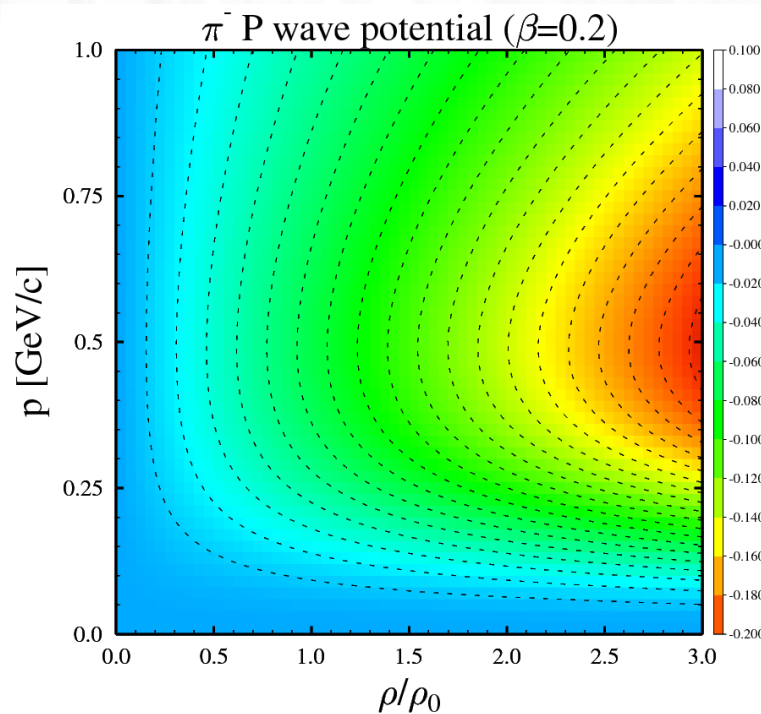
P wave  
(continued)

**Parametrization:** 
$$V(\pi^- ; u, \beta, k) = \frac{k^2}{1 + k^2/\Lambda_1^2 + k^4/\Lambda_2^4} (b_{11} u + b_{21} u \beta)$$

$$\Lambda_1 = 2.138 m_\pi \quad \Lambda_2 = 3.551 m_\pi$$

$$b_{11} = -0.180 m_\pi^{-1} \quad b_{21} = 0.011 m_\pi^{-1}$$

fitted in the region:  $0 < u < 3.0$ ;  $-0.5 < \beta < 0.5$ ;  $0.0 \text{ GeV}/c < k < 0.75 \text{ GeV}/c$



