



# Constraining the nuclear matter equation of state around twice saturation density

by A. Le Fèvre<sup>1</sup>, Y. Leifels<sup>1</sup>, W. Reisdorf<sup>1</sup>, J. Aichelin<sup>2</sup>, Ch. Hartnack<sup>2</sup>

<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

<sup>2</sup>SUBATECH, UMR 6457, Ecole des Mines de Nantes - IN2P3/CNRS - Université de Nantes, France

arXiv:1501.05246, submitted to NPA



# Constraining the nuclear matter equation of state around twice saturation density

by A. Le Fèvre<sup>1</sup>, Y. Leifels<sup>1</sup>, W. Reisdorf<sup>1</sup>, J. Aichelin<sup>2</sup>, Ch. Hartnack<sup>2</sup>

<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

<sup>2</sup>SUBATECH, UMR 6457, Ecole des Mines de Nantes - IN2P3/CNRS - Université de Nantes, France

- ▶ Introduction.
- ▶ Analysis and results.
- ▶ Simulations: the scenario.
- ▶ Summary and discussion.



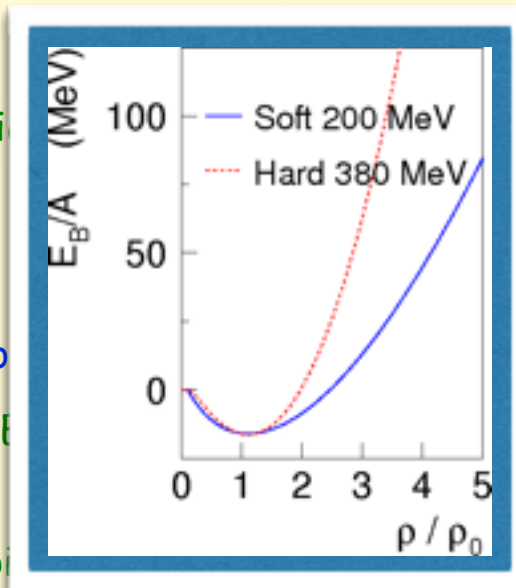
# Introduction



NGC 1952, Crab Nebula pulsar neutron star imaged by the NASA/ESA Hubble Space Telescope

## ▶ The equation of state (EOS) of nuclear matter:

- ▶ of fundamental interest
- ▶ object of intense theoretical studies
- ▶ an important ingredient in the study of:
  - ▶ compact stars <sup>[1]</sup>
  - ▶ core collapse supernovae
- ▶ The calculation of the nuclear EOS <sup>[2]</sup>, is a very complex task.
- ▶ Nuclear physics based on empirical data and the most 'fundamental' theory of nuclear forces requires a confrontation with empirical facts.
- ▶ 1st method, from astrophysicists: from 'neutron' star masses and radii. **But missing:**
  - ▶ precise model-independent radii,
  - ▶ composition of the matter in the centre of the stars.



studies

physical phenomena such as:

such as very recently attempted in

the most 'fundamental' theory of

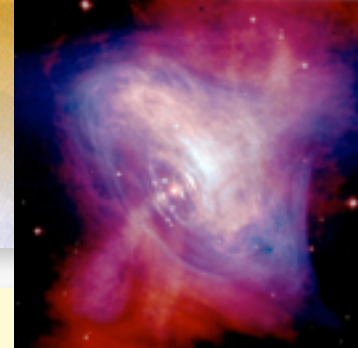
[1] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485.

[2] A. Burrows, Rev. Mod. Phys. 85 (2013) 245.

[3] A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111 (2013) 032501



# Introduction



*NGC 1952, Crab Nebula  
pulsar neutron star imaged by  
the NASA/ESA Hubble Space  
Telescope*

- ▶ The equation of state (EOS) of nuclear matter:
  - ▶ of fundamental interest
  - ▶ object of intense theoretical efforts since several decades
  - ▶ an important ingredient in modeling fascinating astrophysical phenomena such as:
    - ▶ compact stars<sup>[1]</sup>
    - ▶ core collapse supernovae<sup>[2]</sup>
- ▶ The calculation of the nuclear EOS from first principles, such as very recently attempted in [3], is a very complex task.
- ▶ Nuclear physics based on empirical observations => even the most 'fundamental' theory of nuclear forces requires a confrontation with empirical facts.
- ▶ 1st method, from astrophysicists: from 'neutron' star masses and radii. **But missing:**
  - ▶ precise model-independent radii,
  - ▶ composition of the matter in the centre of the stars.

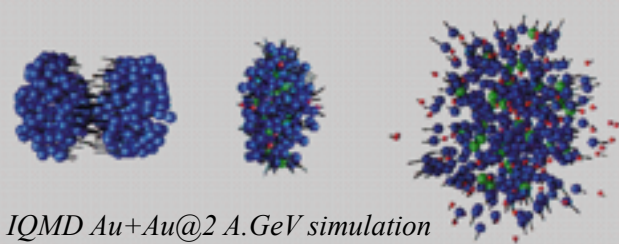
[1] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485.

[2] A. Burrows, Rev. Mod. Phys. 85 (2013) 245.

[3] A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111 (2013) 032501



# Introduction



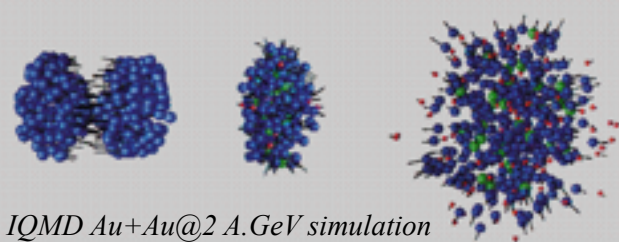
*IQMD Au+Au@2 A.GeV simulation*

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.





# Introduction



*IQMD Au+Au@2 A.GeV simulation*

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.

## Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\Phi - \Phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left( 1 + 2 \sum_{n \geq 1} v_n \cos n(\Phi - \Phi_R) \right)$$

$Y$  = rapidity

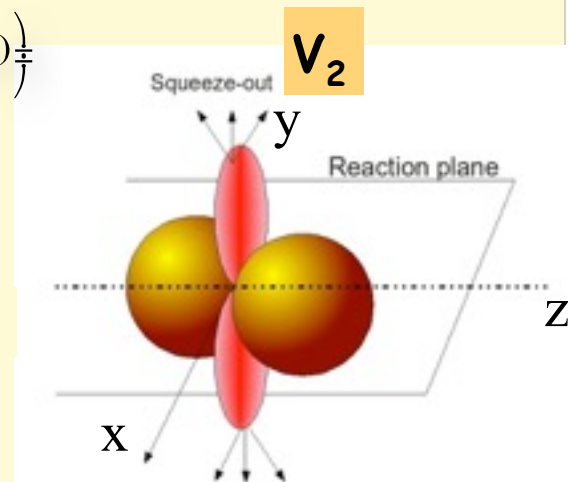
$p_t$  = transverse momentum

$\Phi_R$  = reaction plane azimuthal angle

$V_1$  = 'side/directed flow',  $\cos(\Phi - \Phi_R)$  mode

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

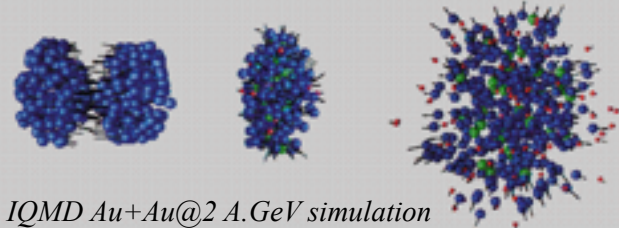
'Elliptic flow':  $\cos(2(\Phi - \Phi_R))$  mode, competition between 'in-plane' ( $V_2 > 0$ ) and 'out-of-plane' ejection ( $V_2 < 0$ ).





# Introduction

IQMD Au+Au@2 A.GeV simulation



- Alternative method: in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.

## Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\Phi - \Phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left( 1 + 2 \sum_{n \geq 1} v_n \cos n(\Phi - \Phi_R) \right)$$

$y$  = rapidity

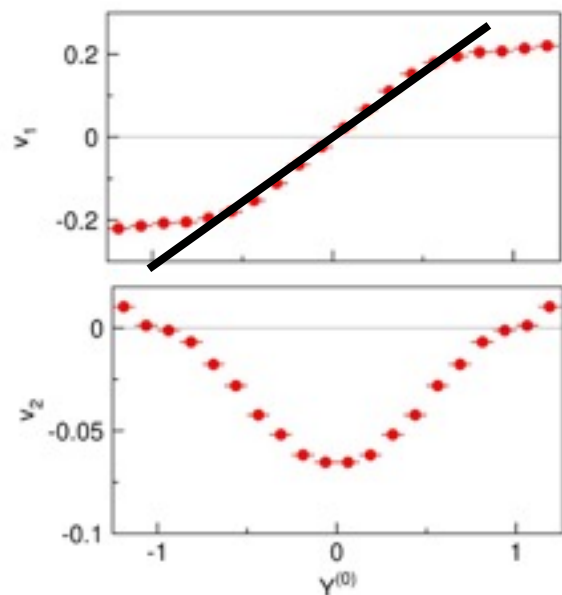
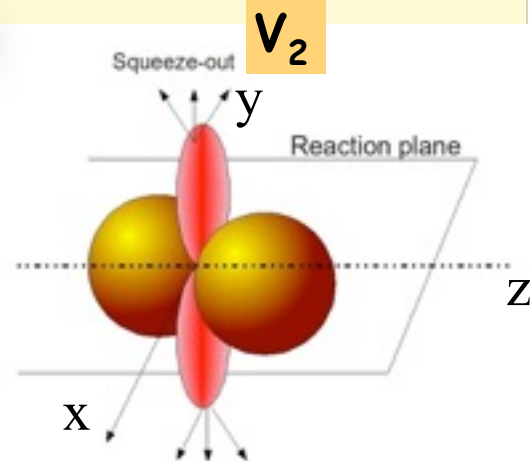
$p_t$  = transverse momentum

$\Phi_R$  = reaction plane azimuthal angle

$V_1$  = 'side/directed flow',  $\cos(\Phi - \Phi_R)$  mode

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

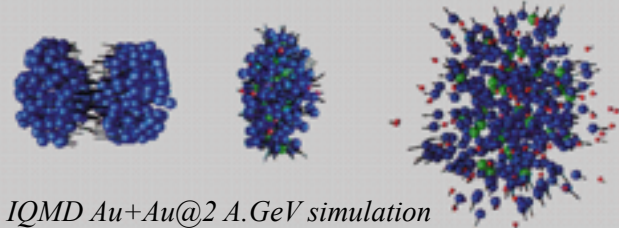
'Elliptic flow':  $\cos(2(\Phi - \Phi_R))$  mode, competition between 'in-plane' ( $V_2 > 0$ ) and 'out-of-plane' ejection ( $V_2 < 0$ ).





# Introduction

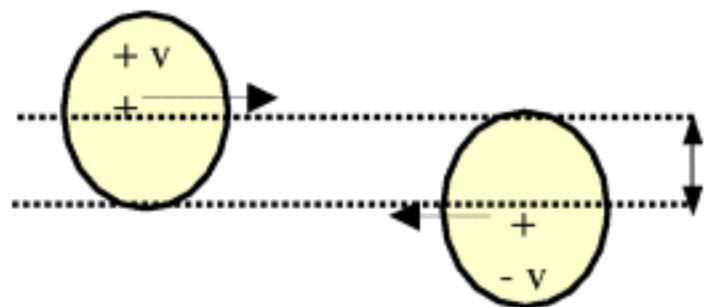
*IQMD Au+Au@2 A.GeV simulation*



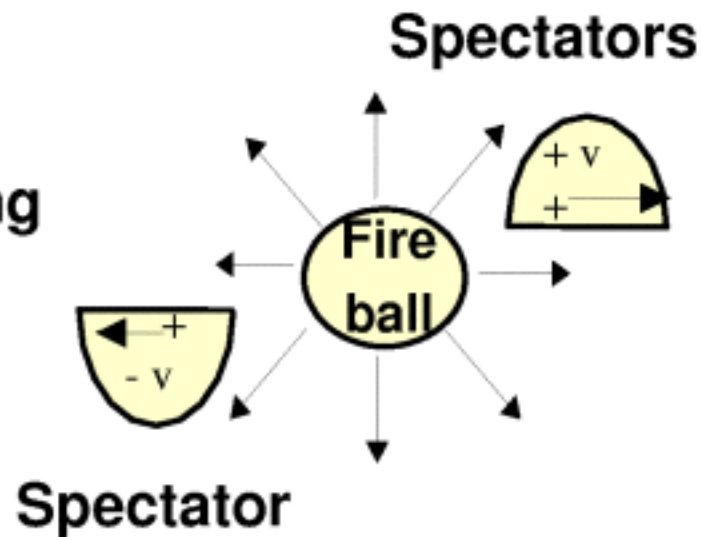
- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.
  - ▶ flow method: limited to  $E_{\text{beam}} < 10 \text{ A.GeV}$  ← some kind of a clock is available (sound velocity versus participant-spectator interaction).

**Before the collision**

**after the collision**



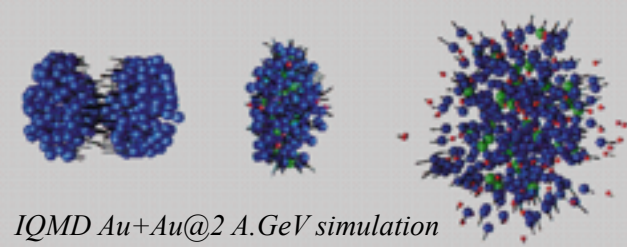
**Overlapping zone**





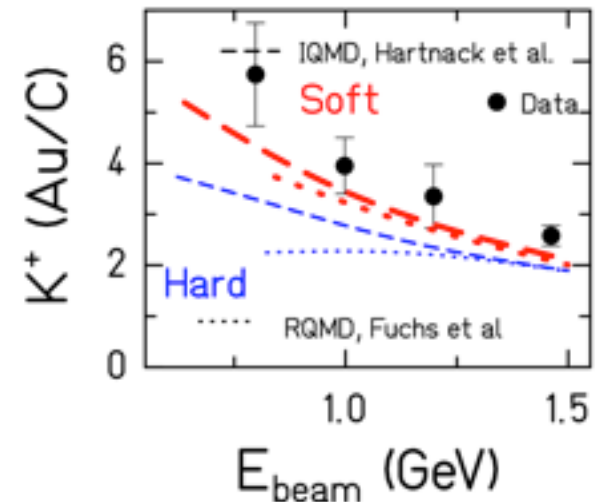


# Introduction



*IQMD Au+Au@2 A.GeV simulation*

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.
  - ▶ flow method: limited to  $E_{\text{beam}} < 10 \text{ A.GeV}$  ← some kind of a clock is available (sound velocity versus participant-spectator interaction).
  - ▶ KaoS (1990's), C+C, Au+Au,  $K^+$  yields → 'soft' EOS. But:
    - ▶ kaons rare at  $E_{\text{beam}} = 0.8 \text{ A.GeV}$  (max. sensitivity to the EOS).
    - ▶ all 'bulk' observables (multiplicities, clusterisation, stopping, flow) under control in the transport model ?