# Comparison with Nieves et al.

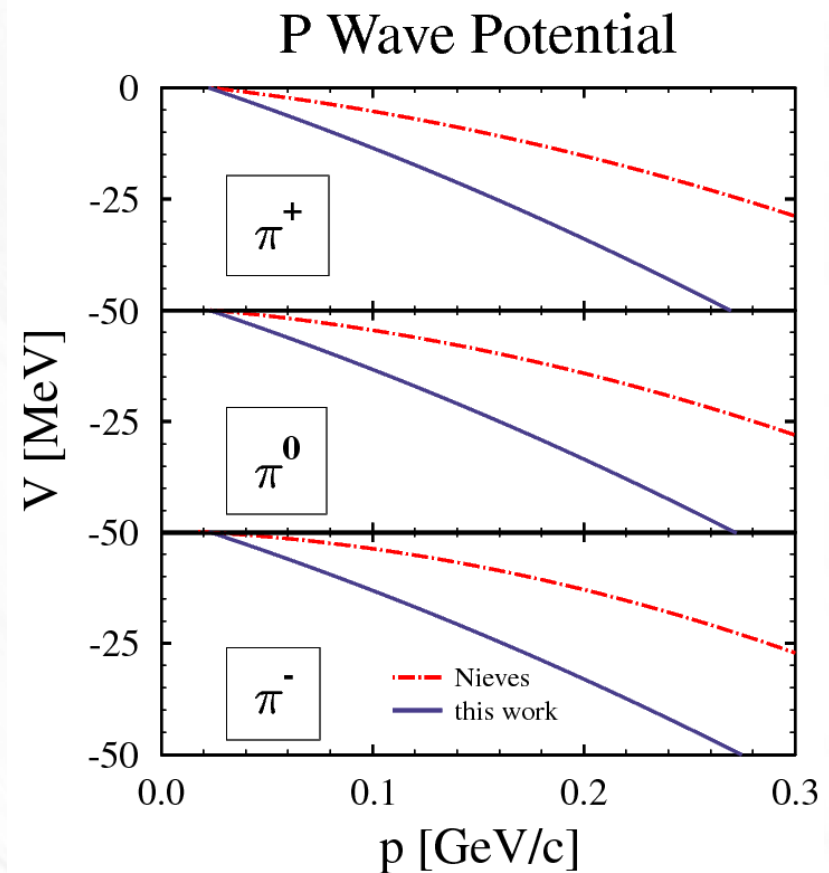
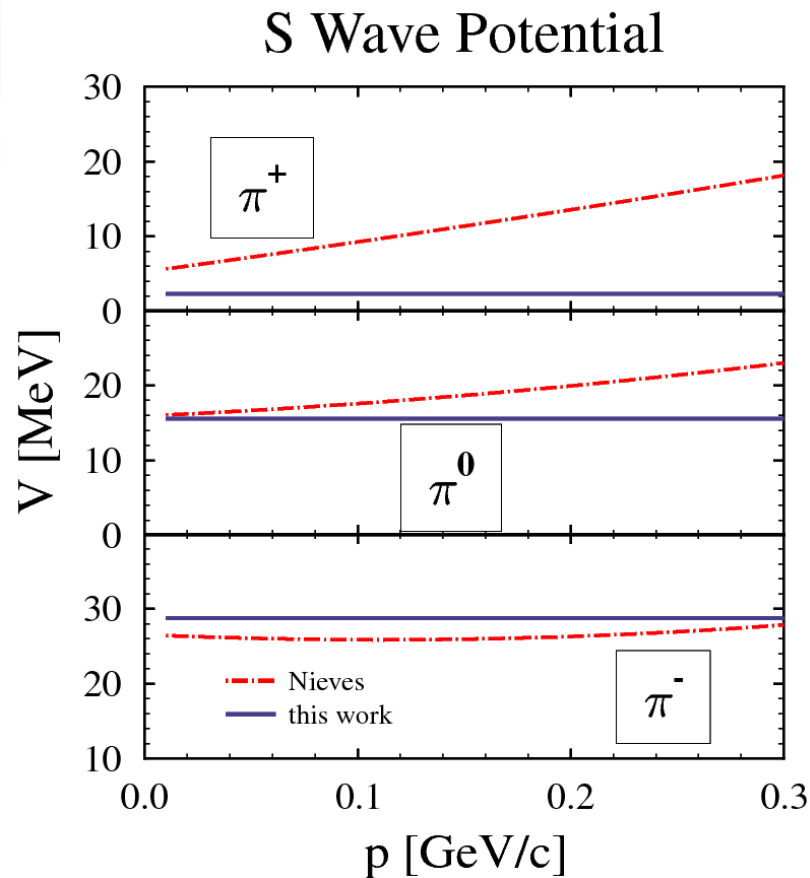
(at saturation density)

J.Nieves et al., NPA 554, 509 (1993)

3-level P wave potential – cold nuclear matter at equilibrium – likely to overestimate  $\pi$ N correlations

W. Eehalt et al. PLB 289, 31 (1993)

G.E. Brown et al. NPA 535, 701 (1991)

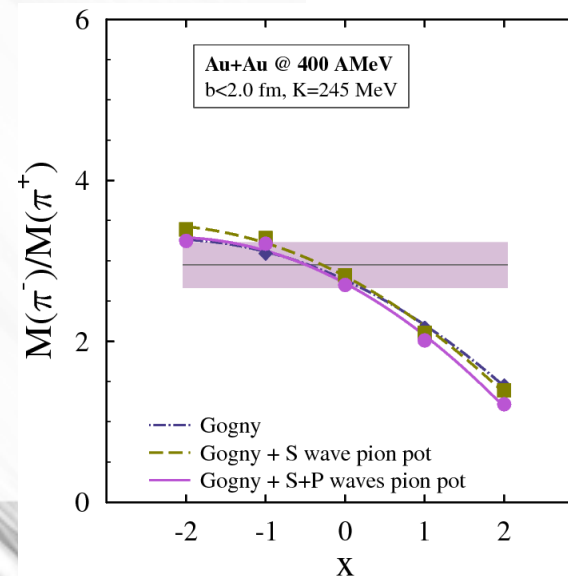
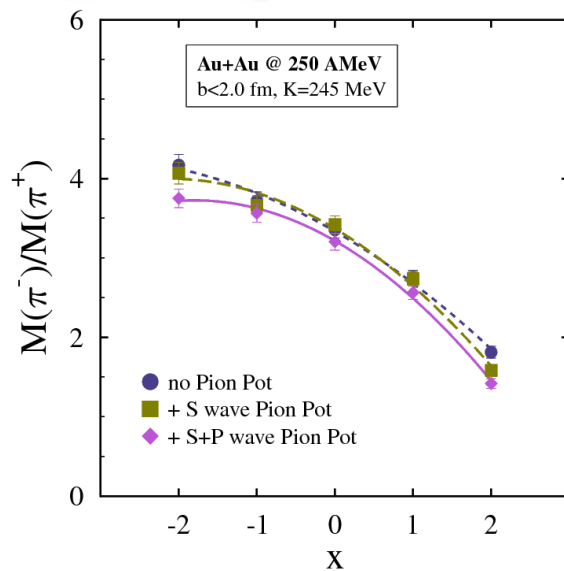
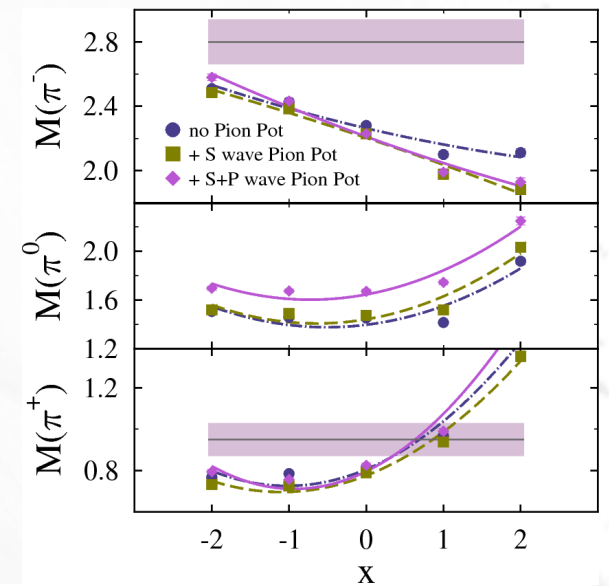
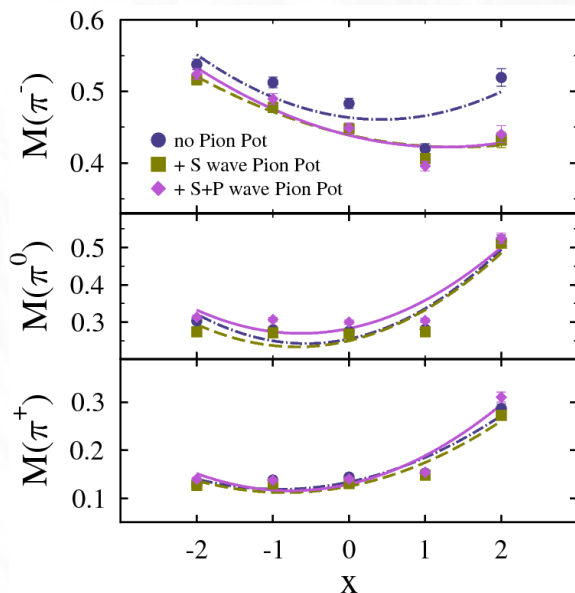


# Multiplicities & Ratio

central collisions ( $b < 2.0$  fm)