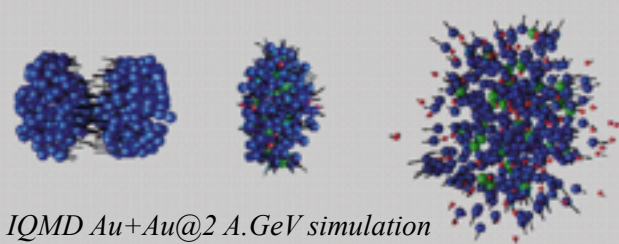


Data: C. Sturm et al., PRL 86 (2001)

39



# Introduction

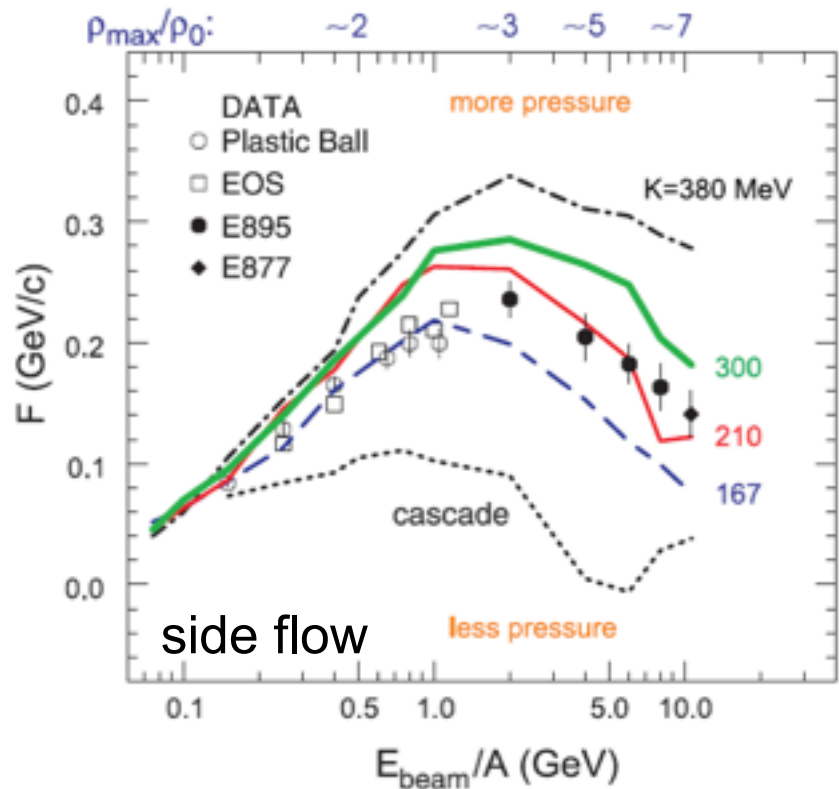
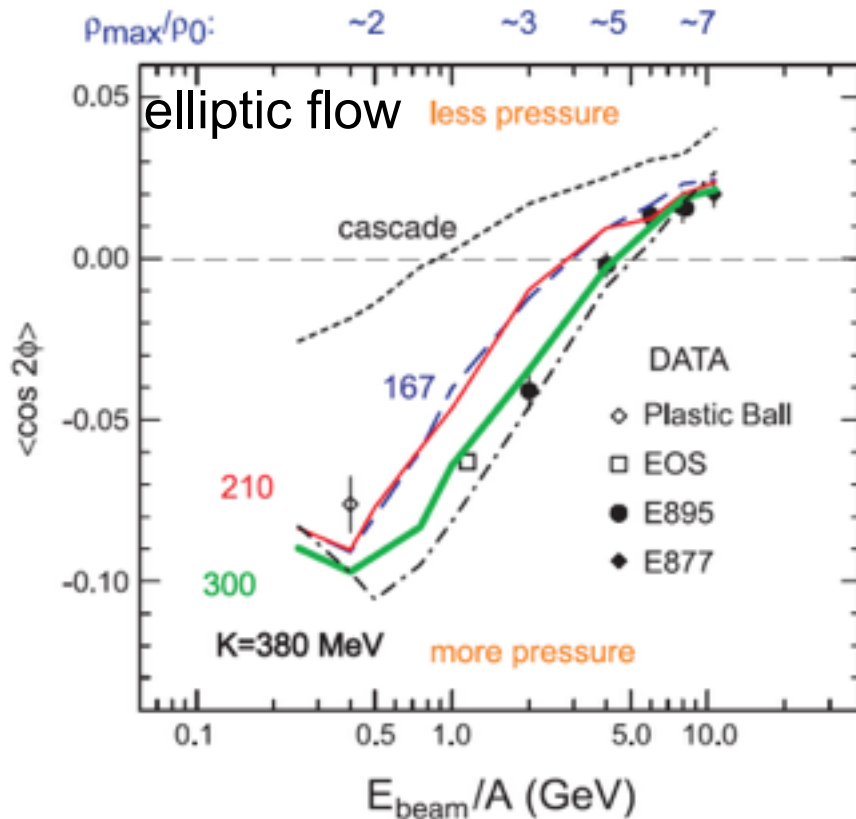


*IQMD Au+Au@2 A.GeV simulation*

- ▶ **Alternative method:** in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.
  - ▶ flow method: limited to  $E_{\text{beam}} < 10 \text{ A.GeV}$  ← some kind of a clock is available (sound velocity versus participant-spectator interaction).
  - ▶ KaoS (1990's), C+C, Au+Au,  $K^+$  yields → 'soft' EOS. **But:**
    - ▶ kaons rare at  $E_{\text{beam}} = 0.8 \text{ A.GeV}$  (max. sensitivity to the EOS).
    - ▶ all 'bulk' observables (multiplicities, clusterisation, stopping, flow) under control in the transport model ?
- ▶ EoS (1996), Au+Au @ 0.25 to 1.15 A.GeV, radial & sideward flow, squeeze-out versus QMD → no strong sensitivity on the nuclear incompressibility  $K_0$ .
- ▶ FOPI (2005), Au+Au @ 0.09-1.5 A.GeV,  $Z=1$  elliptic flow, versus 4 different transport codes → 'no strong constraint on the EOS can be derived at this stage'.
- ▶ BEVALAC & AGS accelerators, proton flows versus transport theories →  $K_0 = 167\text{-}200 \text{ MeV}$  (soft) from  $V_1$ ,  $K_0 = 300 \text{ MeV}$  (semi-stiff) from  $V_2$  → contradictions.



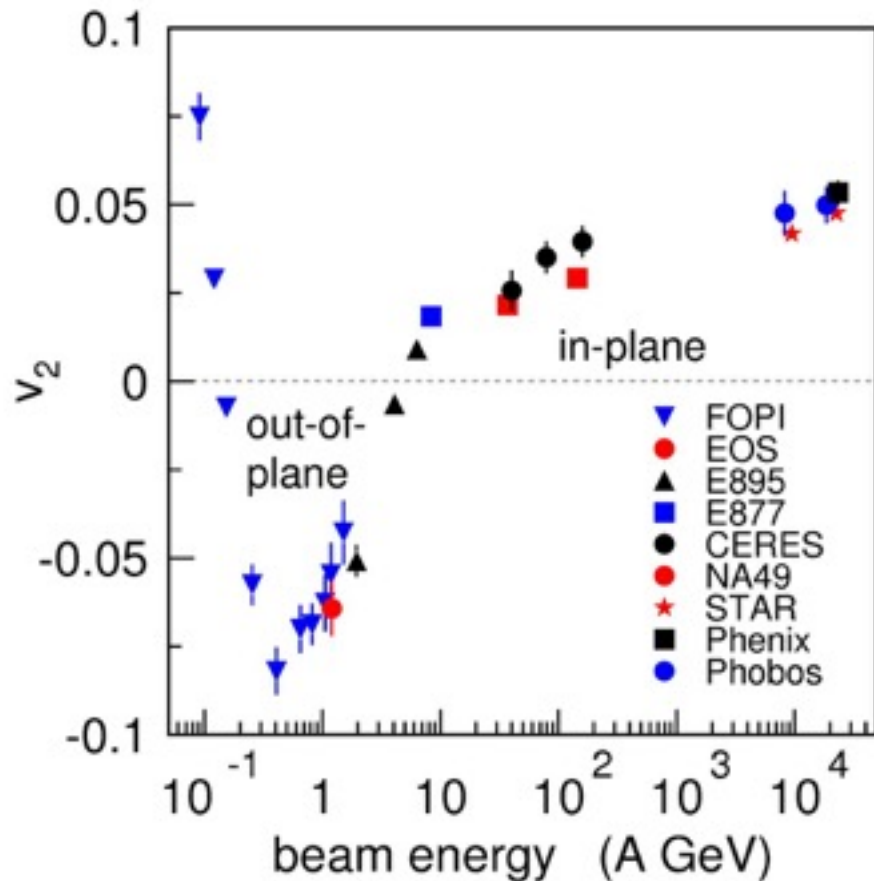
# Elliptic flow and the nuclear matter EOS



P. Danielewicz et al.  
Science 298, 1592 (2002)



# Beam energy dependence of the elliptic flow



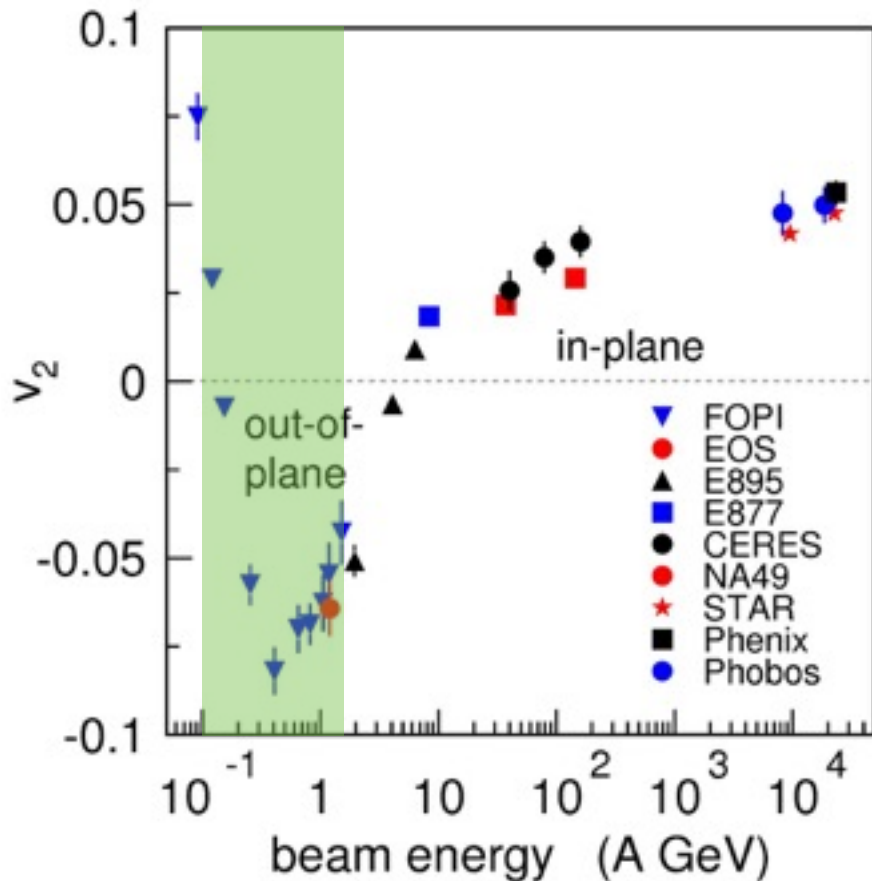
elliptic flow

- pressure gradient of compression zone
- shadowing of spectators
  - attraction due to mean field of nucleons
- at low energies
- at high energies
  - lacking shadowing of spectators





# Beam energy dependence of the elliptic flow



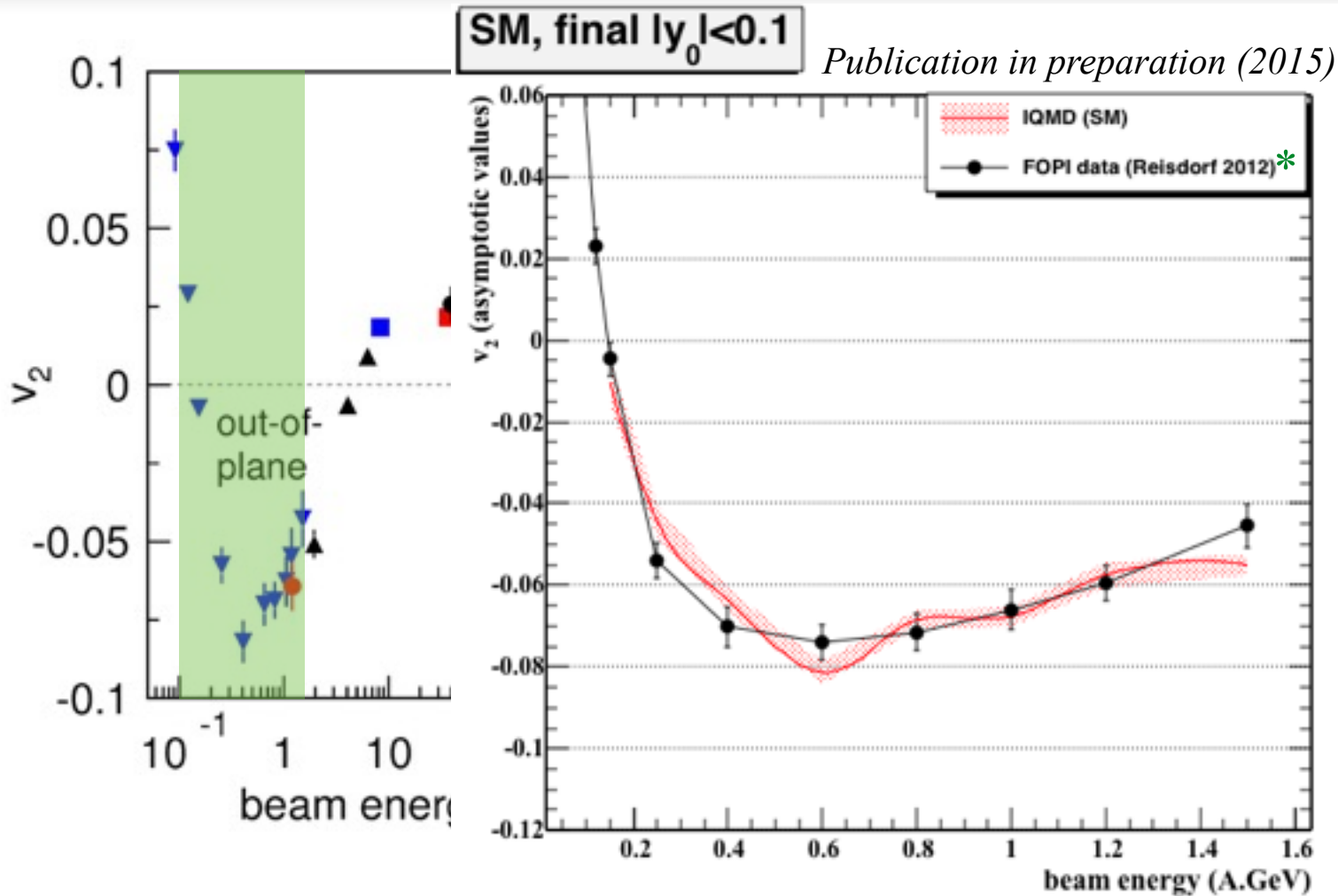
elliptic flow

- pressure gradient of compression zone
- shadowing of spectators
- at low energies
  - attraction due to mean field of nucleons
- at high energies
  - lacking shadowing of spectators