250 AMeV

400 AMeV



# $P_T$ Multiplicity Spectra

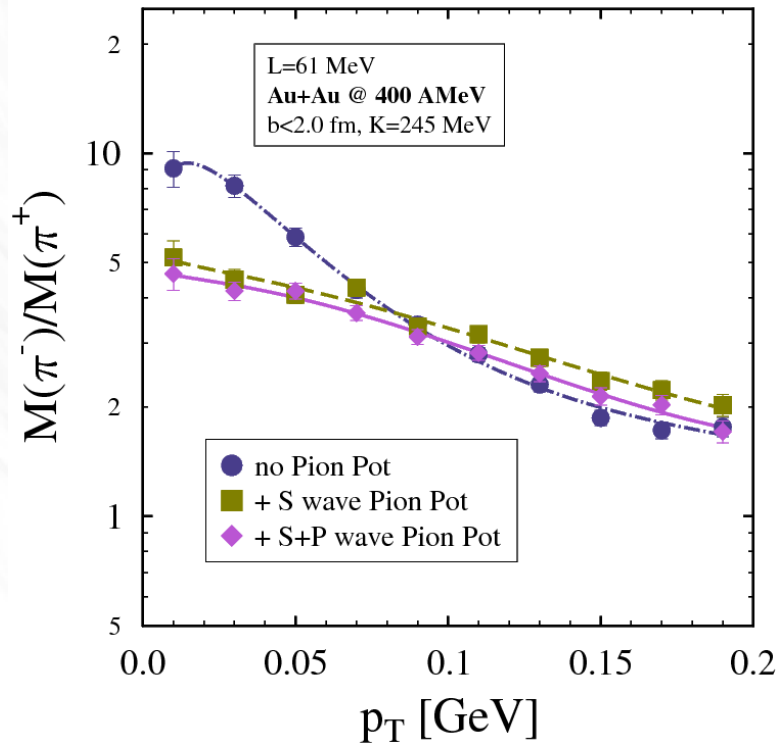
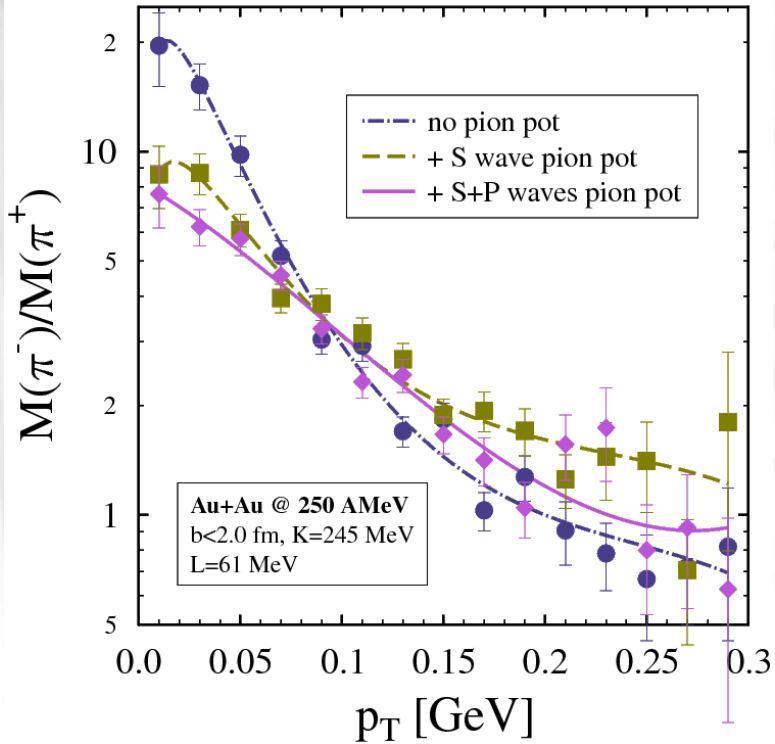
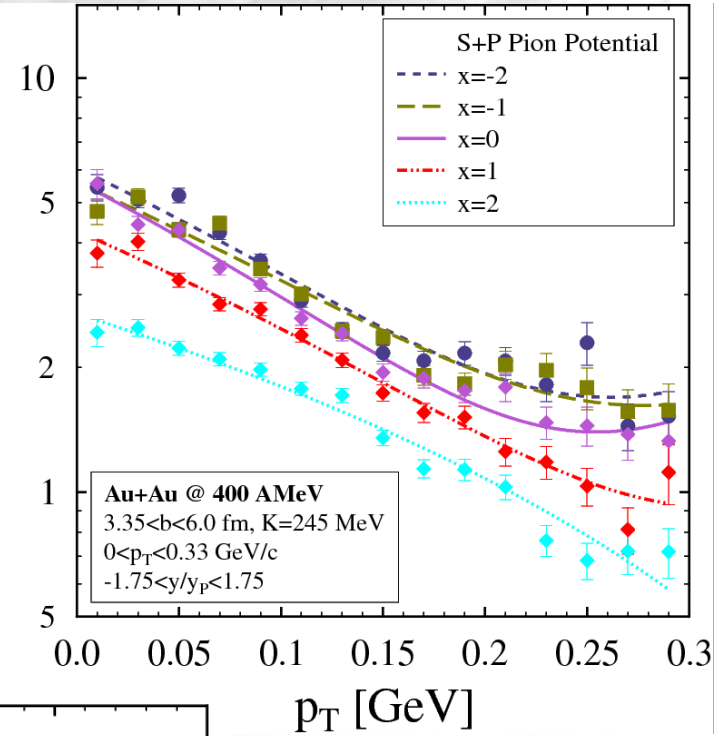
250 AMeV