# Beam energy dependence of the elliptic flow



ression

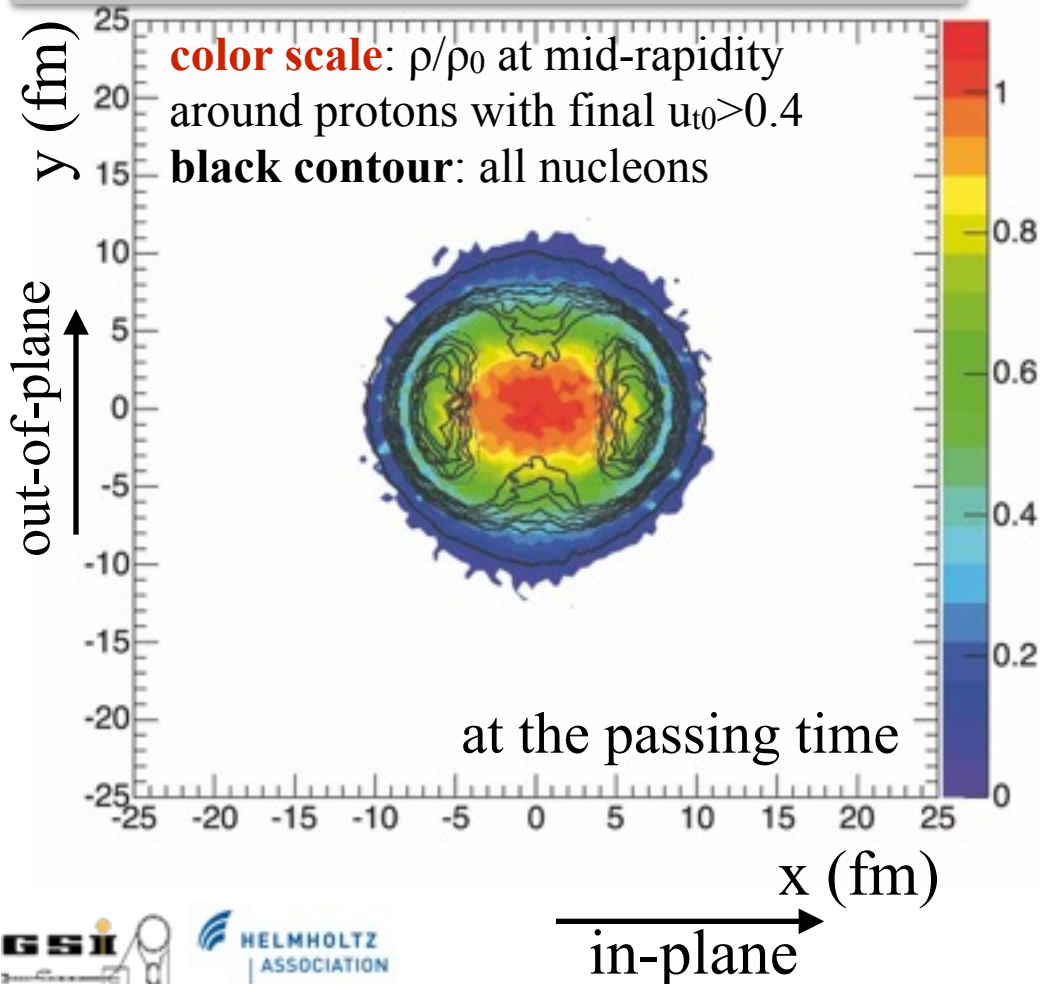
i field of

\*: FOPI data: Reisdorf et al, Nuclear Physics A 876 (2012) 1–60



# Origin of the negative elliptic flow

IQMD (SM) Au+Au @ 1.5 A.GeV,  $b=6$  fm

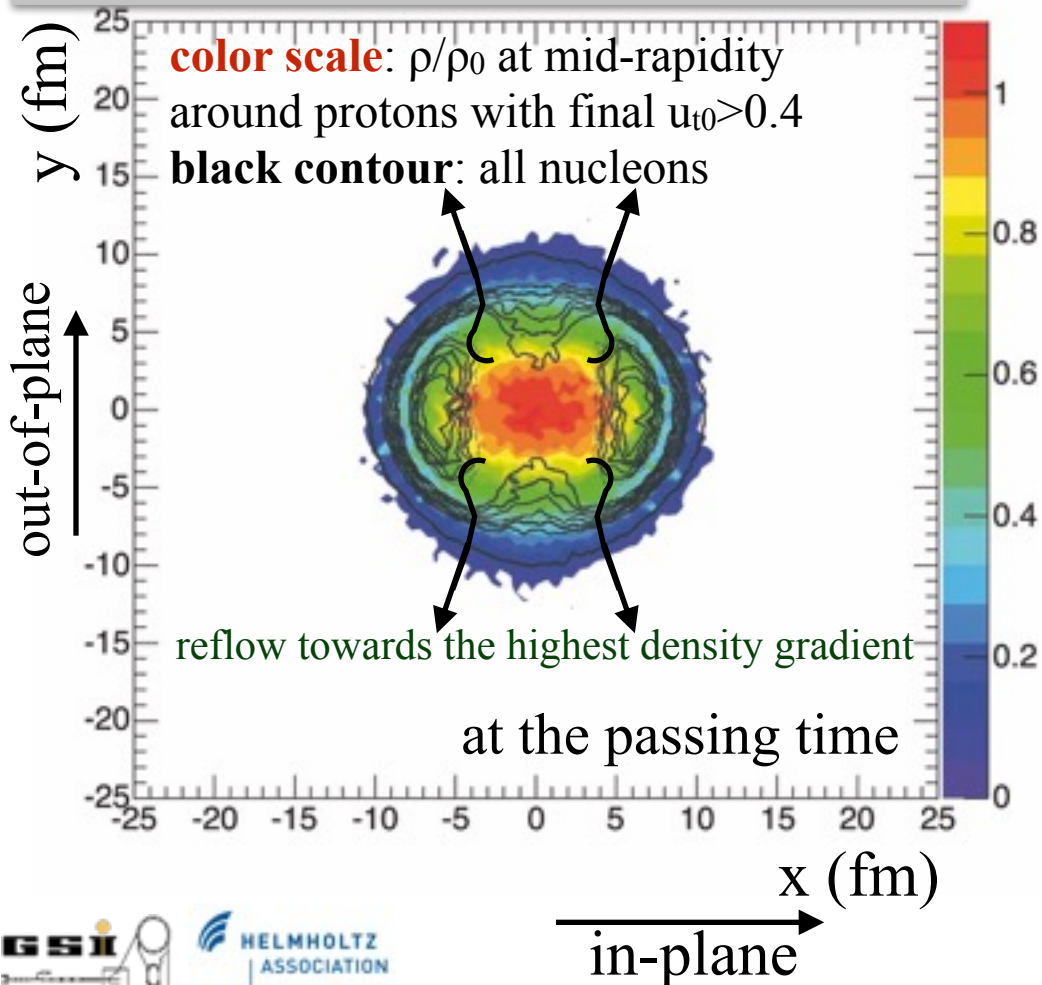


*Publication in preparation (2015)*



# Origin of the negative elliptic flow

IQMD (SM) Au+Au @ 1.5 A.GeV,  $b=6$  fm

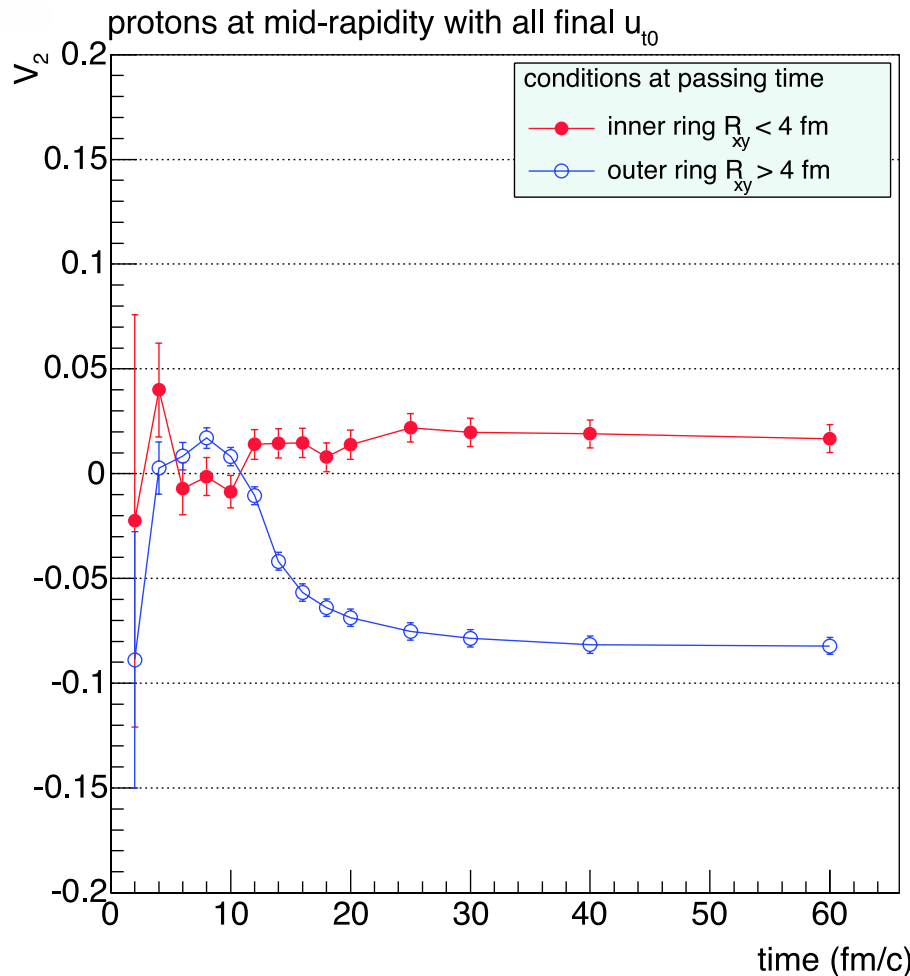
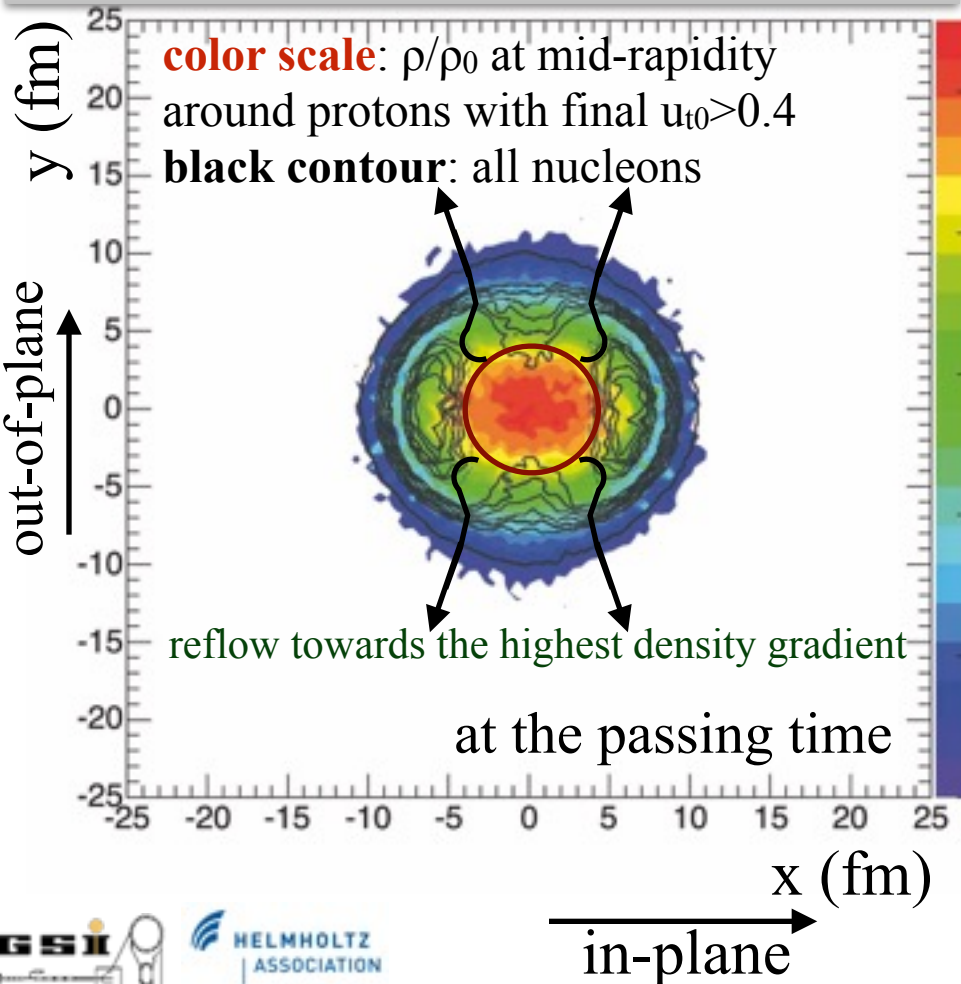


*Publication in preparation (2015)*



# Origin of the negative elliptic flow

IQMD (SM) Au+Au @ 1.5 A.GeV,  $b=6$  fm



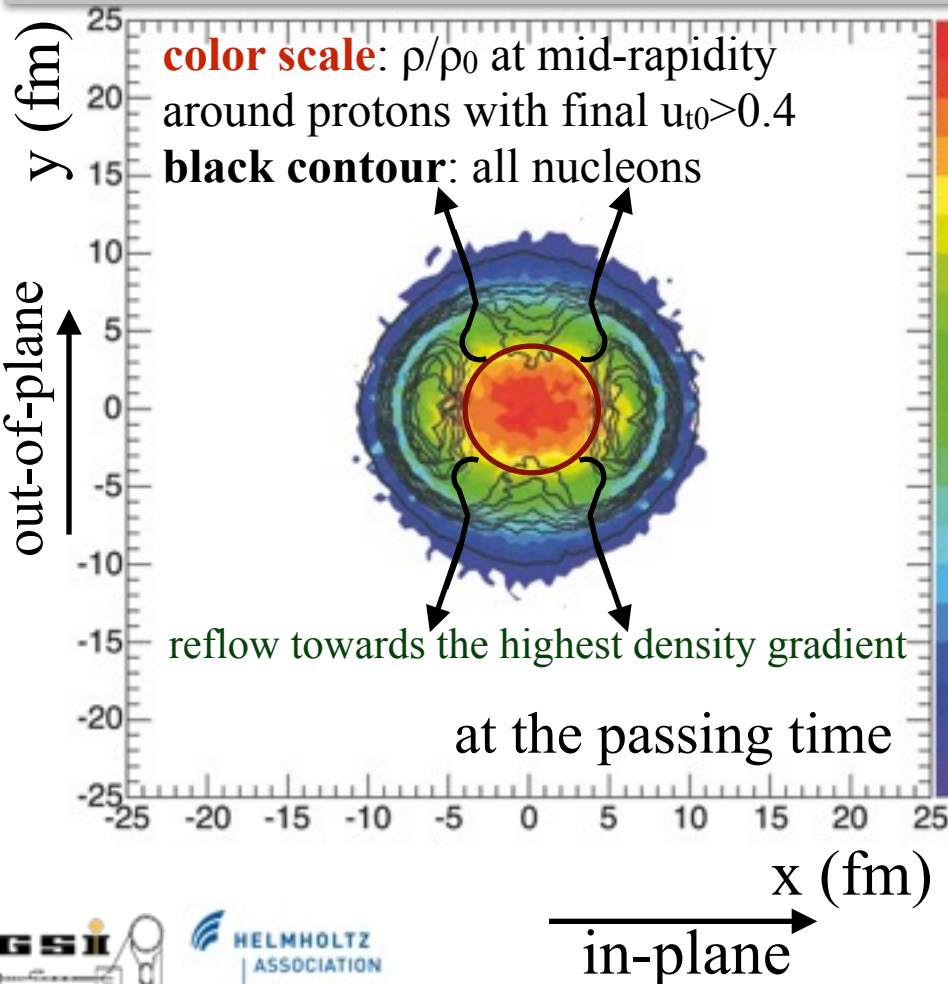
Publication in preparation (2015)



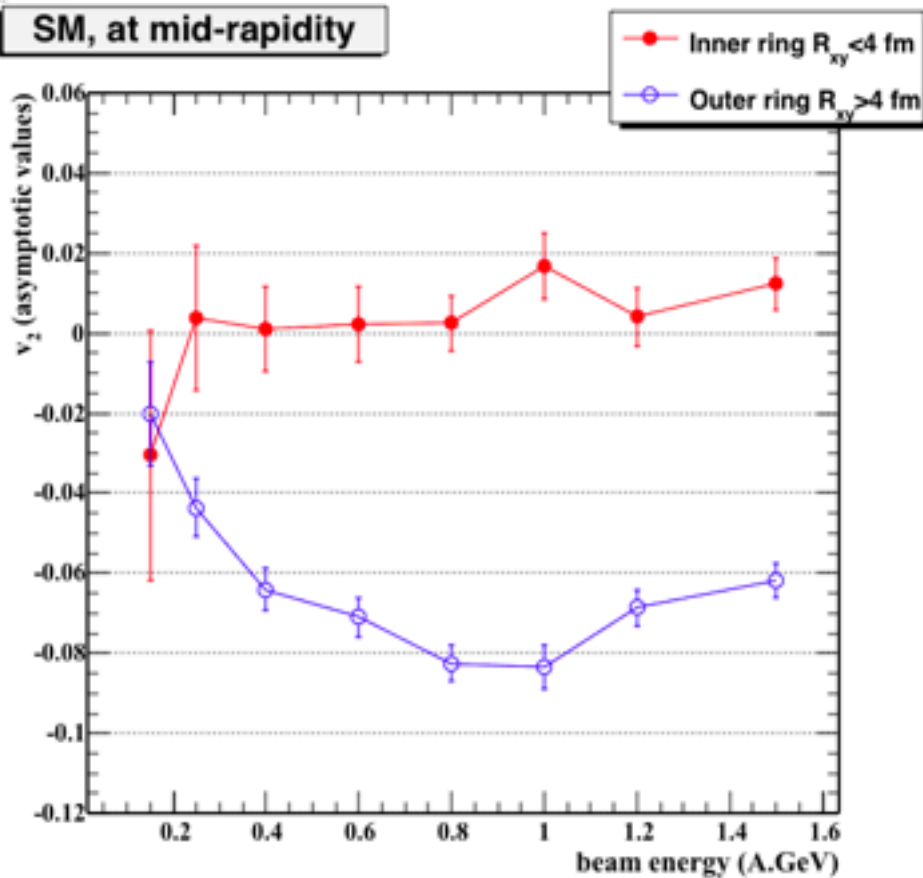


# Origin of the negative elliptic flow

IQMD (SM) Au+Au @ 1.5 A.GeV,  $b=6$  fm



SM, at mid-rapidity



Publication in preparation (2015)





# Introduction

- ▶ **Present work:** improve the situation in the 1 A.GeV regime, from extensive flow data published recently by the **FOPI Collaboration** (Au+Au @ 0.4-1.5 A.GeV) [4]
  - close look at the **elliptic flow data with improvements:**
    - ▶ 1) not only **protons:** d, t,  $^3\text{He}$   $^4\text{He}$  having larger flow signals than single nucleons.
    - ▶ 2) not only **mid-rapidity data:** 80% of the target- projectile rapidity gap.