central  $b < 2.0 \text{ fm}$

400 AMeV

central  $b < 2.0 \text{ fm}$

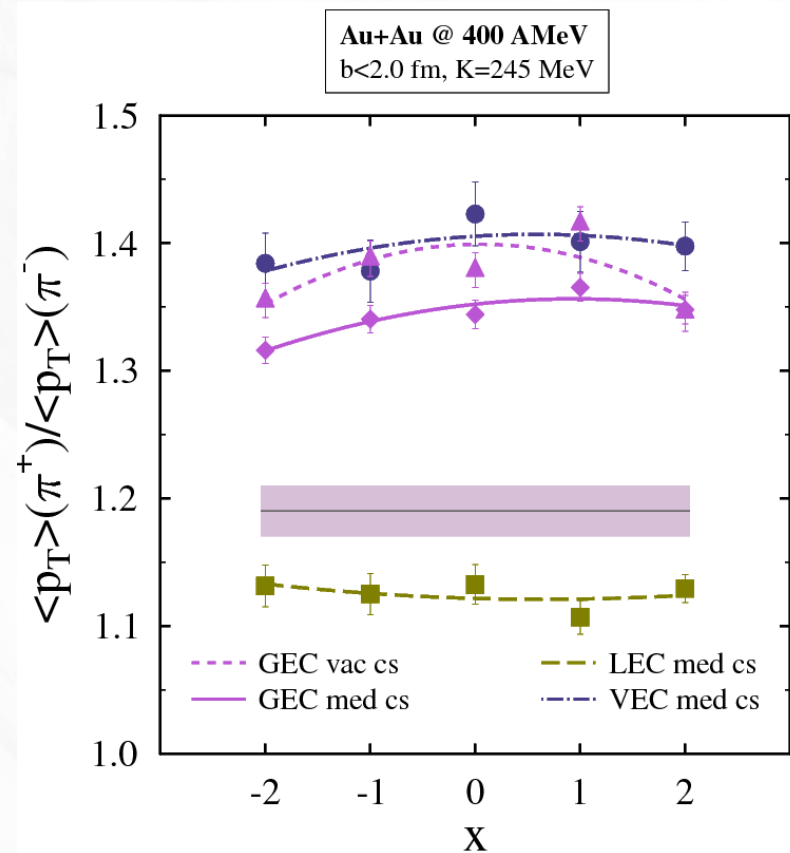
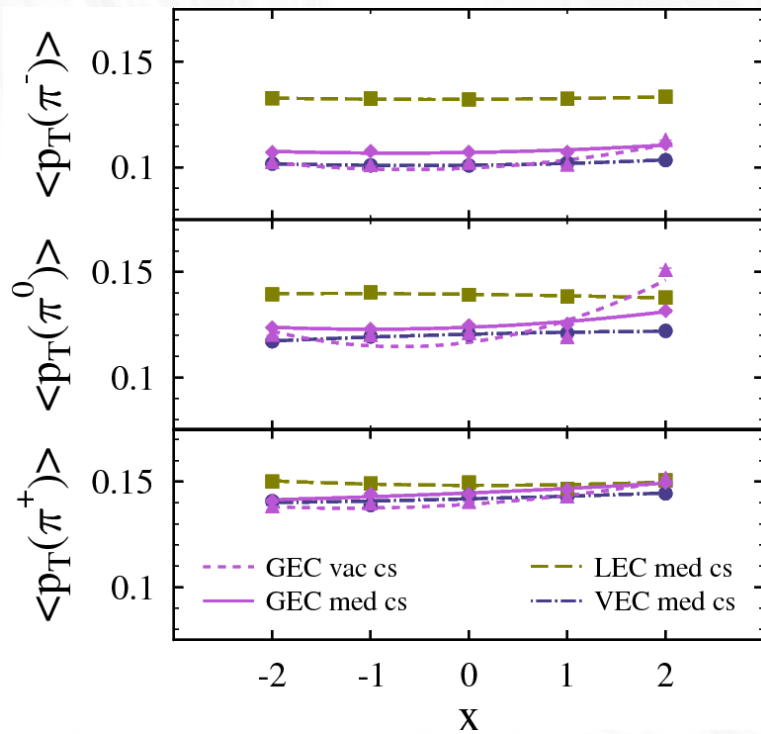
$M(\pi^-)/M(\pi^+)$



# Transverse momentum: $\langle p_T \rangle$

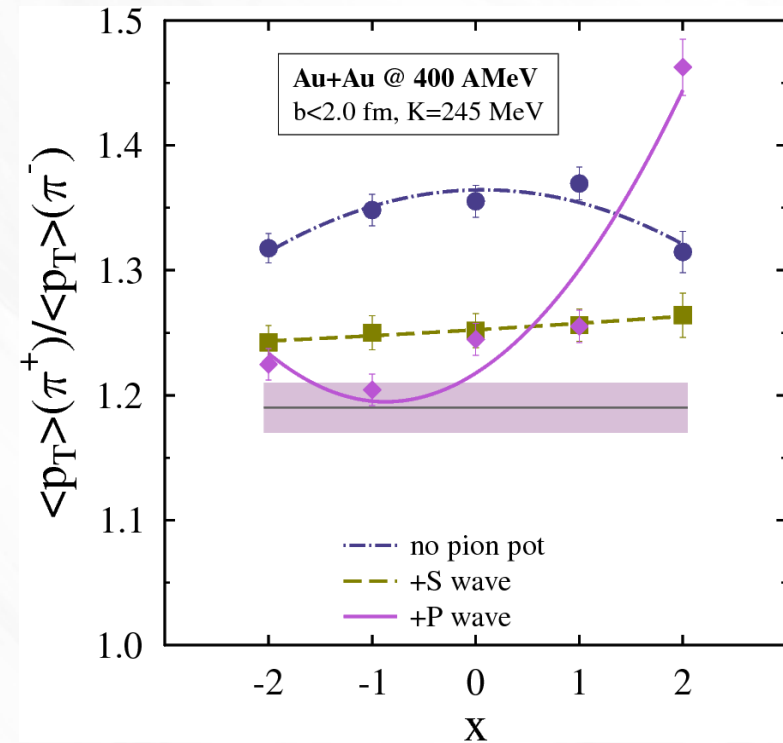
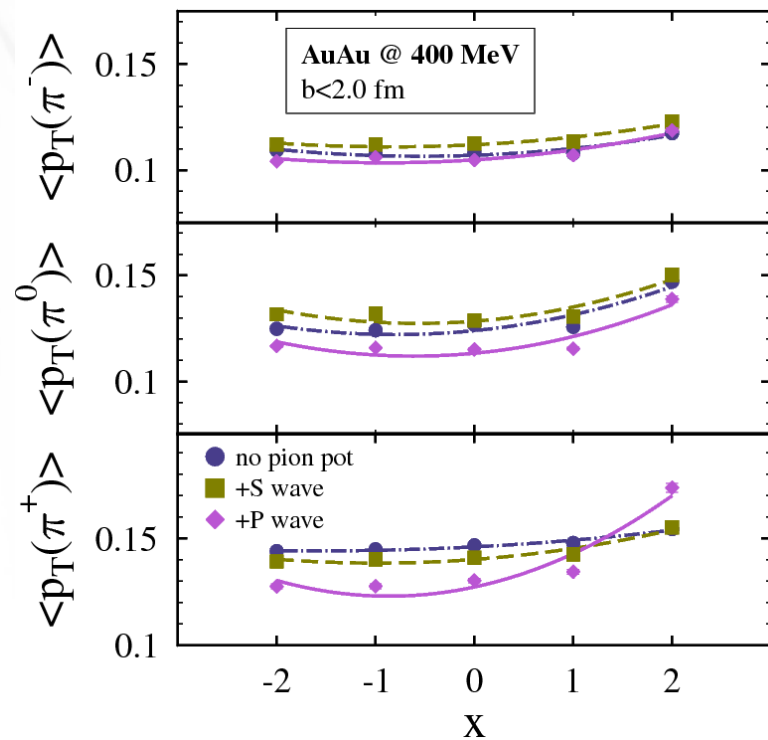
**Impact of:** energy conservation scenario  
in-medium cross-sections