[4] W. Reisdorf, et al. (FOPI Collaboration), Nucl. Phys. A 876 (2012) 1.

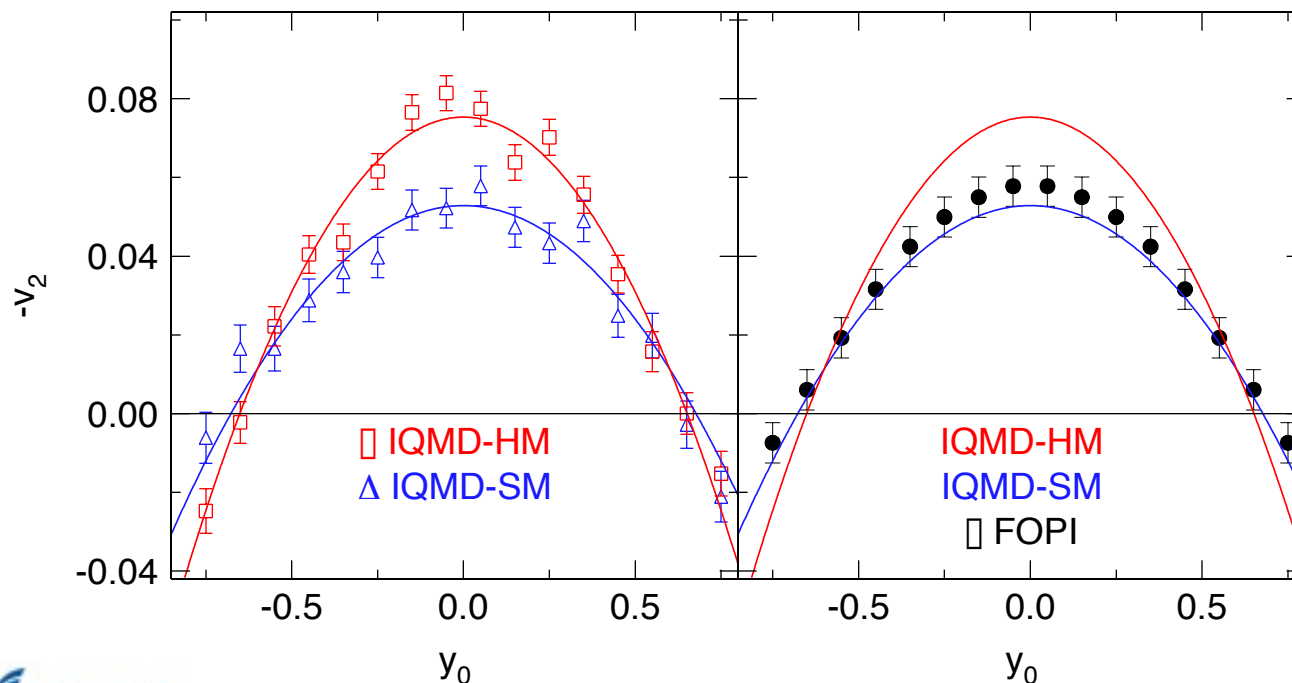


# Analysis and results

After Andronic et al., Phys. Rev. C67 (2003) 034907,  
only the m.d.i. can account for the experimental  
directed flow => we restrict this study to SM and HM.

## Elliptic flow

Au+Au 1.2A GeV  $0.25 < b_0 < 0.45$  protons



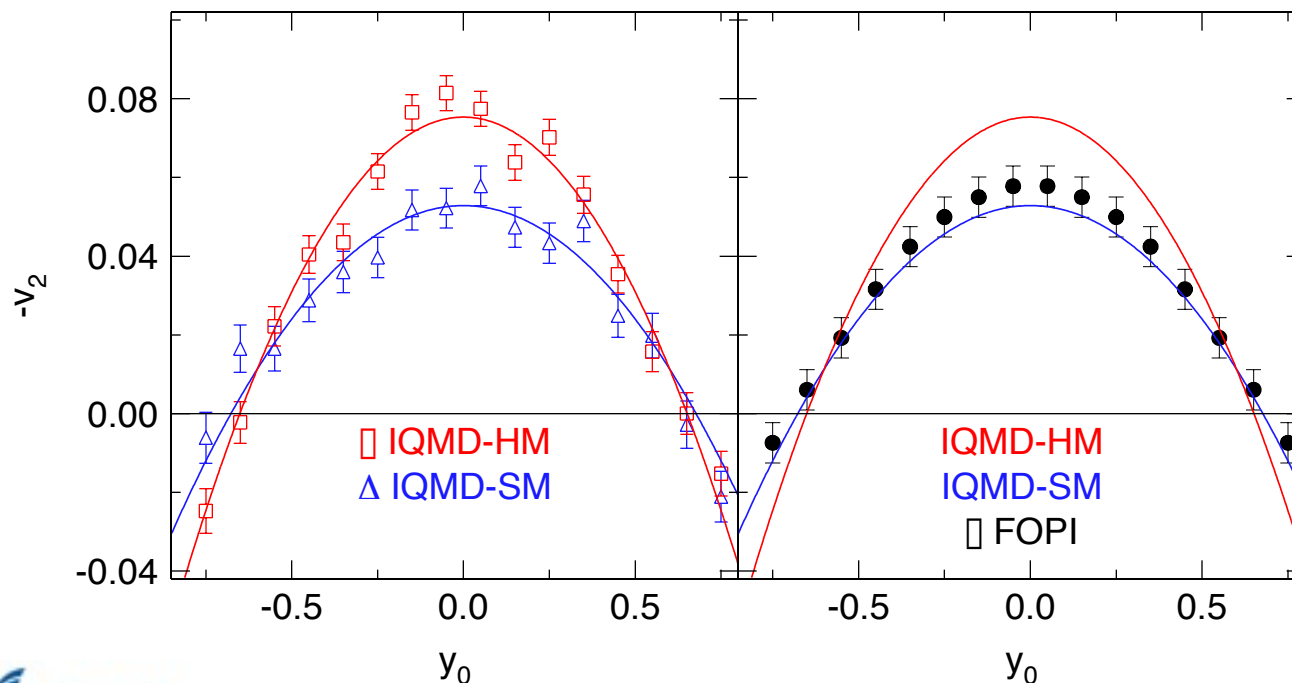


# Analysis and results

After Andronic et al., Phys. Rev. C67 (2003) 034907,  
only the m.d.i. can account for the experimental  
directed flow => we restrict this study to SM and HM.

## Elliptic flow

Au+Au 1.2A GeV  $0.25 < b_0 < 0.45$  protons



$K_0 =$   
380 MeV ('stiff')  
200 MeV ('soft')



# Analysis and results

After Andronic et al., Phys. Rev. C67 (2003) 034907, only the m.d.i. can account for the experimental directed flow => we restrict this study to SM and HM.

Complete shape of  $v_2(y_0)$ :

a new observable:

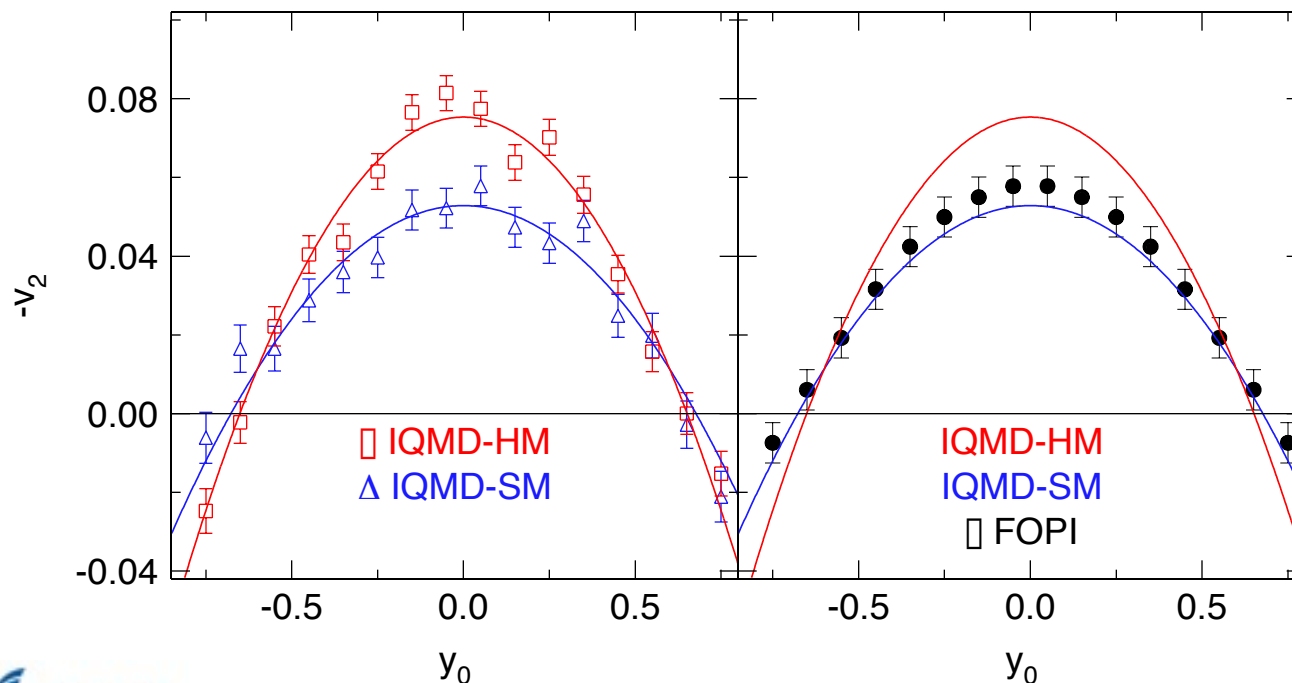
$$v_{2n} = |v_{20}| + |v_{22}|,$$

from fit

$$v_2(y_0) = v_{20} + v_{22} \cdot y_0^2$$

## Elliptic flow

Au+Au 1.2A GeV  $0.25 < b_0 < 0.45$  protons



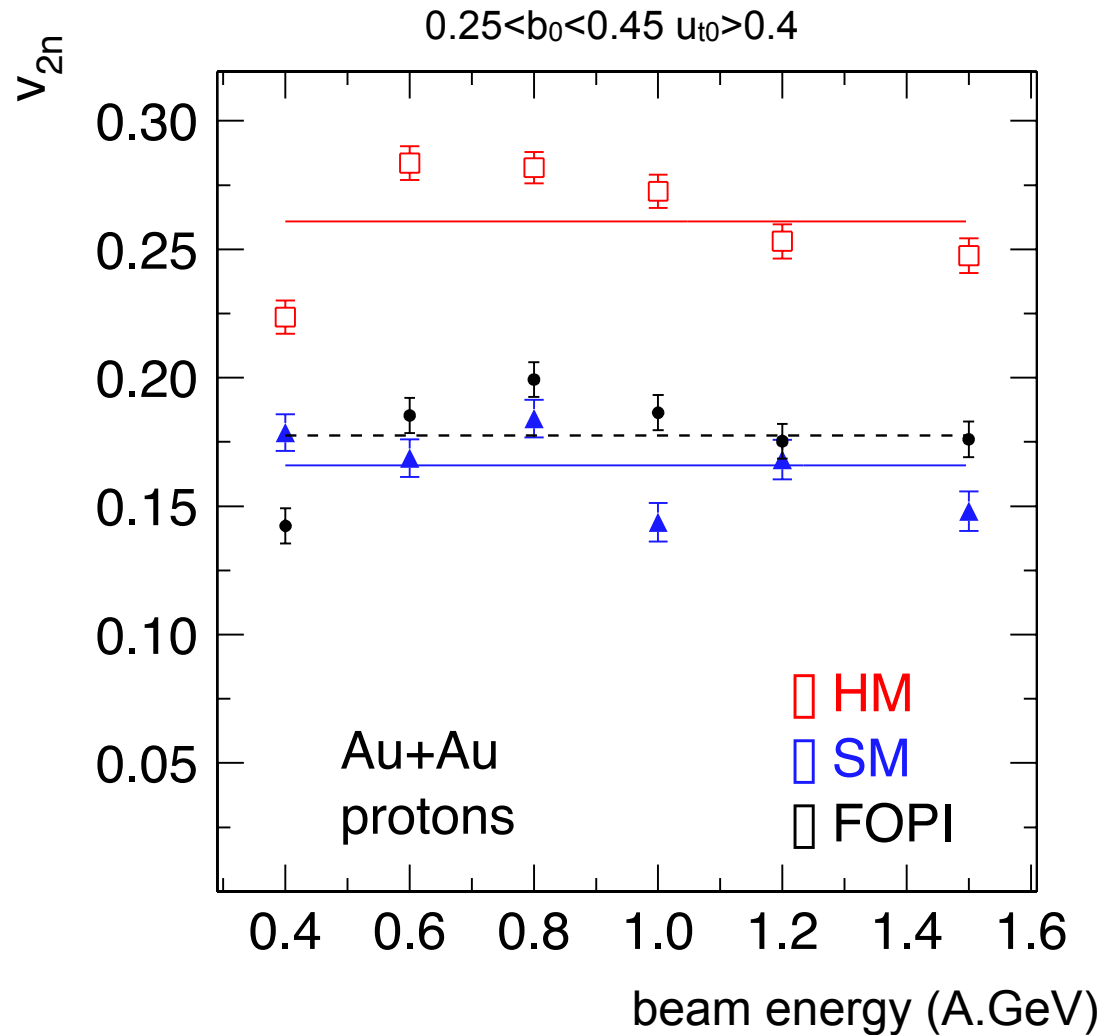
$K_0 =$   
380 MeV ('stiff')  
200 MeV ('soft')





# Analysis and results

→  $v_{2n}(E_{\text{beam}})$  varies by a factor  $\approx 1.6$ ,  $\gg$  measured uncertainty ( $\approx 1.1$ )



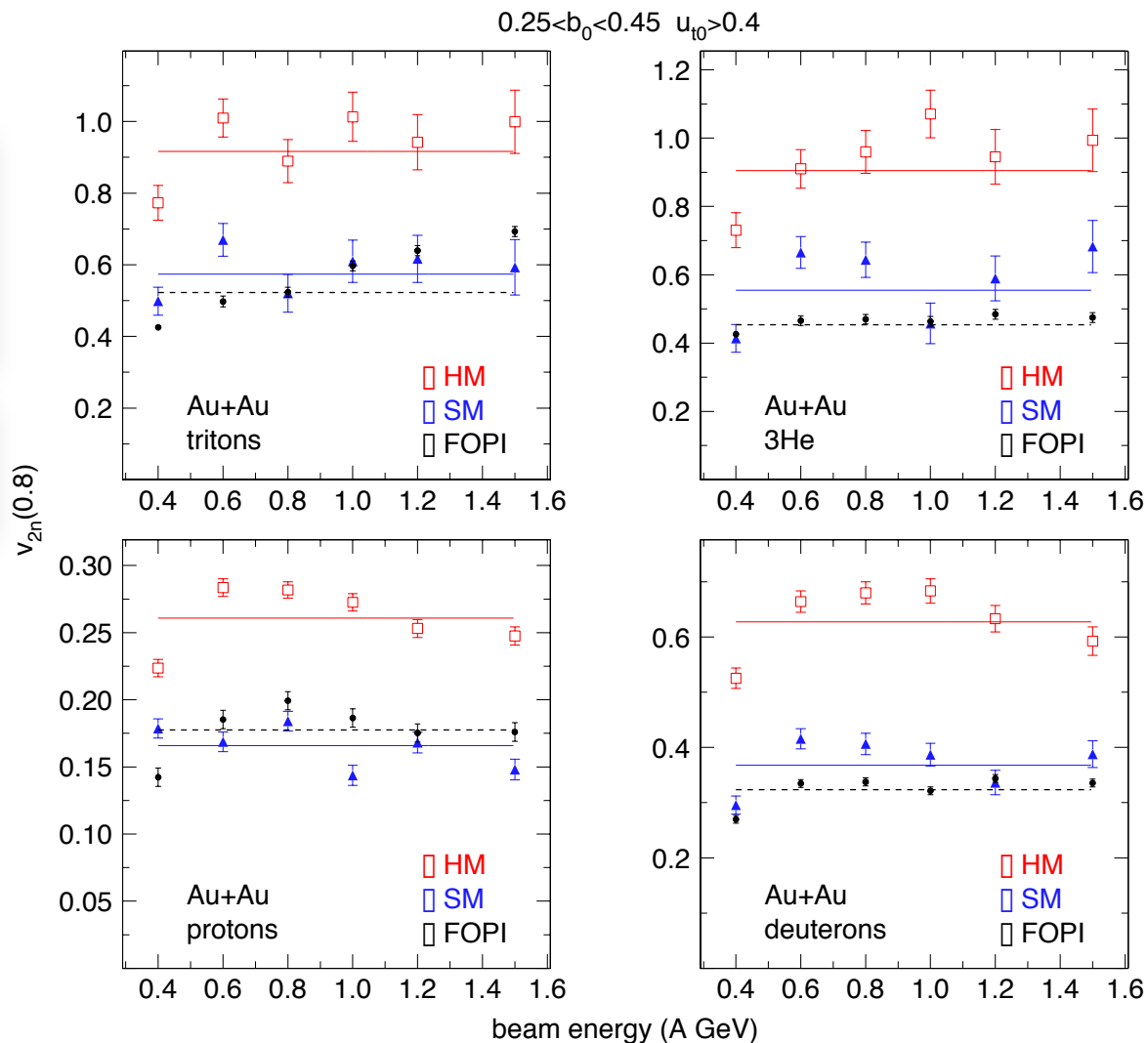




# Analysis and results

→  $v_{2n}(E_{\text{beam}})$  varies by a factor  $\approx 1.6$ ,  $\gg$  measured uncertainty ( $\approx 1.1$ )

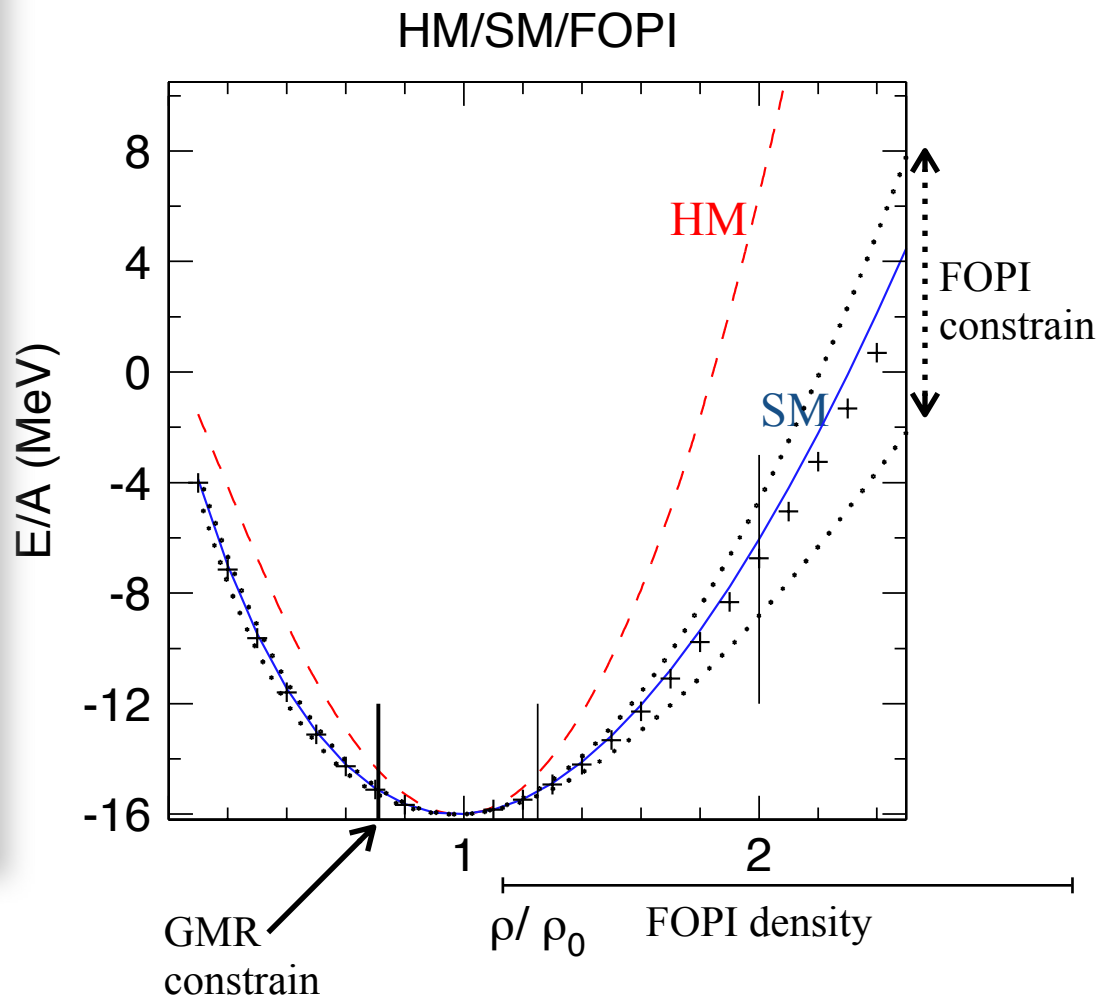
→ clearly favors a 'soft' EOS :  $K_0 = 190 \pm 30 \text{ MeV}$





# Analysis and results

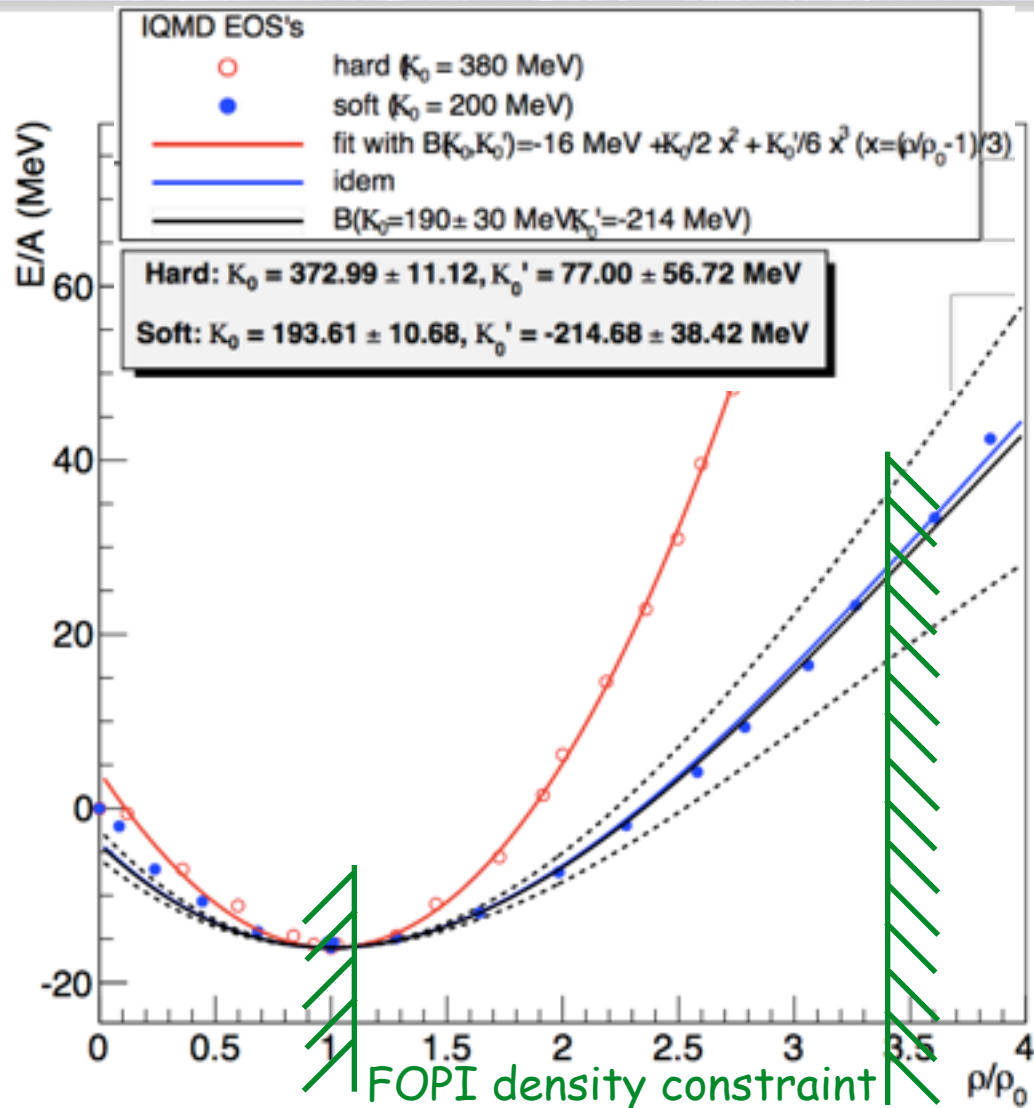
- ▶ Phenomenological EOS  
HM and SM include the saturation point at  $\rho/\rho_0 = 1$ ,  
 $E/A = -16$  MeV by construction.
- ▶ → fixes the absolute position of the curves:
- ▶ the heavy ion data are only sensitive to the shape, i.e. the pressure (derivative).
- ▶ → a stiff EOS, characterised by  $K_0 = 380$  MeV is not in agreement with the flow data in the incident energy range 0.4 - 1.5 A.GeV.





# Analysis and results

**NB:** the qualified EOS has a non quadratic behavior : not only an incompressibility  $K_0 = 190 \pm 30$  MeV, but a skewness  $K_0' = -214 \pm 38$  MeV





# Deducing the probed density





# Deducing the probed density

Purpose = characterise  
which 'typical' densities  
were probed in the FOPI  
experiments  
=> at which time  $V_2$  develops,  
and which conditions  
influence it the most.



# Deducing the probed density

Purpose = characterise  
which 'typical' densities  
where probed in the FOPI  
experiments  
=> at which time  $V_2$  develops,  
and which conditions  
influence it the most.

IQMD transport model<sup>[5,6]</sup>  
various phenomenological  
EOS's:

- » 'stiff' = HM  
(+ momentum dependent),  
 $K_0 = 380$  MeV
- » 'soft' = SM (+ momentum  
dependent),  $K_0 = 200$   
MeV.

Here: protons in Au+Au at  
1.5 A.GeV,  $b=3$  fm

[5] J. Aichelin, Phys. Rep. 202 (1991) 233.

[6] C. Hartnack, et al., Eur. Phys. J. A 1 (1998) 151.



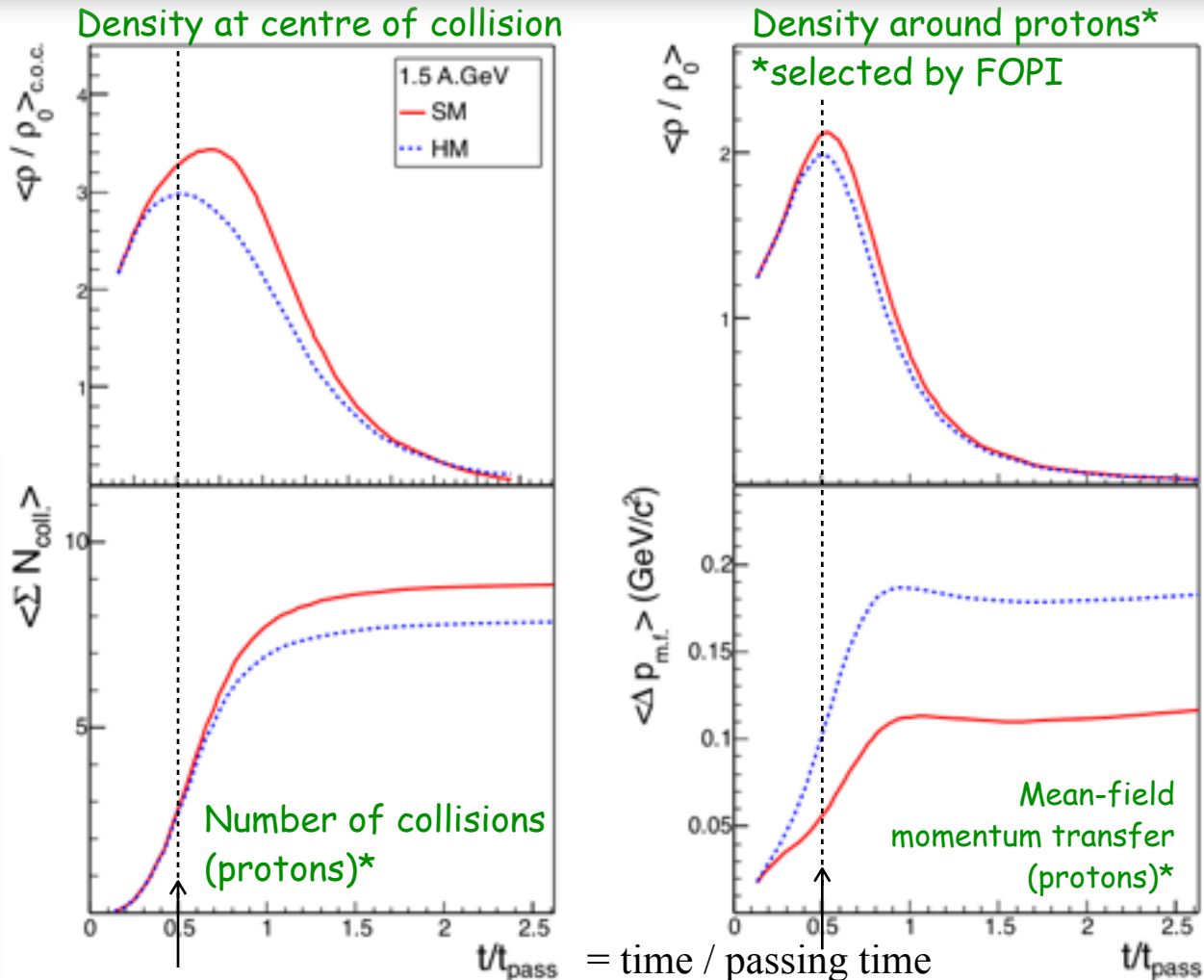
# Deducing the probed density

Purpose = characterise which 'typical' densities where probed in the FOPI experiments  
 => at which time  $V_2$  develops, and which conditions influence it the most.

IQMD transport model<sup>[5,6]</sup>  
 various phenomenological EOS's:

- » 'stiff' = HM (+ momentum dependent),  $K_0 = 380$  MeV
- » 'soft' = SM (+ momentum dependent),  $K_0 = 200$  MeV.

Here: protons in Au+Au at 1.5 A.GeV,  $b=3$  fm



full target-projectile overlap

[5] J. Aichelin, Phys. Rep. 202 (1991) 233.