**pion** – wave function width half of that of the nucleon



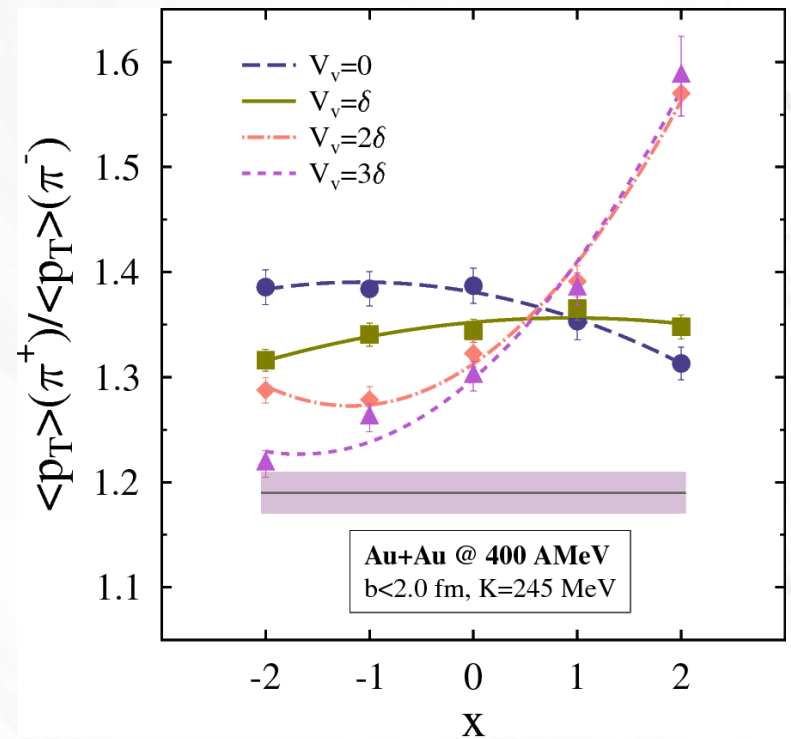
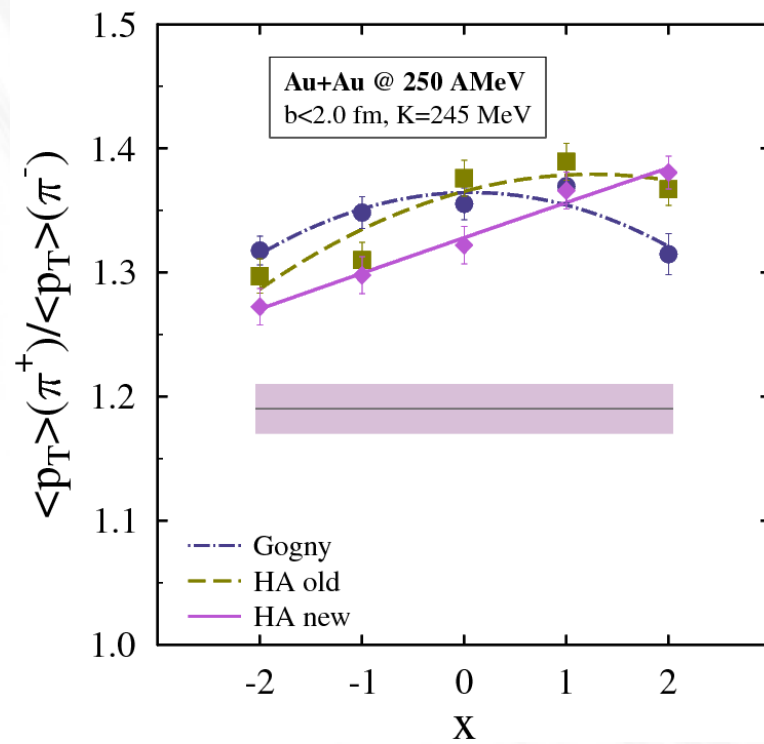
Experimental data (FOPI): [W.Reisdorf et al. NPA 781, 459 \(2007\)](#)