[6] C. Hartnack, et al., Eur. Phys. J. A 1 (1998) 151.



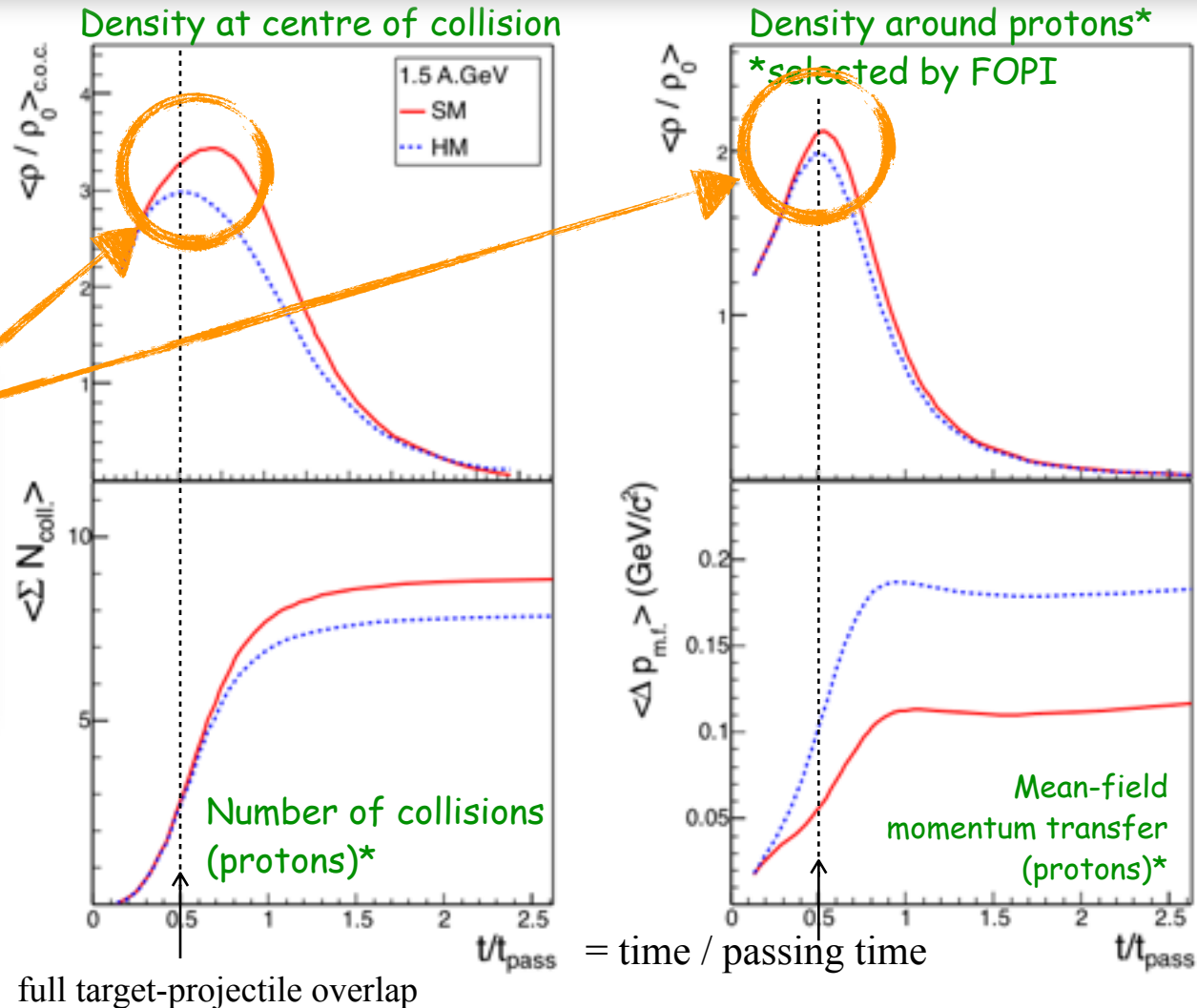


# Deducing the probed density

Purpose = characterise which 'typical' densities where probed in the FOPI experiments  
 => at which time  $V_2$  develops, and which conditions influence it the most.

The highest density phase initiates the high pressure, hence the flow.

Tested: a high density cut-off in the EOS => no elliptic flow.



full target-projectile overlap

= time / passing time



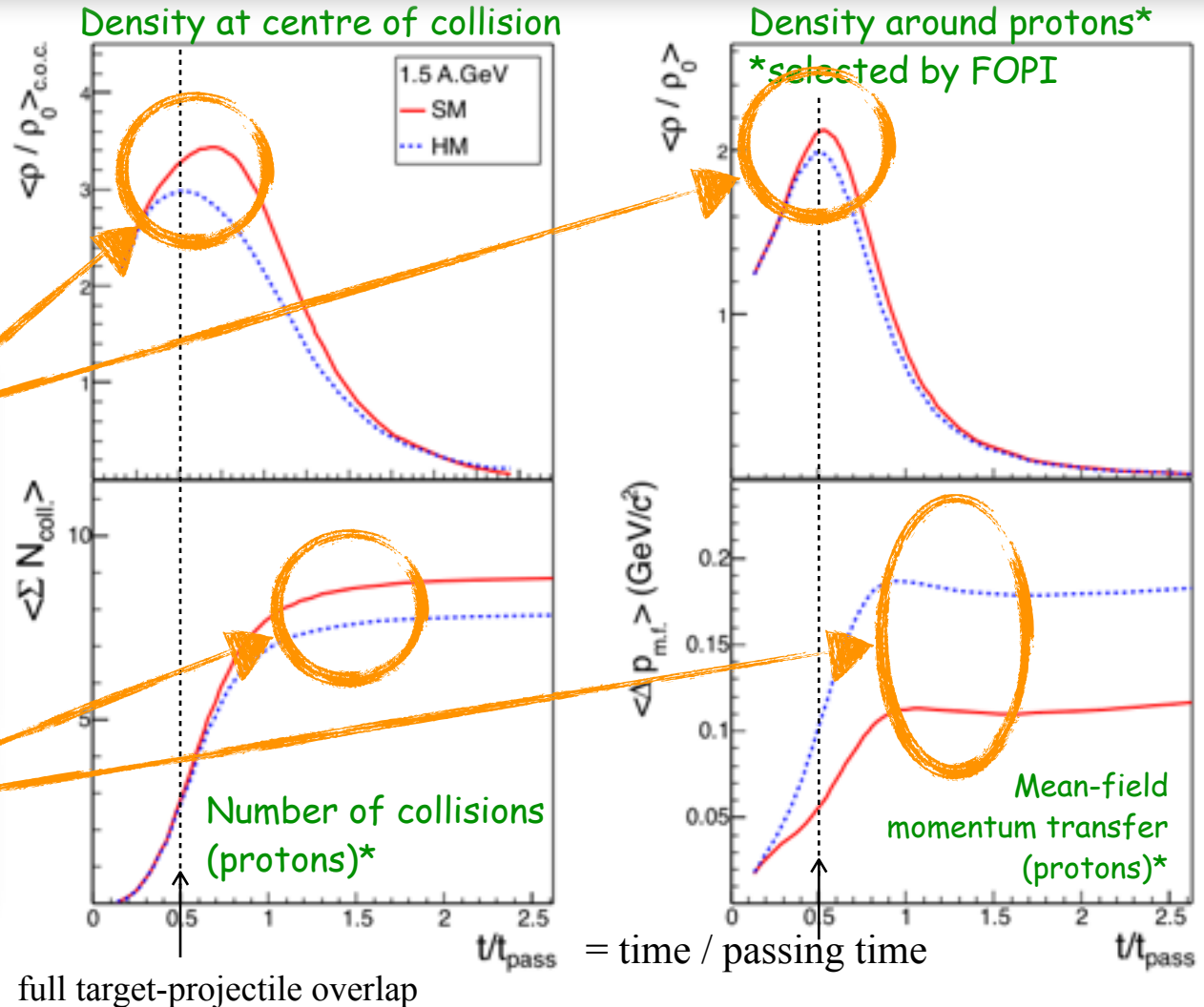
# Deducing the probed density

Purpose = characterise which 'typical' densities where probed in the FOPI experiments  
 => at which time  $V_2$  develops, and which conditions influence it the most.

The highest density phase initiates the high pressure, hence the flow.

Tested: a high density cut-off in the EOS => no elliptic flow.

The (flow) dynamics develops up to later times, hence lower densities.

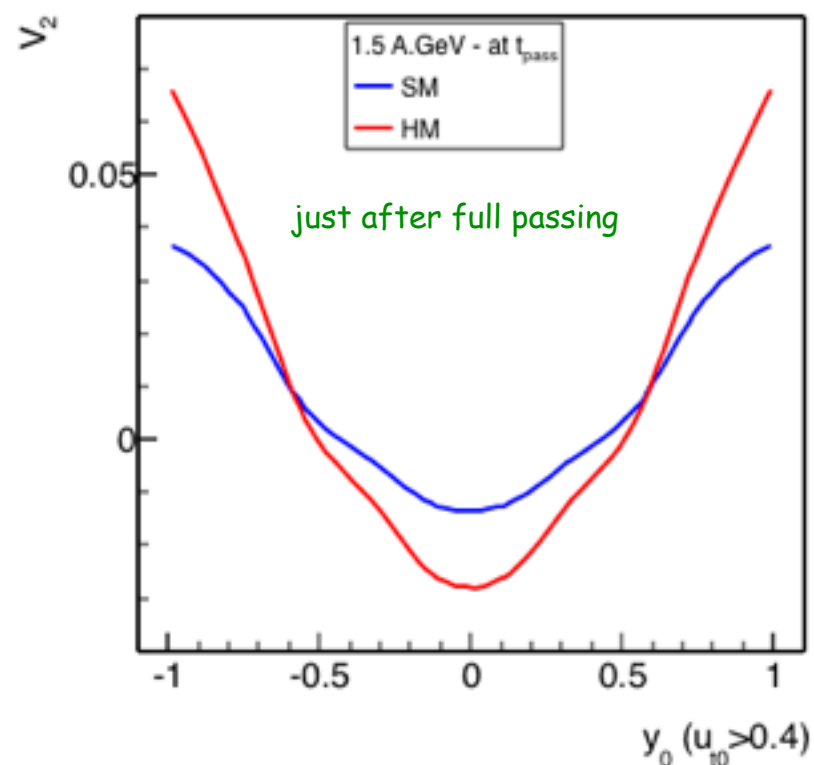
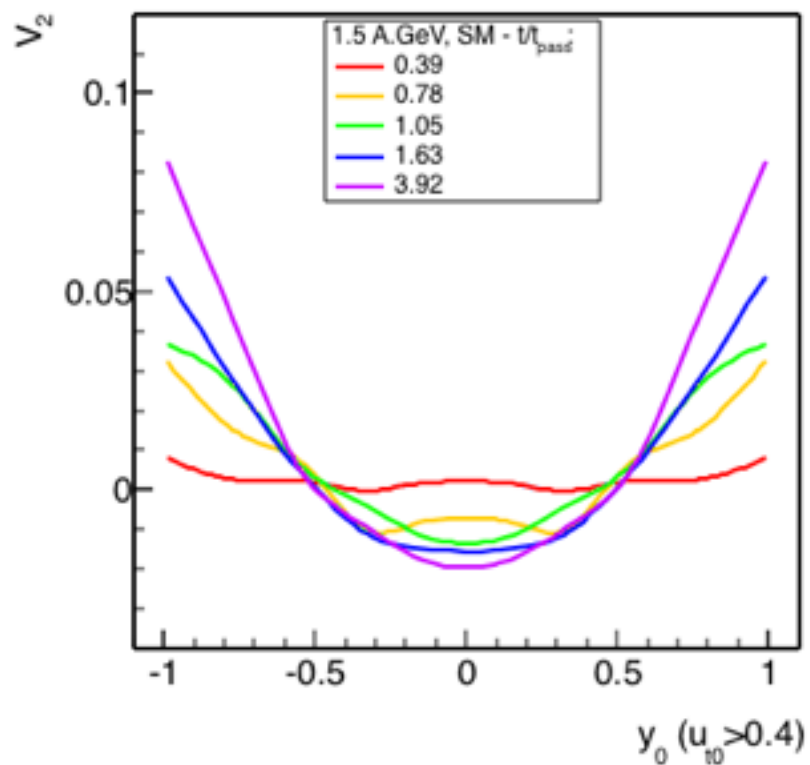


full target-projectile overlap

= time / passing time

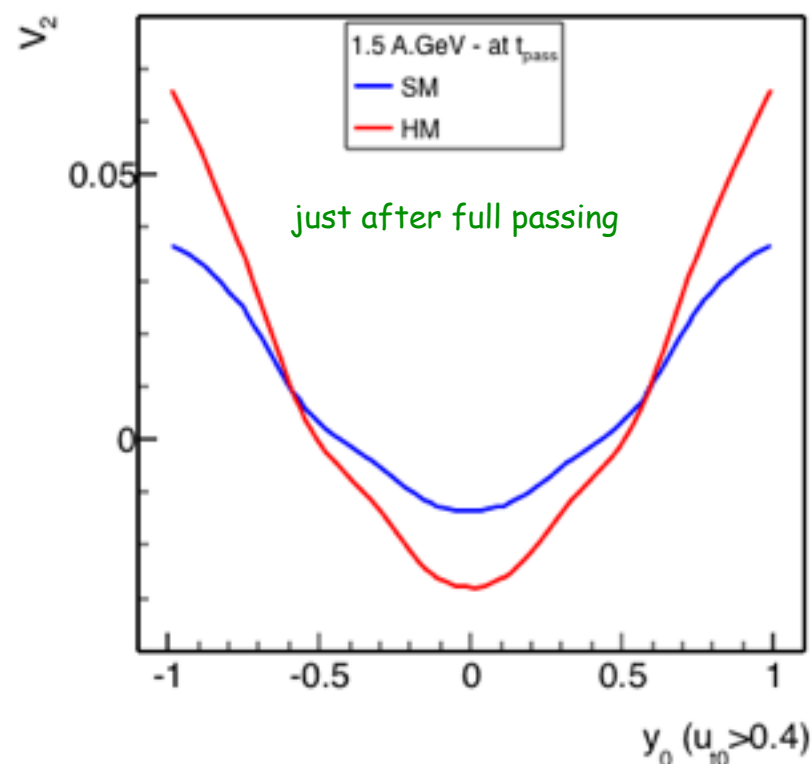
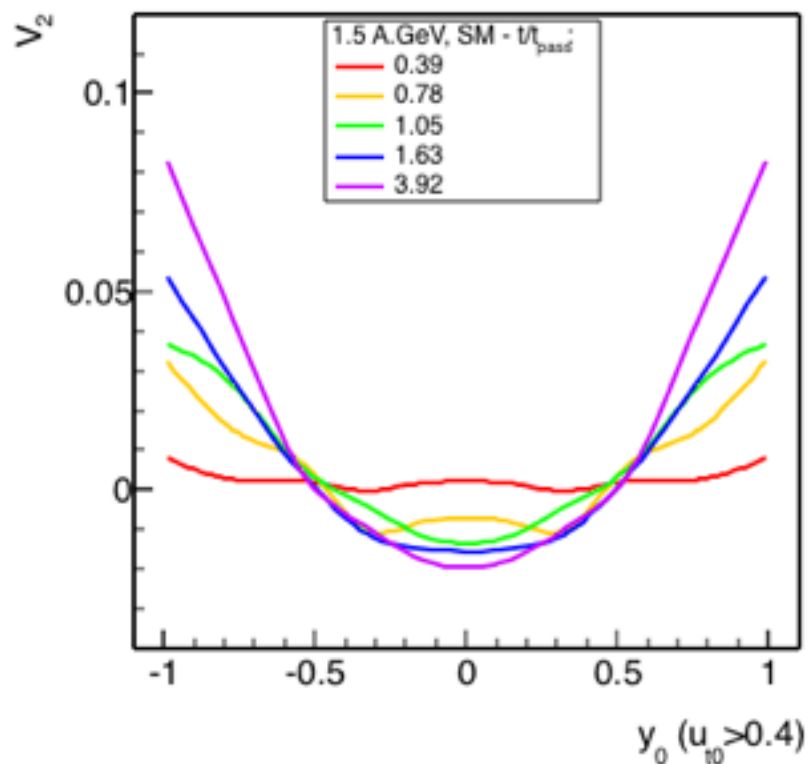


# Simulations: the scenario





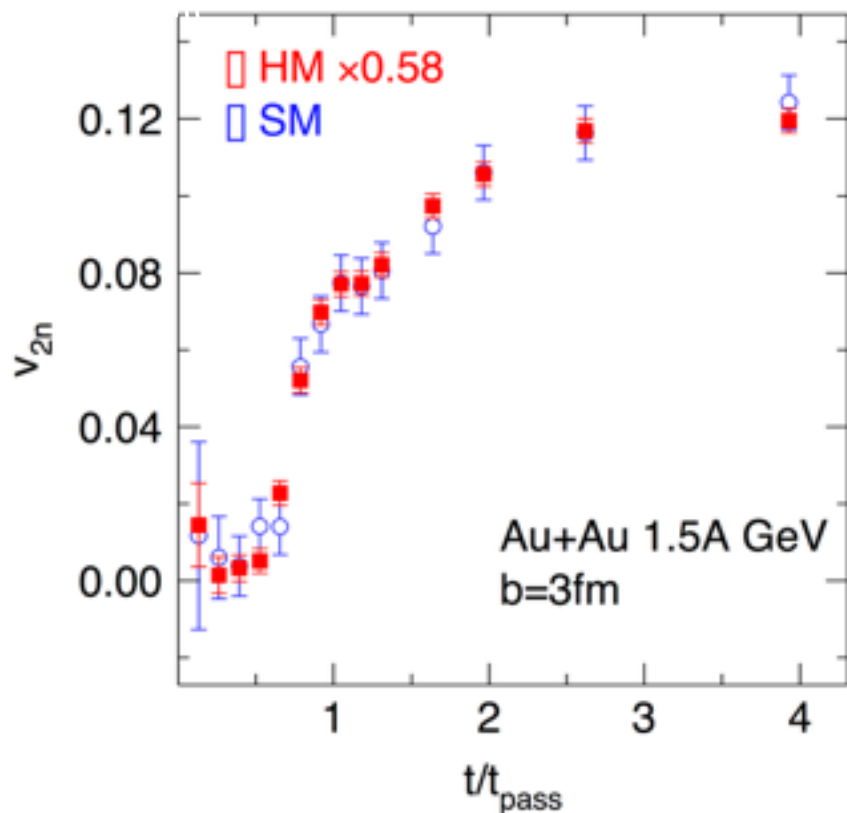
# Simulations: the scenario



- ▶ The elliptic flow at mid-rapidity develops fast: already **stabilised at the passing time**.
- ▶ **At  $t_{\text{pass}}$** , the elliptic flow, in its rapidity dependence, depends already strongly on the EOS.
- ▶ The elliptic flow **around the spectators** ( $|y_0|$  close to 1) stabilises **twice slower**.



# Simulations: the scenario

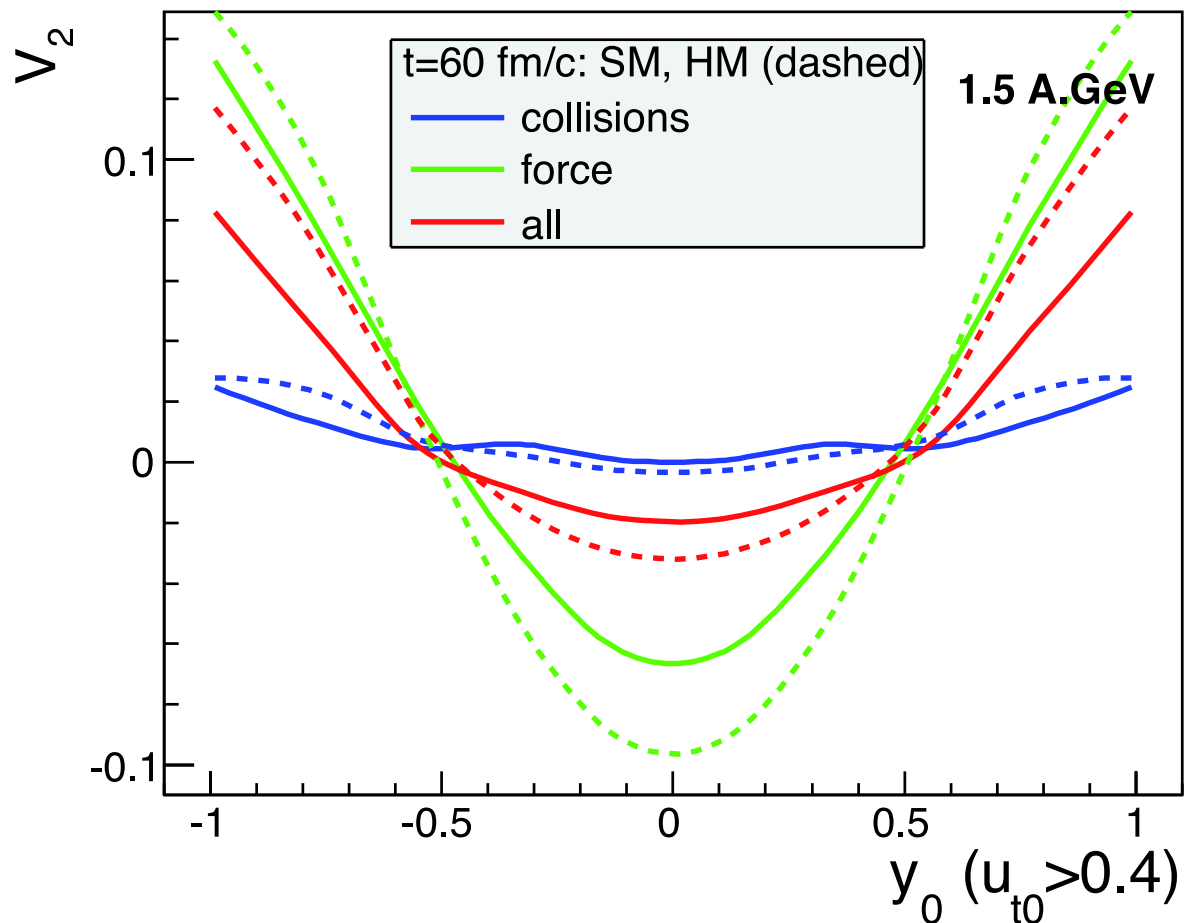


- ▶ The **shape** of its rapidity dependence shows a **universality** with the EOS's (through scaling).





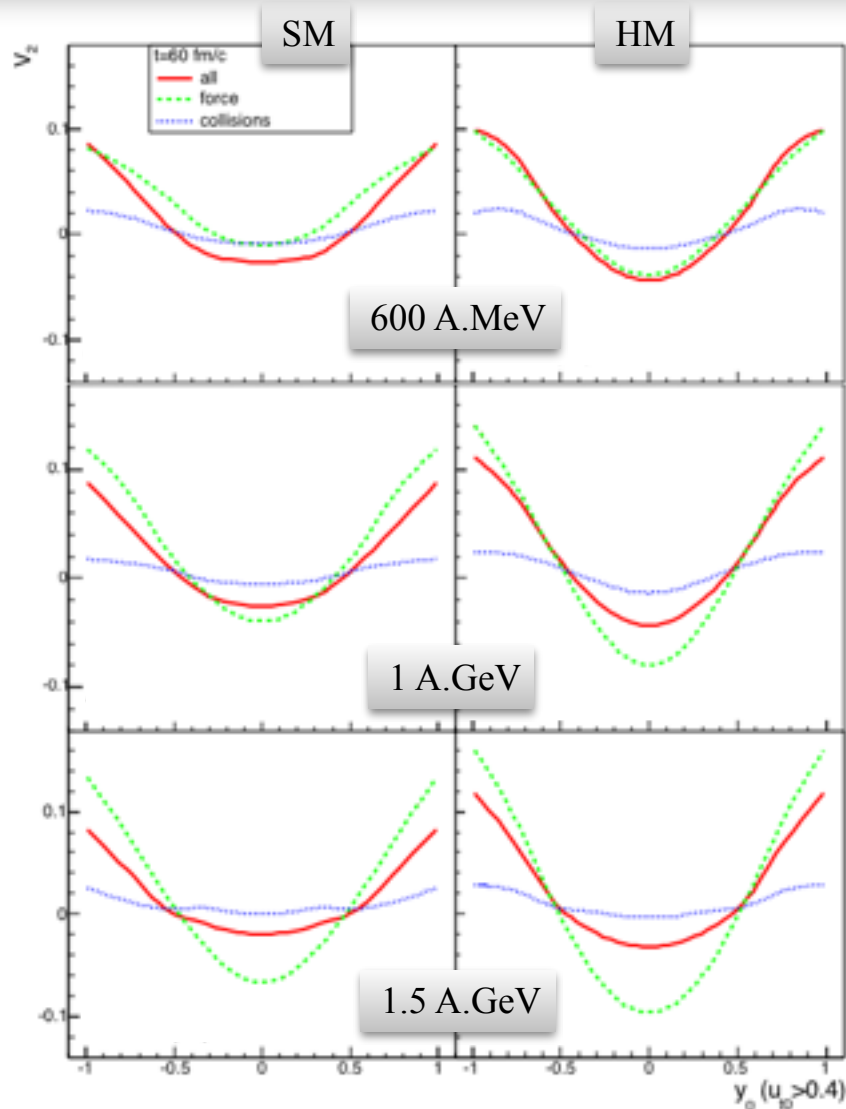
# Simulations: the scenario





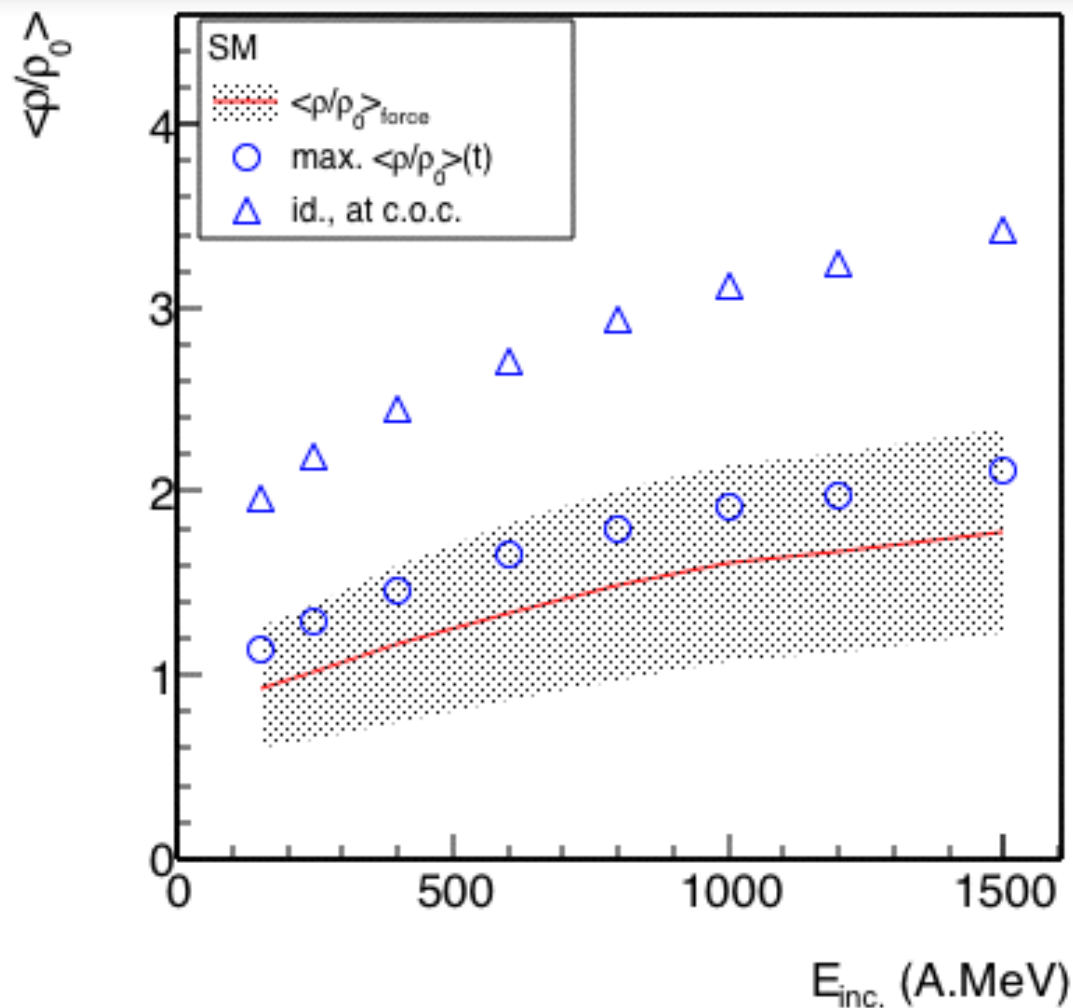
# Simulations: the scenario

- ▶ The elliptic flow in strength and shape is mostly influenced by the force of the mean field (hence EOS).
- ▶ A 'mean' density characterising the development of the elliptic flow can be built from the mean value weighted by this force up to around the passing time.



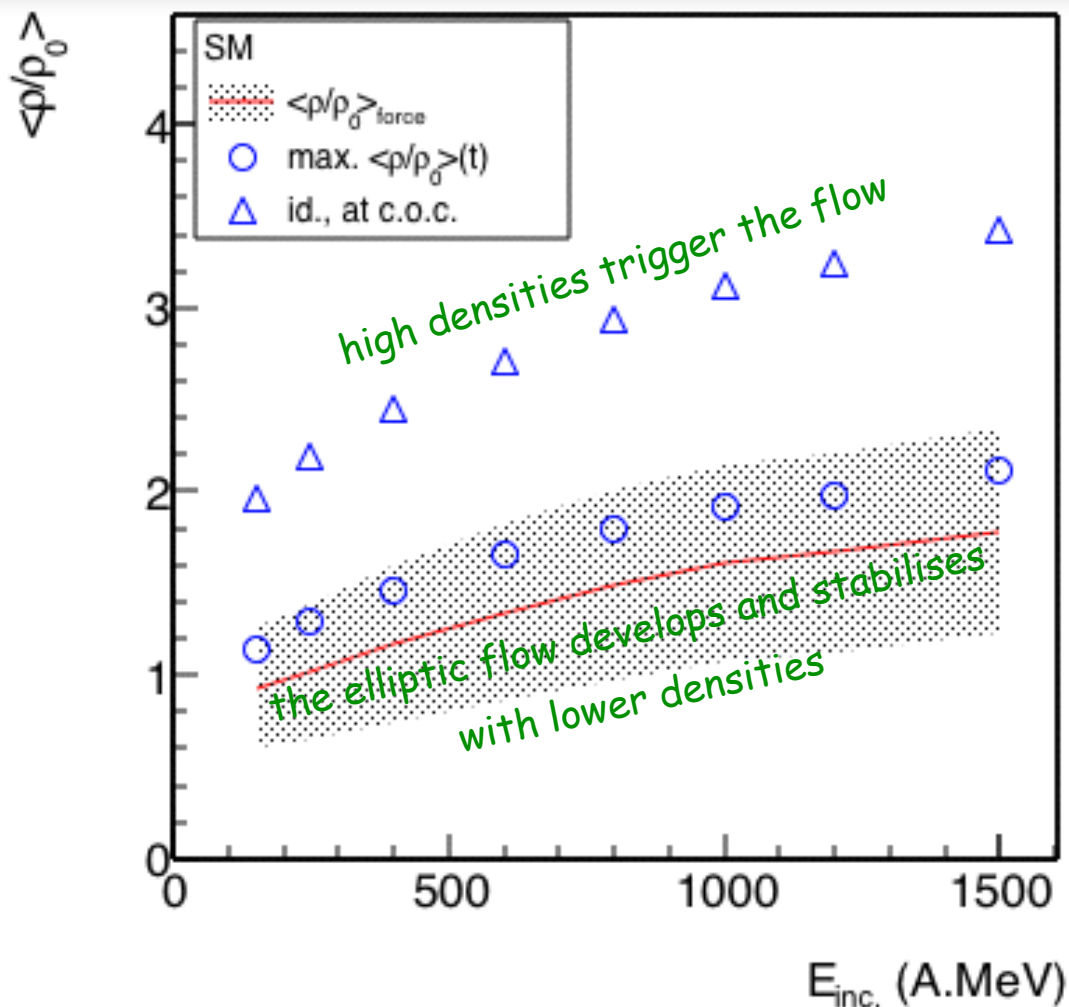


# Simulations: the scenario





# Simulations: the scenario



► In the QMD model, the EOS must be correct over a broad range of densities in order to predict the observed elliptic flow.