# Impact of the Pion Potential





# Optical/Delta Potential Dependence



# Constraints for symmetry energy

$$V(\Delta^{++}) = V_s + \frac{3}{2}V_v$$

$$V(\Delta^+) = V_s + \frac{1}{2}V_v$$

$$V(\Delta^0) = V_s - \frac{1}{2}V_v$$

$$V(\Delta^-) = V_s - \frac{3}{2}V_v$$

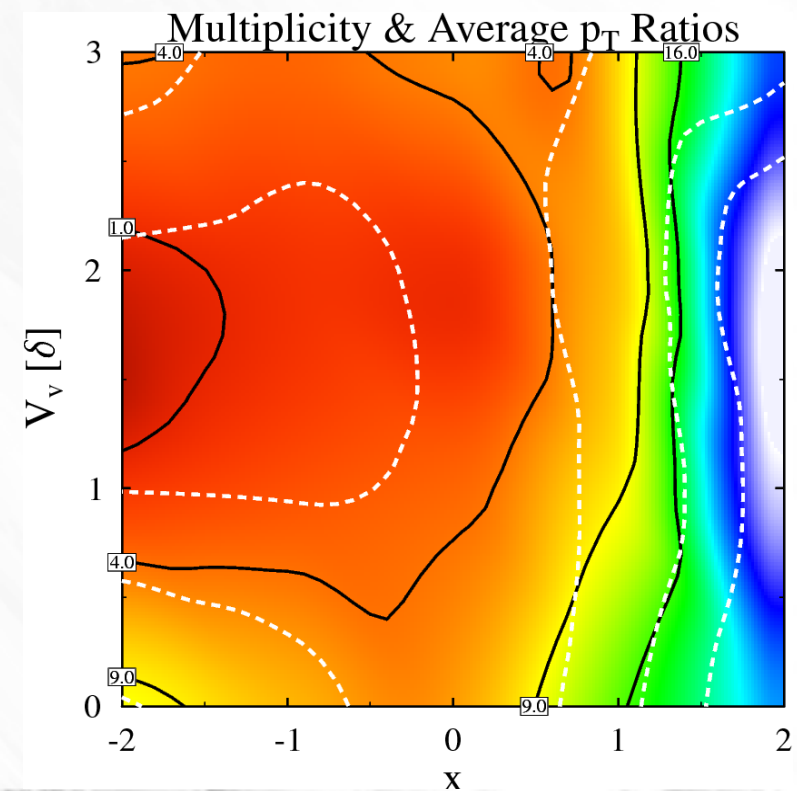
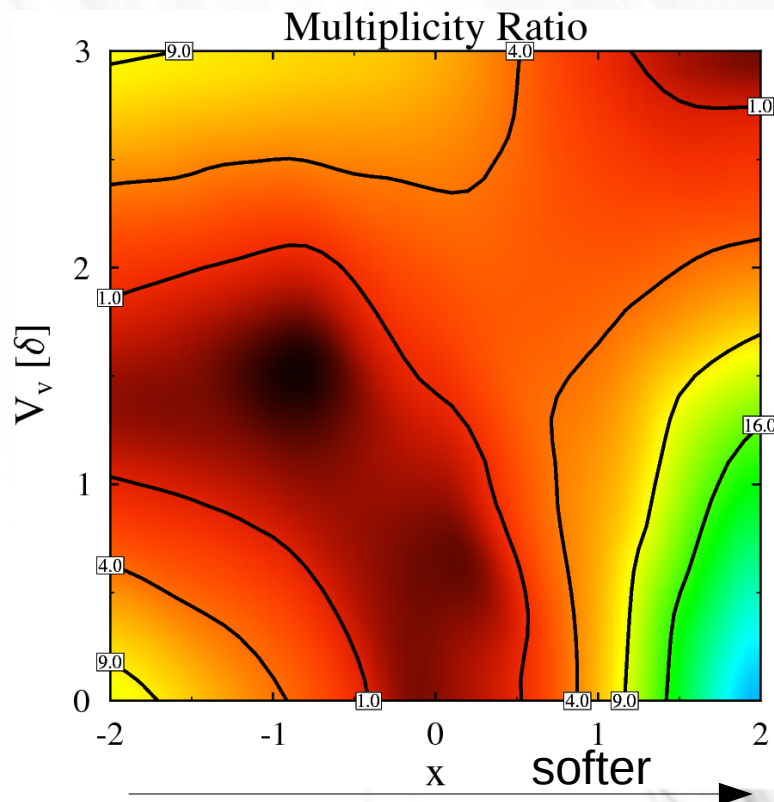
$$V_s = \frac{1}{2}(V_n + V_p)$$

$$\delta = \frac{1}{3}(V_n - V_p)$$

Lower limits for L:

$$L \geq 10 \text{ MeV} \quad (3\sigma)$$

$$L \geq 30 \text{ MeV} \quad (2\sigma)$$



Nucleon Optical Potentials:

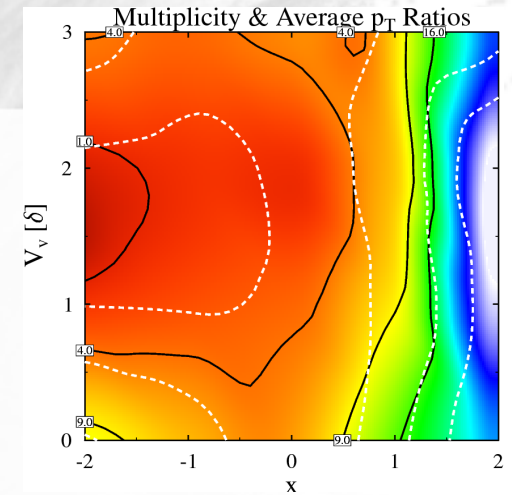
Gogny: Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

HA: Hartnack, Aichelin, PRC 49, 2901 (1994)

# Conclusions

To be able to constrain SE from pion observables in HIC:

- enlarge the list of observable included in the fit  
(+pion  $\langle p_T \rangle$  ratios)
- include pion optical potential



**Impact of pion optical potential:**

- multiplicity ratios – small
- $p_T$  multiplicity spectra – important reduction/enhancement at low/high  $p_T$   
(factor of 2 at 250 AMeV)
- $\langle p_T \rangle$  - both S and P wave contributions are important
- $\langle p_T \rangle$  ratios – S wave has an important impact

**Average  $p_T$  ratios:**

- **important impact from** in-medium effects on cross-sections,  
in-medium delta potential, nucleon optical potential, pion potential
- **large differences** between the VEC, LEC and GEC scenarios

**Constraint for SE:** a lower limit on L can be extracted from comparison with FOPI data

$$L \geq 10 \text{ MeV} \quad (3\sigma) \quad L \geq 30 \text{ MeV} \quad (2\sigma)$$

**To do list:** - impact of in-medium cross-sections

- energy dependence of S wave pion potential
- model dependence due to nucleon optical potential
- include in the constraint the individual  $\langle p_T \rangle$  rather than their ratio