► The density range, relevant to the EOS evidenced by the FOPI Collaboration, spans in the range  $\rho \approx (1 - 3) \rho_0$ .



# Summary and discussion





# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 A \text{ GeV}$ , close to threshold).



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 A \text{ GeV}$ , close to threshold).
- ▶ Both methods lead to the same conclusion (with same transport model IQMD).



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 \text{ A.GeV}$ , close to threshold).
- ▶ Both methods lead to the same conclusion (with same transport model IQMD).
- ▶ Convincing conclusions on basic nuclear properties imply a successful simulation:
  - ▶ of the full set of experimental observables
  - ▶ with the same code
  - ▶ using the same physical and technical parameters.





# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 \text{ A.GeV}$ , close to threshold).
- ▶ Both methods lead to the same conclusion (with same transport model IQMD).
- ▶ Convincing conclusions on basic nuclear properties imply a successful simulation:
  - ▶ of the full set of experimental observables
  - ▶ with the same code
  - ▶ using the same physical and technical parameters.
- ▶ Has been reached for a number of observables, for some other data not yet the case.



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 \text{ A.GeV}$ , close to threshold).
- ▶ Both methods lead to the same conclusion (with same transport model IQMD).
- ▶ Convincing conclusions on basic nuclear properties imply a successful simulation:
  - ▶ of the full set of experimental observables
  - ▶ with the same code
  - ▶ using the same physical and technical parameters.
- ▶ Has been reached for a number of observables, for some other data not yet the case.
- ▶ Radial flow of the light clusters was well reproduced, but insensitive to the EOS.



# Summary and discussion

- ▶ A single parameter  $v_{2n}$ , characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- ▶ Relevant density range: estimated from the simulations to span  $\rho = (1 - 3)\rho_0$ .
- ▶ The 'flow method': competitive and complementary with the 'kaon method' (which is as discriminating only for the narrow energy range  $E_{\text{beam}} = 0.8 \text{ A.GeV}$ , close to threshold).
- ▶ Both methods lead to the same conclusion (with same transport model IQMD).
- ▶ Convincing conclusions on basic nuclear properties imply a successful simulation:
  - ▶ of the full set of experimental observables
  - ▶ with the same code
  - ▶ using the same physical and technical parameters.
- ▶ Has been reached for a number of observables, for some other data not yet the case.
- ▶ Radial flow of the light clusters was well reproduced, but insensitive to the EOS.
- ▶ Pion yields: differ only by about 10% between HM and SM options, imply high experimental accuracy and better transport model predictions (elementary pion cross sections not precisely known).



# Summary and discussion



# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).





# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
- ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by independent experimental efforts, and similar confrontation to transport models.



# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
- ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by **independent experimental efforts, and similar confrontation to transport models.**
- ▶ Several issues need further efforts by the community: momentum dependences, clean Lorentz covariance at beam energies exceeding  $1 \text{ A.GeV}$ , clusterisation and entropy balances, in-medium nucleon-nucleon reactions...



# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
- ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by independent experimental efforts, and similar confrontation to transport models.
- ▶ Several issues need further efforts by the community: momentum dependences, clean Lorentz covariance at beam energies exceeding  $1 \text{ A.GeV}$ , clusterisation and entropy balances, in-medium nucleon-nucleon reactions...
- ▶ The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.



# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
- ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by **independent experimental efforts**, and **similar confrontation to transport models**.
- ▶ Several issues need further efforts by the community: momentum dependences, clean Lorentz covariance at beam energies exceeding  $1 \text{ A.GeV}$ , clusterisation and entropy balances, in-medium nucleon-nucleon reactions...
- ▶ The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.
- ▶ Beyond  $4 \text{ A.GeV}$ , **other ideas** are needed to extract EOS information from heavy ion data.



# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
- ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by **independent experimental efforts**, and similar confrontation to transport models.
- ▶ Several issues need further efforts by the community: momentum dependences, clean Lorentz covariance at beam energies exceeding  $1 \text{ A.GeV}$ , clusterisation and entropy balances, in-medium nucleon-nucleon reactions...
- ▶ The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.
- ▶ Beyond  $4 \text{ A.GeV}$ , **other ideas** are needed to extract EOS information from heavy ion data.
- ▶ **Main conclusion**: we believe we can say that the feasibility of establishing reasonably tight empirical constraints on the nuclear EOS has been demonstrated.





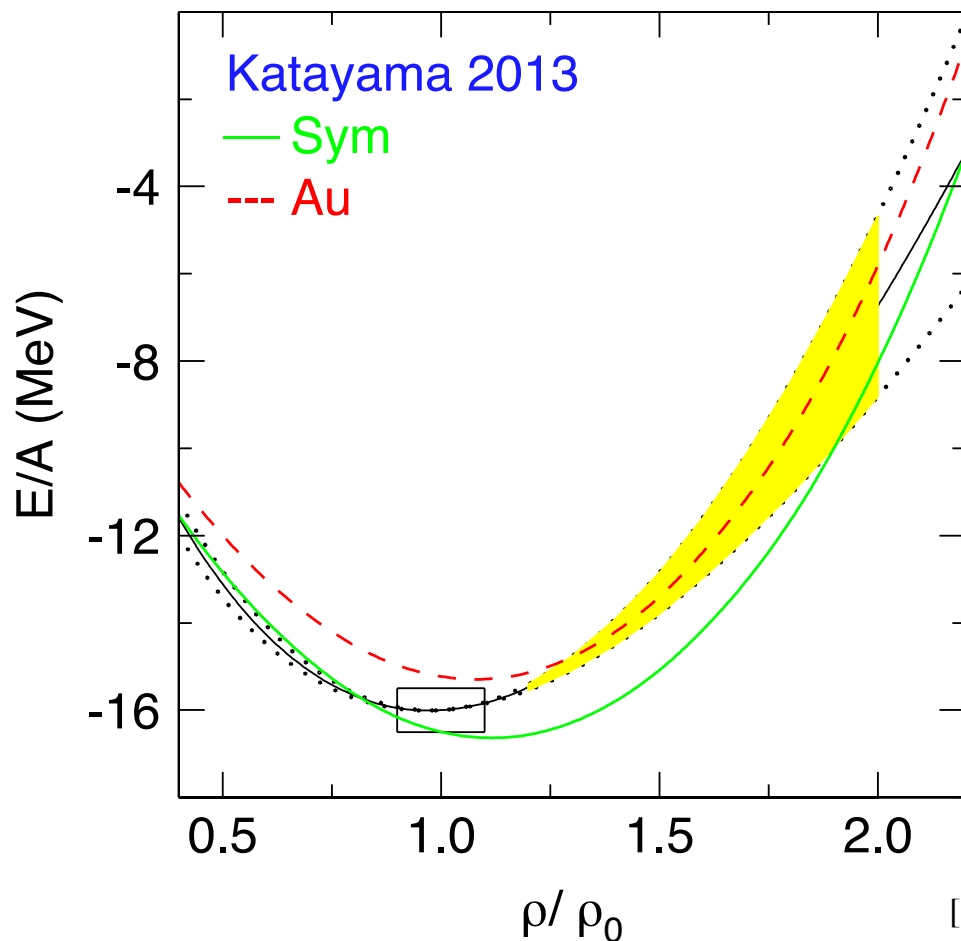
# Summary and discussion

- ▶ Sensitivity of the proton elliptic flow method: in the range  $E_{\text{beam}} = 0.4 \text{ A.GeV}$  (below, energy/nucleon EOS is too flat  $\rightarrow$  low pressure) to  $4 \text{ A.GeV}$  (above, participant-spectator clock effect versus shadowing disappears).
  - ▶ Although IQMD successful to conclude at a 'soft' EOS, need for confirmation by **independent experimental efforts**, and similar confrontation to transport models.
  - ▶ Several issues need further efforts by the community: momentum dependences, clean Lorentz covariance at in-medium nucleon-nucleon interaction and entropy balances,
- Thank you for your attention!*
- ▶ The spectator clock can presumably be used to try to extend improved EOS constraints to densities (3-4  $\rho_0$ ) in future accelerator systems such as FAIR.
  - ▶ Beyond  $4 \text{ A.GeV}$ , **other ideas** are needed to extract EOS information from heavy ion data.
  - ▶ **Main conclusion:** we believe we can say that the feasibility of establishing reasonably tight empirical constraints on the nuclear EOS has been demonstrated.



# Comparison to microscopic calculations

(three representative microscopic calculations compared with our new constraints)



Dirac-Brueckner-Hatree-Fock (DBHF) calculation<sup>[10]</sup> using the Bonn A<sup>[11]</sup> nucleon-nucleon potential

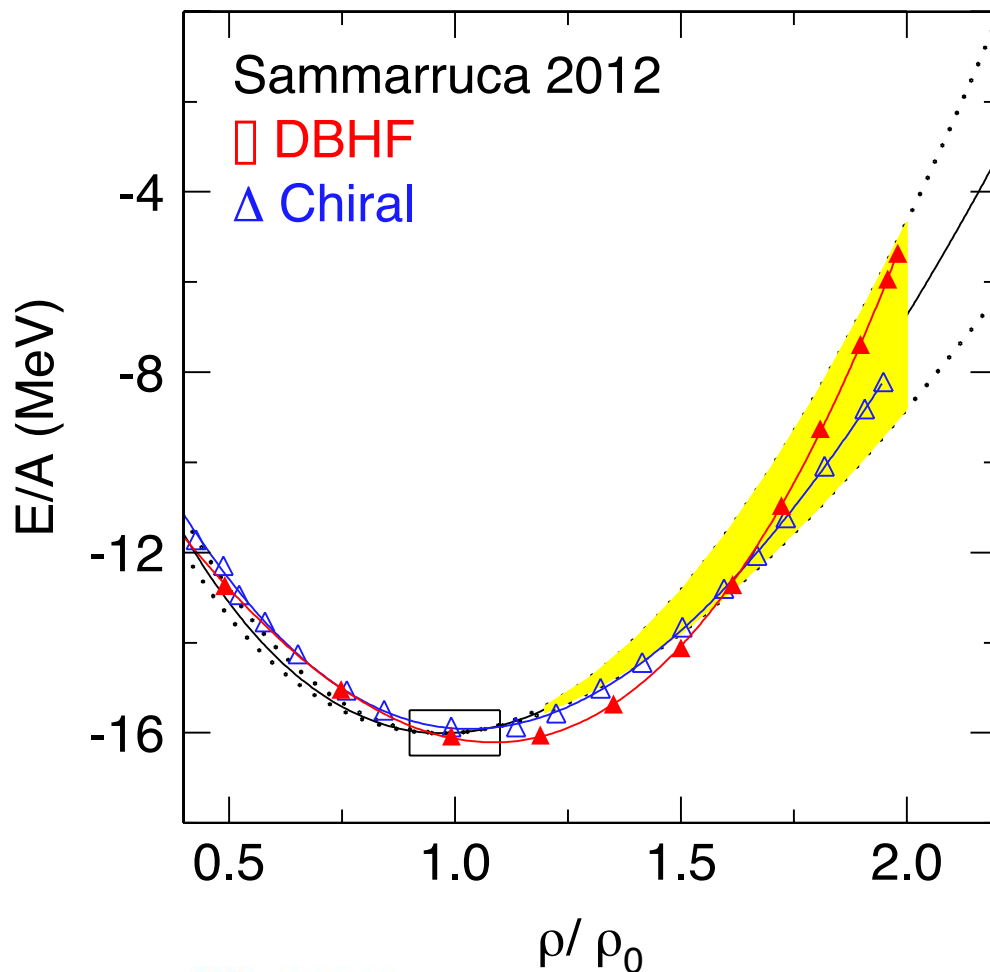
[10] R. Brockmann, R. Machleidt, Phys. Rev. C 42 (1990) 1965.

[11] T. Katayama, K. Saito, Phys. Rev. C 88 (2013) 035805.



# Comparison to microscopic calculations

(three representative microscopic calculations compared with our new constraints)



2 symmetric nuclear matter EOS's from [12]:

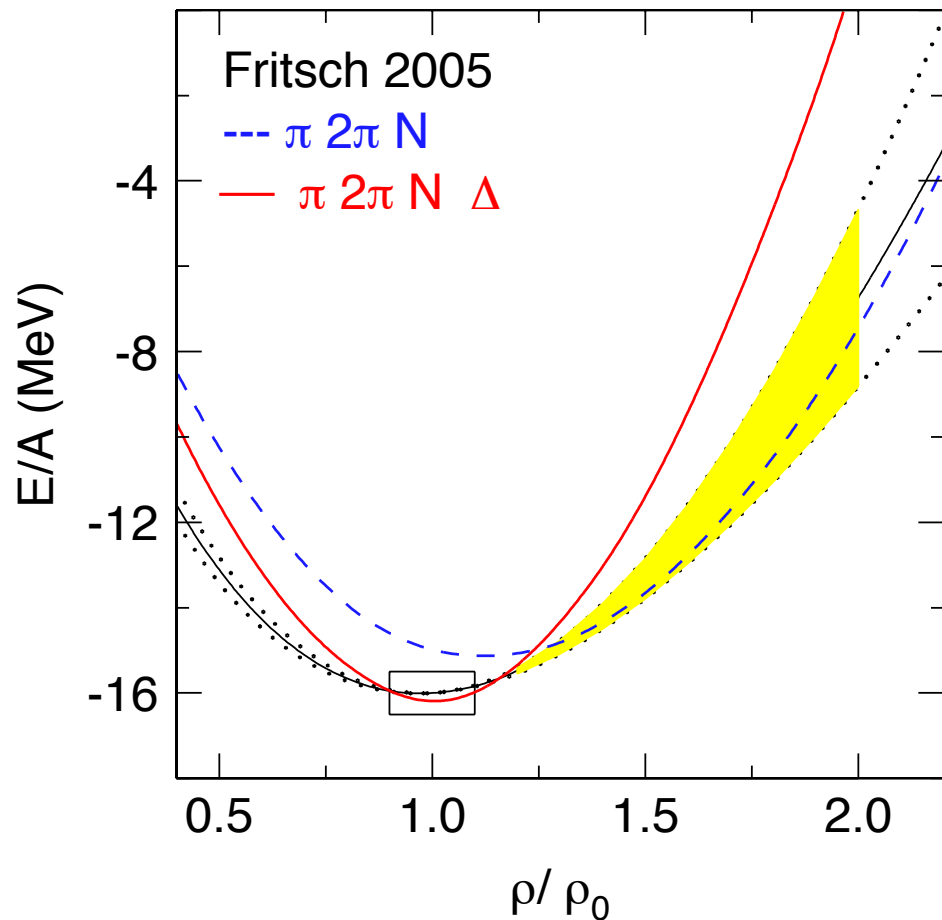
- 1) 'DBHF' = meson theoretic potential together with the DBHF method
- 2) 'Chiral' = use of effective field theory (EFT) with density dependent interactions derived from leading order chiral three-nucleon forces.

[12] P. Danielewicz, G. Odyniec, Phys. Lett. B 157 (1985) 168.



# Comparison to microscopic calculations

(three representative microscopic calculations compared with our new constraints)



Using the chiral approach<sup>[13]</sup>: 2 rather different EOS's including or not virtual  $\Delta$  excitations.

- » the virtual  $\Delta$ -excitations help locate the EOS at the right horizontal place around  $\rho = 0.16 \text{ fm}^{-3}$ .
- » the  $\Delta$  leads to a rather marked stiffening of the EOS ( $K_0 = 304 \text{ MeV}$ )
- » because 'cold' EOS ?
- » finite temperature in the reaction  $\Rightarrow$  the  $\Delta$  are real rather than virtual. The theoretical ' $\Delta$  stiffness' could then be a dispersion effect rapidly changing with temperature.

[13] S. Fritsch, N. Kaiser, W. Weise, Nucl. Phys. A 750 (2005) 259.